Math 110 – College Algebra

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Graphs of Equations

First, we should understand that an equation is a way to describe a bunch of points that have the same relationship between x and y.

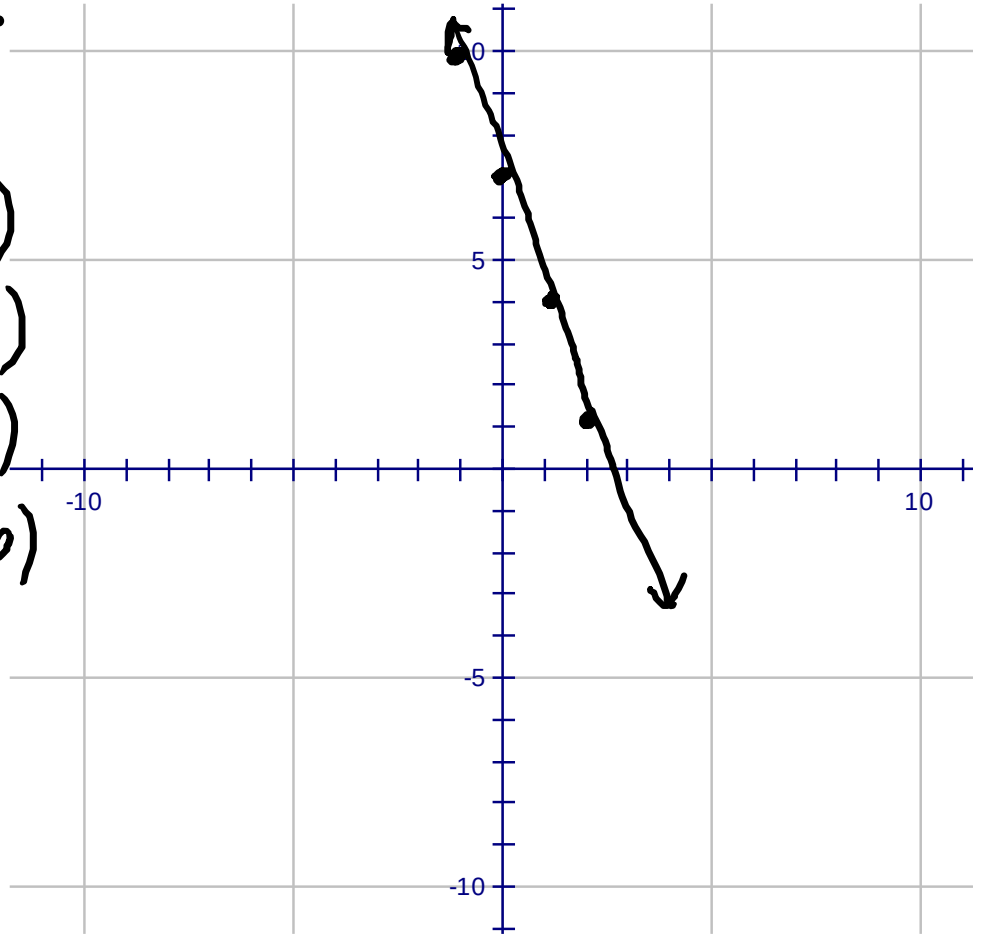
- I could list the points (1, 3), (4, 12), (-2, -6), (1.8, 5.4), (-¼, -¾)
- Or, I could describe them all by writing

$$y = 3x$$

To graph an equation, an easy way is the t-chart.

