

Other Equations

Ex. Solve $3x^4 = 48x^2$

$$3x^4 - 48x^2 = 0$$

$$3x^2(x^2 - 16) = 0$$

$$3x^2(x+4)(x-4) = 0$$

$$\frac{3x^2}{3} = \frac{0}{3}$$
$$\sqrt{x^2} = \sqrt{0}$$
$$x = 0$$

$$x+4=0$$
$$x=-4$$

$$x-4=0$$
~~$$x+4=0$$~~
$$x=4$$

Ex. Solve $\underbrace{x^3 - 3x^2} + \underbrace{3x - 9} = 0$

$$x^2(x-3) + 3(x-3) = 0$$

$$(x-3)(x^2 + 3) = 0$$

$$x-3=0$$

$$x = 3$$

$$x^2 + 3 = 0$$

$$\sqrt{x^2} = \sqrt{-3}$$

$$x = \pm i\sqrt{3}$$

Ex. Solve $x^4 - 3x^2 + 2 = 0$ $\xrightarrow{y=x^2}$ $y^2 - 3y + 2 = 0$
 $(x^2-1)(x^2-2) = 0$ \longleftarrow $(y-1)(y-2) = 0$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

$$x^2 - 2 = 0$$

$$x^2 = 2$$

$$x = \pm \sqrt{2}$$

When solving equations with radicals, absolute value, or variables on the bottom, check all answers.

Ex. Solve $\sqrt{2x+7} - \cancel{x} = 2$

$$\begin{aligned} (x+2)^2 &= (x+2)(x+2) \\ &= x^2 + 2x + 2x + 4 \\ &= x^2 + 4x + 4 \end{aligned}$$

x = -3

$$\begin{aligned} \sqrt{2(-3)+7} - (-3) &= 2 \\ \sqrt{-6+7} + 3 &= 2 \\ \sqrt{1} + 3 &= 2 \end{aligned}$$

$$\begin{aligned} \sqrt{2x+7} &= (x+2)^2 \\ 2x+7 &= x^2 + 4x + 4 \\ 0 &= x^2 + 2x - 3 \end{aligned}$$

x = 1

$$\begin{aligned} \sqrt{2(1)+7} - 1 &= 2 \\ \sqrt{2+7} - 1 &= 2 \\ \sqrt{9} - 1 &= 2 \end{aligned}$$

$$\begin{aligned} 0 &= (x+3)(x-1) \\ x+3 &= 0 & x-1 &= 0 \\ \cancel{x} &= -3 & \boxed{x} &= 1 \end{aligned}$$

Ex. Solve $\sqrt{2x-5} - \sqrt{x-3} = 1$

$$\begin{aligned} \underline{x=7} \\ \sqrt{2(7)-5} - \sqrt{7-3} &= 1 \\ \sqrt{14-5} - \sqrt{4} &= 1 \\ \sqrt{9} - \sqrt{4} &= 1 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \underline{x=3:} \\ \sqrt{2(3)-5} - \sqrt{3-3} &= 1 \\ \sqrt{6-5} - \sqrt{0} &= 1 \\ \sqrt{1} - \sqrt{0} &= 1 \quad \checkmark \end{aligned}$$

$$(\sqrt{2x-5}) = (\sqrt{x-3} + 1)^2$$

$$2x-5 = x-3 + 2\sqrt{x-3} + 1$$

$$2x-5 = x-2 + 2\sqrt{x-3}$$

$$(x-3)^2 = (2\sqrt{x-3})^2$$

$$x^2 - 6x + 9 = 4(x-3)$$

$$x^2 - 6x + 9 = 4x - 12$$

$$x^2 - 10x + 21 = 0$$

$$(x-3)(x-7) = 0$$

$$x-3=0$$

$$x=3$$

$$x-7=0$$

$$x=7$$

$$(\sqrt{x-3} + 1)(\sqrt{x-3} + 1)$$

$$= x-3 + 2\sqrt{x-3} + 1$$

$$(x-3)(x-3)$$

$$= x^2 - 6x + 9$$

Ex. Solve $\frac{2}{x} = \frac{3}{x-2} - 1$

$$\frac{2}{\cancel{x}} \cdot \frac{\cancel{x}(x-2)}{1} = \frac{3}{\cancel{x-2}} \cdot \frac{\cancel{x}(x-2)}{1} - 1 \cdot x(x-2)$$

$$2(x-2) = 3x - x(x-2)$$

$$2x - 4 = 3x - x^2 + 2x$$

$$2x - 4 = -x^2 + 5x$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$\cancel{x-4=0}$$
$$x=4$$

$$\cancel{x+1=0}$$
$$x=-1$$

Check:

