Ex. Factor

a) $x^2 + 5x + 6$ = $(x + 2)(x + 3)$

add. mult.

First
Outer
Inner
Last

b) $x^2 + 3x - 40$ = $(x + 8)(x + -5)$

c) $5x^2 - 17x + 6$ = $(5x + -2)(1x + -3)$

5 6

-2 -17 -15

d) $9x^2 - 25$ = $(3x+5)(3x-5)$.

$(3x)^2 - (5)^2$

Quadratic Equations

Ex. Solve $2x^2 + 9x + 7 = \cancel{3}$

$$-3 \qquad -3$$

$$2x^2 + 9x + 4 = 0$$

$$2 \qquad 4$$

$$(2x + 1)(1x + 4) = 0$$

$$1 \qquad 9 \qquad 8$$

$$2x + \cancel{1} = 0 \qquad\qquad x + \cancel{4} = 0$$

$$-1 \quad -1 \qquad\qquad -4 \quad -4$$

$$2x = -1$$

$$x = -\frac{1}{2} \qquad\qquad x = -4$$

Ex. Solve $6x^2 - 3x = 0$

$$3x(2x - 1) = 0$$

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{0}{3} \qquad 2x - 1 = 0$$

$$x = 0 \qquad 2x = 1$$

$$x = \frac{1}{2}$$

Ex. Solve $\sqrt{(x-3)^{\cancel{2}}} = \sqrt{7}$

$x - \cancel{3} = \pm\sqrt{7}$

$\cancel{+3} \quad +3$

$\boxed{x = 3 \pm \sqrt{7}}$

$3 + \sqrt{7}$

$3 - \sqrt{7}$

"Completing the Square" means making the equation look like $(x + b)^2 = c$

<u>Ex.</u> Solve $x^2 + 2x - 6 = 0$ by completing the square

$+6 \quad +6$

$$x^2 + 2x + \underline{1} = 6 + \underline{1}$$

$$\sqrt{(x + \underline{1})^2} = \sqrt{7}$$

$$x + 1 = \pm\sqrt{7}$$

$$\boxed{x = -1 \pm \sqrt{7}}$$

Ex. Solve $3x^2 - 4x - 5 = 0$ by completing the square

Thm. The Quadratic Formula

The solutions to $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ex. Solve $x^2 + 3x = 9$

$-9 \quad -9$

$$1x^2 + 3x - 9 = 0$$

$a = 1$

$b = 3$

$c = -9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4(1)(-9)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{9 + 36}}{2} = \frac{-3 \pm \sqrt{45}}{2} = \boxed{\frac{-3 \pm 3\sqrt{5}}{2}}$$

$$\sqrt{45} = 3\sqrt{5}$$

45

9 5

3 3

Ex. The number of internet users in the US can be modeled by the equation

$$I = -1.163t^2 + 17.19t + 125.9$$

where t is years since 2000. Using this model, find the year when the number of users will surpass 180.

Complex Numbers

$i = \sqrt{-1}$

$i^2 = -1$

You learned that we can't take the square root of a negative number.

→ Guess again…

The solution to $x^2 + 1 = 0$ is $x = \sqrt{-1}$ which we define as " i ".

→4 is called a <u>real number</u>

→5i is called an <u>imaginary number</u>

$\triangle + \square i$

→4 + 5i is called a <u>complex number</u>

When adding, combine like terms.

<u>Ex.</u> $(4 + 7i) + (1 - 6i)$ $= \boxed{5 + i}$

<u>Ex.</u> $(3 + 2i) + (4 - 5i) - (7 + i)$ $= \boxed{0 - 4i}$

$3 + 2i + 4 - 5i - 7 - i$

$\underline{\text{Ex.}}$ $(2 - i)(4 + 3i) = 8 + 6i - 4i \underbrace{-3i^2}_{+3}$ $= \boxed{11 + 2i}$

$\underline{\text{Ex.}}$ $(3 + 2i)(3 - 2i) = 9 \underline{-6i} + \underline{6i} \underbrace{-4i^2}_{+4}$ $= \boxed{13}$

$\underline{\text{Ex.}}$ $(3 + 2i)^2 = (3+2i)(3+2i) = 9 + 6i + 6i + \underbrace{4i^2}_{-4}$

$= \boxed{5 + 12i}$

$3 + 2i$ and $3 – 2i$ are complex conjugates because their product was a real number.

Def. $a + bi$ and $a – bi$ are complex conjugates.

When you have a fraction that involves complex number, it is not OK to have any complex numbers on the bottom.

→ We can use the complex conjugate to "rationalize" the fraction.

Ex. Simplify $\dfrac{(2+3i)}{(4-2i)} \dfrac{(4+2i)}{(4+2i)}$

$$= \frac{8+4i+12i+\cancel{6i^2}\,-6}{16+8i-8i-\cancel{4i^2}\,+4}$$

$$= \frac{2+16i}{20}$$

$$= \frac{2}{20}+\frac{16}{20}i$$

$$= \boxed{\frac{1}{10}+\frac{4}{5}i}$$

$$= \triangle + \square i$$

Ex. Simplify $\dfrac{7}{6+i}\cdot\dfrac{(6-i)}{(6-i)} = \dfrac{42-7i}{36-\underline{6i}+\underline{6i}-\cancel{i^2}}$

$+1$

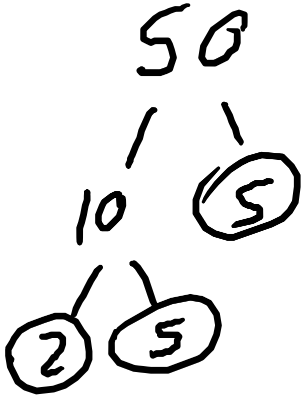
$= \dfrac{42-7i}{37}$

$= \dfrac{42}{37} - \dfrac{7}{37}i$

We should express square roots of negative numbers as complex numbers.

$$\sqrt{-16} = i\sqrt{16} = 4i$$

$$\sqrt{-50} = i\sqrt{50} = 5i\sqrt{2}$$



$$\sqrt{-3}\sqrt{-12} = i\sqrt{3} \cdot i\sqrt{12} = i^2\sqrt{36} = -6$$

Ex. Solve $3x^2 - 2x + 5 = 0$

$a = 3$
$b = -2$
$c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(5)}}{2(3)}$$

$$= \frac{2 \pm \sqrt{4 - 60}}{6} = \frac{2 \pm \sqrt{-56}}{6}$$

$$= \frac{2 \pm i\sqrt{56}}{6} = \frac{2 \pm 2i\sqrt{14}}{6}$$

$$= \frac{\cancel{2}(1 \pm i\sqrt{14})}{\cancel{6}_{3}} = \boxed{\frac{1 \pm i\sqrt{14}}{3}}$$