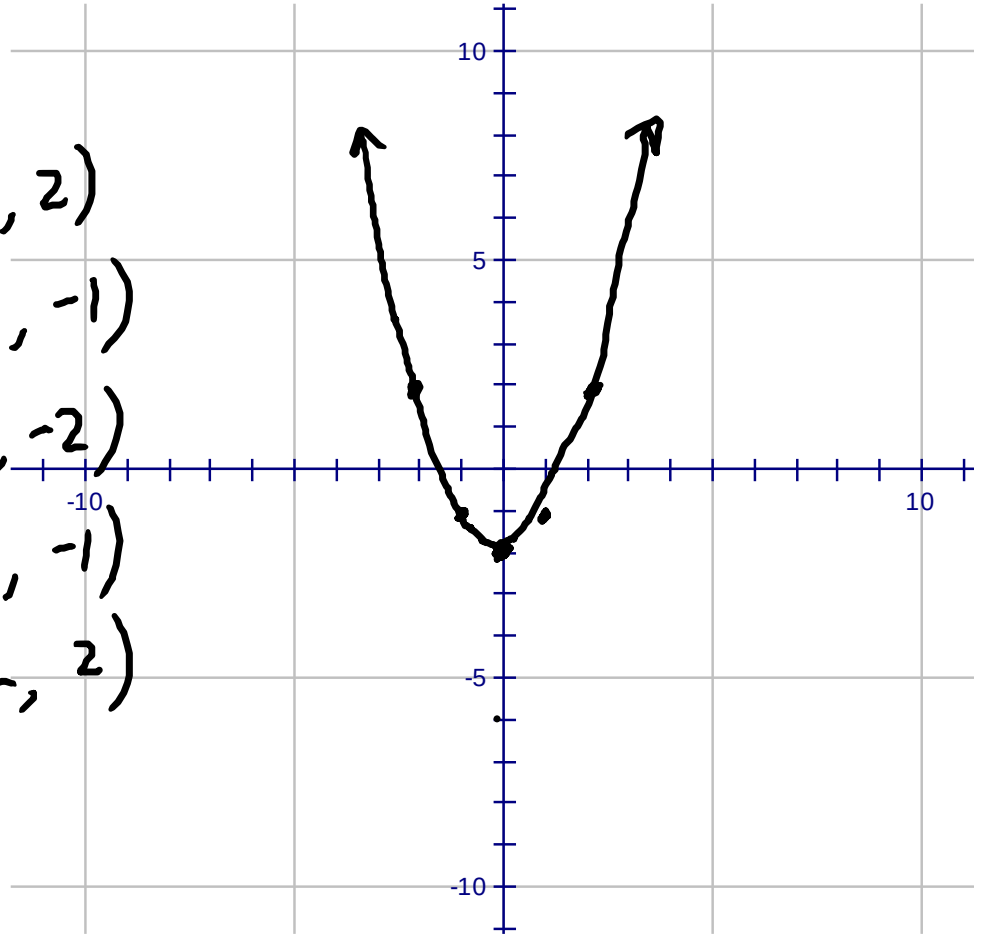
Ex. Graph $y = 7 - 3x$.

x	$y = 7 - 3x$	
2	$7 - 3(2) = 7 - 6 = 1$	$(2, 1)$
1	$7 - 3(1) = 7 - 3 = 4$	$(1, 4)$
0	$7 - 3(0) = 7 - 0 = 7$	$(0, 7)$
-1	$7 - 3(-1) = 7 + 3 = 10$	$(-1, 10)$
-2		