Neither answer
makes us divide
by 0 \Rightarrow both
answers

Ex. Solve $|x^2 - 3x| = -4x + 6$

$$x^2 - 3x = -4x + 6$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$x+3=0$$

$$x = -3$$

$$|(-3)^2 - 3(-3)| = -4(-3) + 6$$

$$|9 + 9| = 12 + 6$$

✓

$$x-2=0$$

$$x = 2$$

$$|2^2 - 3(2)| = -4(2) + 6$$

$$|4 - 6| = -8 + 6$$

✗

$$x^2 - 3x = -(-4x + 6)$$

$$x^2 - 3x = 4x - 6$$

$$x^2 - 7x + 6 = 0$$

$$(x-6)(x-1) = 0$$

$$x-6=0$$

$$x = 6$$

$$|6^2 - 3(6)| = -4(6) + 6$$

$$|36 - 18| = -24 + 6$$

✗

$$x-1=0$$

$$x = 1$$

$$|1^2 - 3(1)| = -4(1) + 6$$

$$|1 - 3| = -4 + 6$$

✓

Compound Interest

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

A = amount in account

P = principal (original amount)

r = annual interest rate (as a decimal)

n = number of compoundings annually

t = years that have passed

Ex. When you were born, your grandparents deposited \$5000 = P in an account that compounds interest quarterly ^{$n=4$} . On your $t = 25^{\text{th}}$ birthday, the value of the investment is \$25,062.59 = A . What is the annual interest rate on the account?

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$\frac{25062.59}{5000} = \frac{5000}{5000} \left(1 + \frac{r}{4} \right)^{4 \cdot 25}$$

$$\left[\frac{25062.59}{5000} \right]^{\frac{1}{100}} = \left[\left(1 + \frac{r}{4} \right)^{100} \right]^{\frac{1}{100}}$$

$$1.01625 = 1 + \frac{r}{4}$$

$$4 \cdot (.01625) = \frac{r}{4} \cdot 4$$

$$r = .065$$

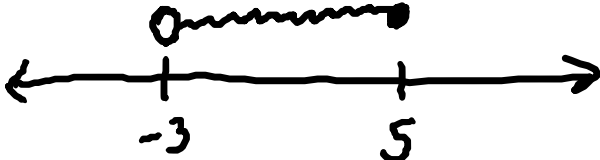

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Inequalities

This means that we'll use $<$ $>$ \leq and \geq

Ex. Express each interval as an inequality.

Is it bounded?

- a. $(-3, 5]$  $-3 < x \leq 5$
- b. $(-3, \infty)$  $x > -3$
- c. $[0, 2]$
- d. $(-\infty, \infty)$

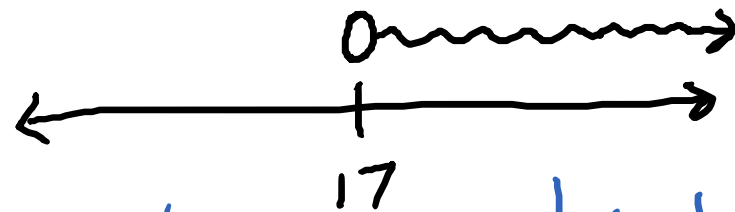
To solve inequalities, change them to equalities and then check the intervals.

[If you want treat as an equality, remember that multiplying or dividing by a negative switches the inequality]

Ex. Solve $5x - 7 > 3(x + 9)$

$$\begin{aligned}5x - 7 &= 3(x + 9) \\5x - 7 &= 3x + 27 \\2x &= 34 \\x &= 17\end{aligned}$$

$x > 17$
$(17, \infty)$



test $x=0$

$$\begin{aligned}5(0) - 7 &> 3(0 + 9) \\-7 &> 27\end{aligned}$$

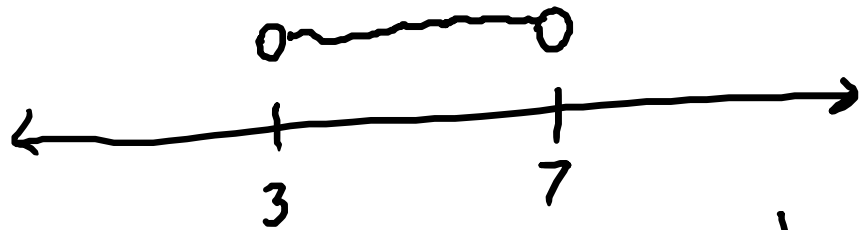


test $x=18$

$$\begin{aligned}5(18) - 7 &> 3(18 + 9) \\90 - 7 &> 3(27) \\83 &> 81\end{aligned}$$