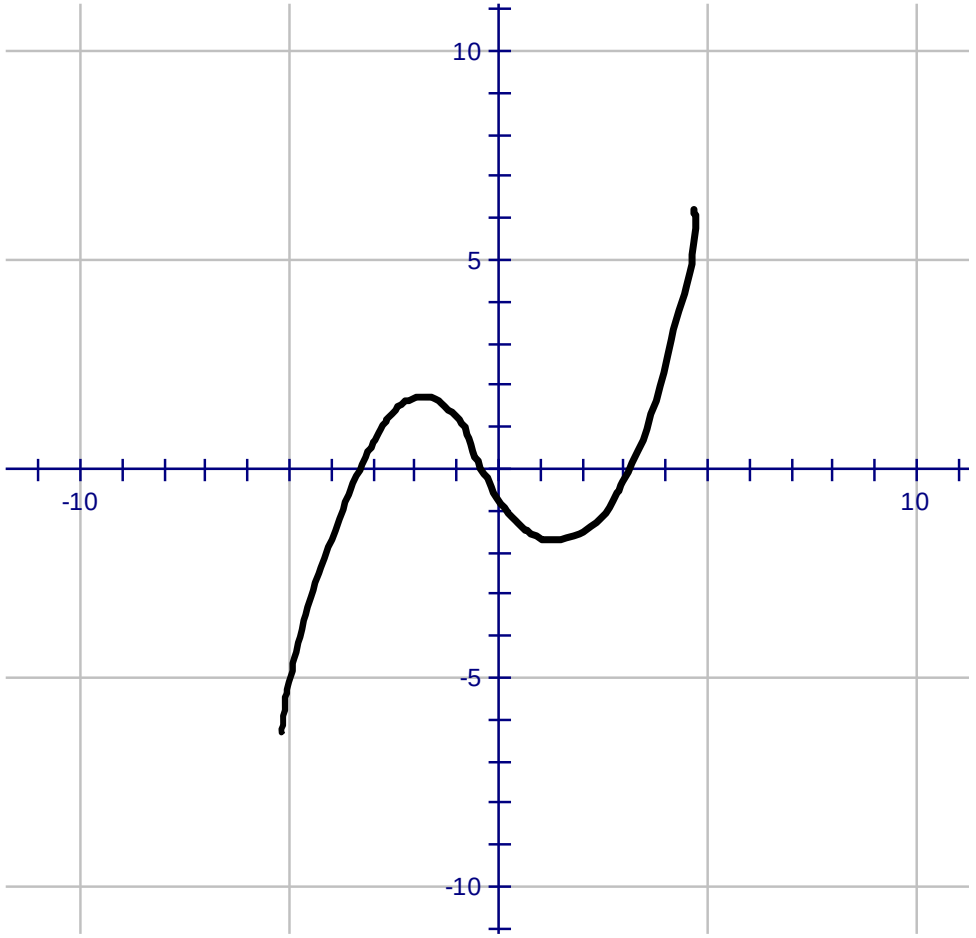


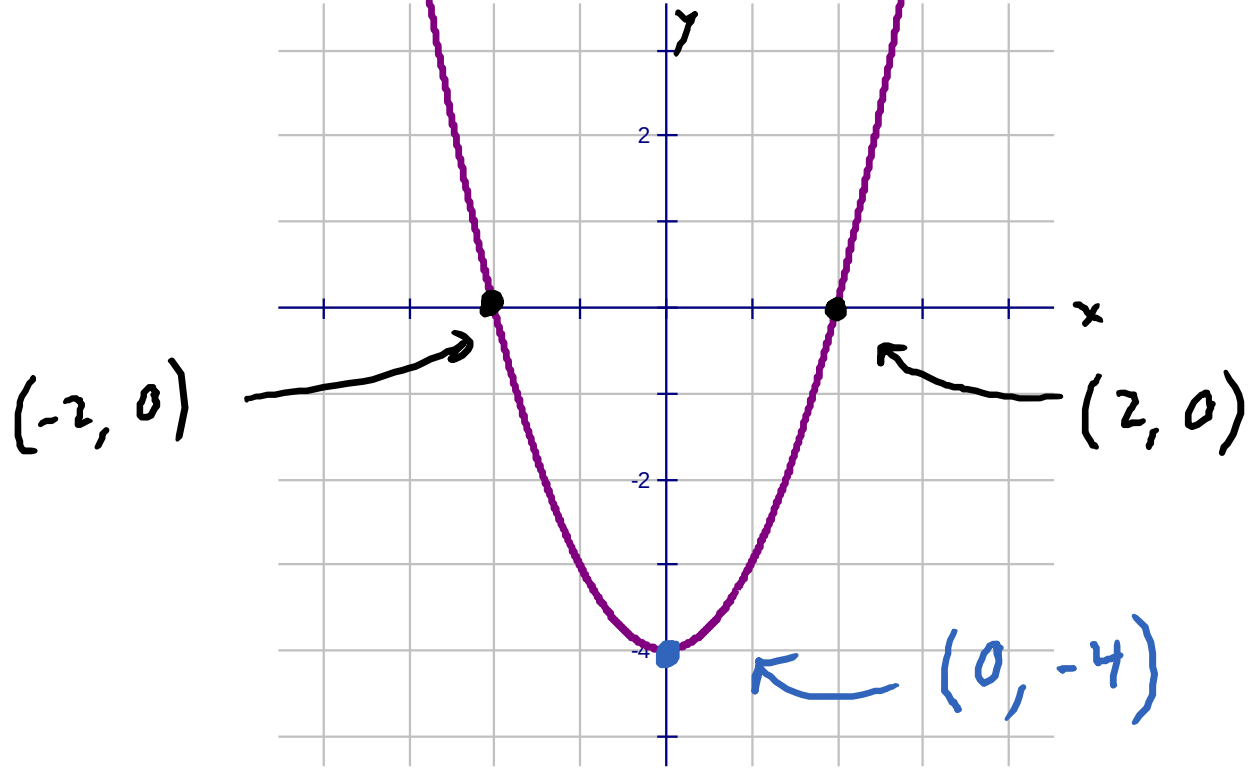
Ex. Graph $y = x^2 - 2$

x	$y = x^2 - 2$	
2	$(2)^2 - 2 = 4 - 2 = 2$	$(2, 2)$
1	$(1)^2 - 2 = 1 - 2 = -1$	$(1, -1)$
0	$(0)^2 - 2 = 0 - 2 = -2$	$(0, -2)$
-1	$(-1)^2 - 2 = 1 - 2 = -1$	$(-1, -1)$
-2	$(-2)^2 - 2 = 4 - 2 = 2$	$(-2, 2)$



Ex. Graph $y = x^3 - 2x$ with your calculator





x-intercept is where the graph crosses the *x*-axis → $y = 0$

y-intercept is where the graph crosses the *y*-axis → $x = 0$

Ex. Find the x- and y-intercepts of the line $3x + 2y = 6$, then graph the line.

x-int.: $y = 0$

$3x + 2(0) = 6$

$\frac{3x}{3} = \frac{6}{3}$

$x = 2$

$(2, 0)$

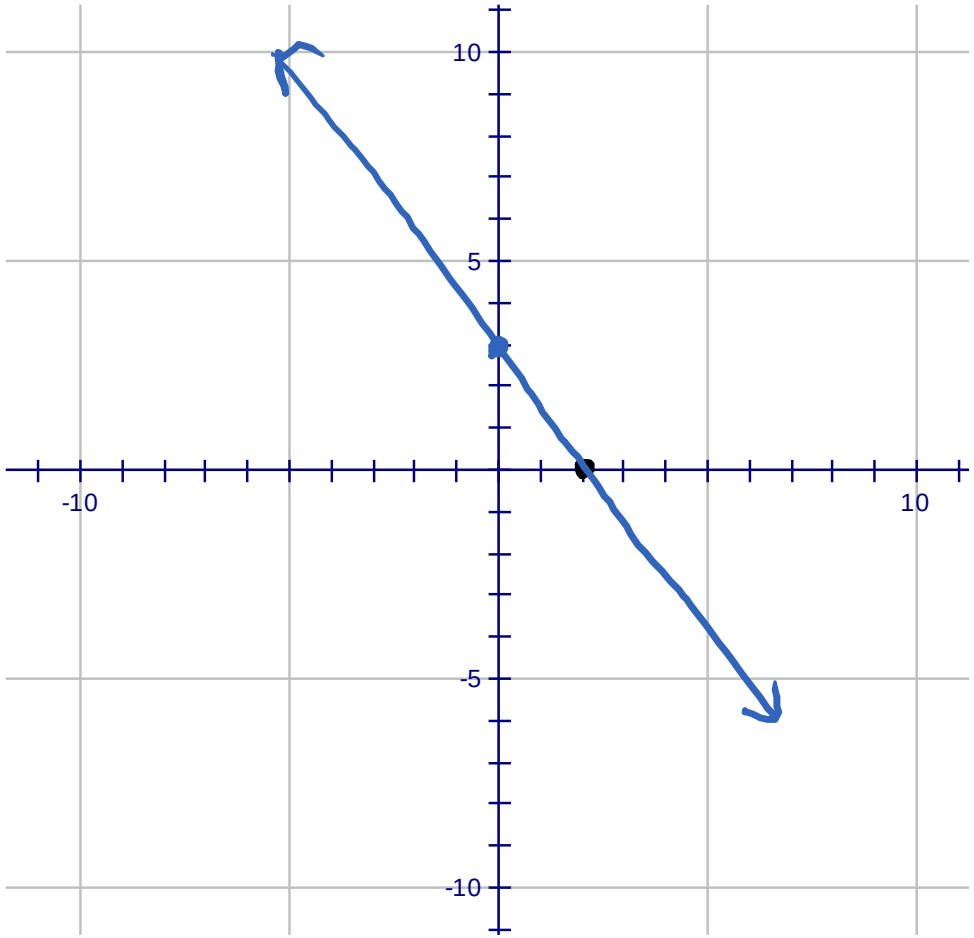
y-int.: $x = 0$

$3(0) + 2y = 6$

$\frac{2y}{2} = \frac{6}{2}$

$y = 3$

$(0, 3)$



Thm. The equation of a circle is

$$(x-h)^2+(y-k)^2=r^2$$

where the center is (h, k) and the length of the radius is r.

Distance Formula

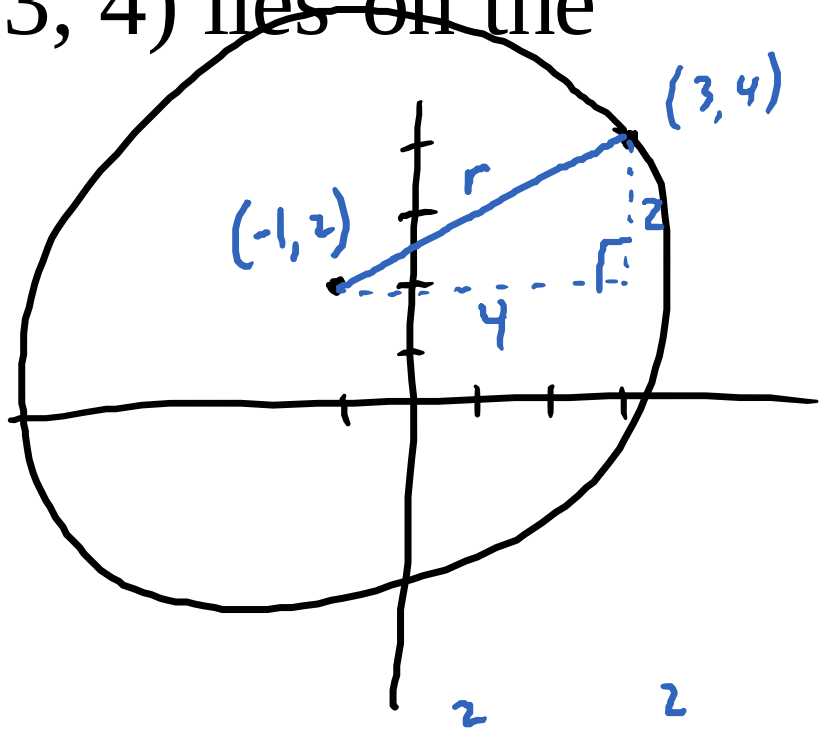
$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Ex. Write the equation of the circle with center (-1, 2) if the point (3, 4) lies on the circle.

$(x-h)^2+(y-k)^2=r^2$

$(x--1)^2+(y-2)^2=(\sqrt{20})^2$

$(x+1)^2+(y-2)^2=20$



$4^2+2^2=r^2$

$16+4=r^2$

$\sqrt{20}=\sqrt{r^2}$

$\sqrt{20}=r$

Ex. Solve $3(7x + 1) = 6(x - 1) + 4$

$21x + 3 = 6x - 6 + 4$

$21x + 3 = 6x - 2$

$-6x \qquad -6x$

$15x + 3 = -2$

$-3 \qquad -3$

$\frac{15x}{15} = \frac{-5}{15}$

$x = -\frac{1}{3}$

Ex. Solve $\frac{x}{3}+\frac{3x}{4}=2$

$$\frac{x}{3}\cdot\frac{12}{1}+\frac{3x}{4}\cdot\frac{12}{1}=2\cdot(12)$$

$$4x+9x=24$$

$$\frac{13x}{13}=\frac{24}{13}$$

$$x=\frac{24}{13}$$

Ex. Solve $\frac{1}{x-2} = \frac{3}{x+2}$

$1(x+2) = 3(x-2)$

$x+2 = 3x-6$

$-x \quad -x$

$2 = 2x-6$

$+6 \quad +6$

$\frac{8}{2} = \frac{2x}{2}$

$x = 4$

$\frac{1}{2} \times \frac{5}{10}$

$1 \cdot 10 = 2 \cdot 5$

Check

$\frac{1}{4-2} = \frac{3}{4+2}$

$\frac{1}{2} = \frac{3}{6}$ ✓

Ex. The number y (in millions) of female participants in high school athletic programs in the United States from 1989 to 2002 can be approximated by the linear model $y = 0.085t + 1.83$, where $t = 0$ represents 1990.

a) Find the y-intercept of the model. What does it represent?

$t = 0 \Rightarrow$ 1990

$y = 0.085(0) + 1.83$

$y = 1.83$ mil. part.

Represents # of part. in 1990.

Ex. The number y (in millions) of female participants in high school athletic programs in the United States from 1989 to 2002 can be approximated by the linear model $y = 0.085t + 1.83$, where $t = 0$ represents 1990.

b) Find the year in which there will be 3 million participants.

$t = ?$

$y = 3$

$$y = 0.085t + 1.83$$

$$3 = 0.085t + 1.83$$

$$-1.83 \qquad -1.83$$

$$\frac{1.17}{.085} = \frac{0.085t}{.085}$$

$$t = 13.76 \rightarrow 2004$$

Ex. You have a job for which your annual salary will be $32,300. This includes a year-end bonus of $500. You will be paid twice a month. What is your pay (before taxes) for each paycheck?

x = amt. of each paycheck

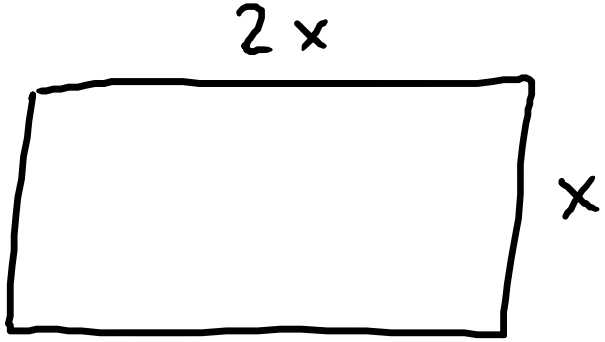
$$24x + 500 = 32300$$
$$-500 \qquad -500$$
$$24x = 31800$$
$$\frac{24x}{24} = \frac{31800}{24}$$
$$x = \$1325$$

Translating Key Words and Phrases

Key Words and Phrases	*Verbal Description*	*Algebraic Expression or Equation*
Equality:		
Equals, equal to, is, are, was, will be, represents	• The sale price S is \$10 less than the list price L.	$S = L - 10$
Addition:		
Sum, plus, greater than, increased by, more than, exceeds, total of	• The sum of 5 and x • Seven more than y	$5 + x$ or $x + 5$ $7 + y$ or $y + 7$
Subtraction:		
Difference, minus, less than, decreased by, subtracted from, reduced by, the remainder	• The difference of 4 and b • Three less than z	$4 - b$ $z - 3$
Multiplication:		
Product, multiplied by, twice, times, percent of	• Two times x • Three percent of t	$2x$ $0.03t$
Division:		
Quotient, divided by, ratio, per	• The ratio of x to 8	$\frac{x}{8}$

from p. 91 in the book

Ex. A rectangular kitchen is twice as long as it is wide. If the perimeter is 84 feet, find the dimensions of the kitchen.



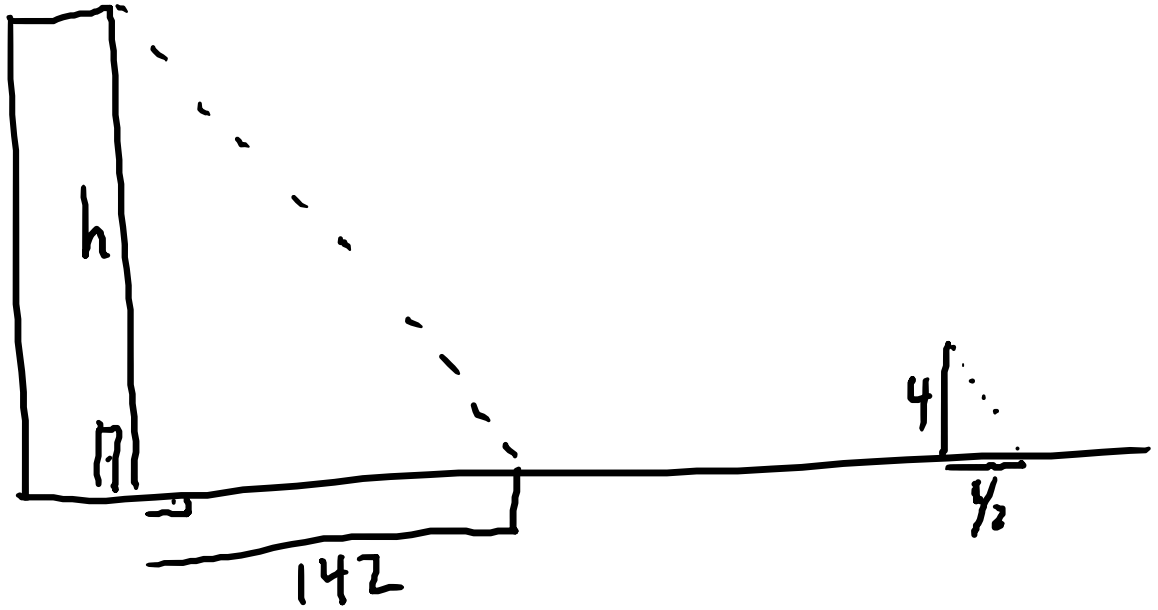
$$x + 2x + x + 2x = 84$$

$$\frac{6x}{6} = \frac{84}{6}$$

$x =$ 14 ft.

$2x =$ 28 ft.

<u>Ex.</u> The shadow of a building is 142 feet long. At the same time, a 4-foot post has a shadow that is 6 in. long. How tall is the building?



$$\frac{h}{142} = \frac{4}{\frac{1}{2}}$$

$$\frac{1}{2}h = (142)(4)$$

$$2 \cdot \frac{1}{2}h = 568 \cdot 2$$

$$h = 1136 \text{ ft.}$$

Ex. You invested $10,000 at 4.5% and 5.5% simple interest. During 1 year, the accounts earned $508.75. How much did you invest in each account?

x = amt. at 4.5% $\longrightarrow$ $.045x$ = int. in acct.

$10000 - x$ = amt. at 5.5% $\longrightarrow$ $.055(10000-x)$ = int. in acct.

$$.045x + .055(10000 - x) = 508.75$$

There are several useful formulas on p. 95

Ex. A cylindrical can has a volume of $200cm^3$ and a radius of 4cm. Find the height of the can.

$$V = \pi r^2 h$$

$$200 = \pi (4)^2 h$$

$$\frac{200}{16\pi} = \frac{16\pi h}{16\pi}$$

$$h = .397$$