Ex. Solve $|x - 5| < 2$



$x = 0$ ↓
 $|0 - 5| < 2$
✗

$x = 5$ ↓
 $|5 - 5| < 2$
✓

$x = 9$ ↓
 $|9 - 5| < 2$
✗

$$|x - 5| = 2$$

$$x - 5 = 2$$

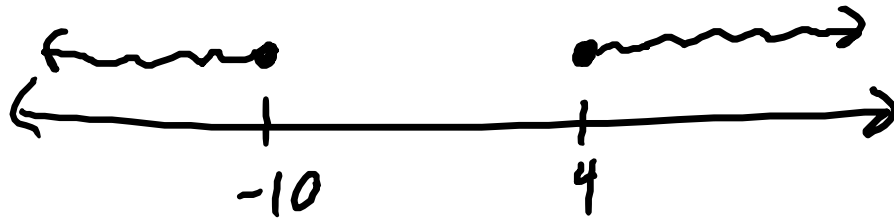
$$x = 7$$

$$x - 5 = -2$$

$$x = 3$$

$$(3, 7)$$

Ex. Solve $|x + 3| \geq 7$



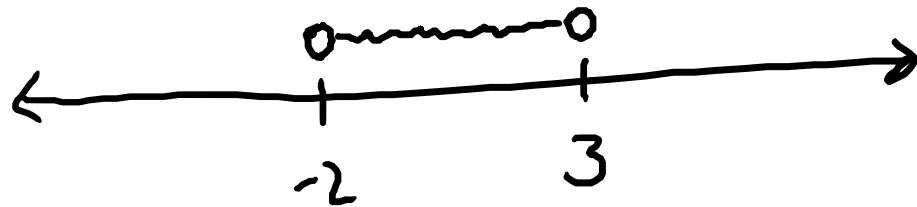
$$(-\infty, -10] \cup [4, \infty)$$

$$x \leq -10 \quad \text{or} \quad x \geq 4$$

More Inequalities

Ex. Solve $x^2 - x - 6 < 0$

$(-2, 3)$



$$\begin{aligned}x^2 - x - 6 &= 0 \\(x-3)(x+2) &= 0 \\x-3 &= 0 & x+2 &= 0 \\x &= 3 & x &= -2\end{aligned}$$

$x = -3$

$$\begin{aligned}(-3)^2 - (-3) - 6 &< 0 \\9 + 3 - 6 &< 0 \\&\times\end{aligned}$$

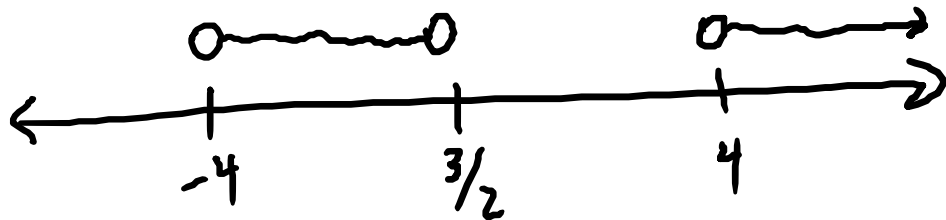
$x = 0$

$$\begin{aligned}0 - 0 - 6 &< 0 \\-6 &< 0 \\&\checkmark\end{aligned}$$

$x = 4$

$$\begin{aligned}4^2 - 4 - 6 &< 0 \\16 - 4 - 6 &< 0 \\&\times\end{aligned}$$

Ex. Solve $2x^3 - 3x^2 - 32x > -48$



$x=0$

$$0 - 0 - 0 > -48$$

$$\boxed{\left(-4, \frac{3}{2}\right) \cup (4, \infty)}$$

$$2x^3 - 3x^2 - 32x = -48$$

$$2x^3 - 3x^2 - 32x + 48 = 0$$

$$x^2(2x-3) - 16(2x-3) = 0$$

$$(2x-3)(x^2-16) = 0$$

$$2x-3=0$$

$$2x=3$$

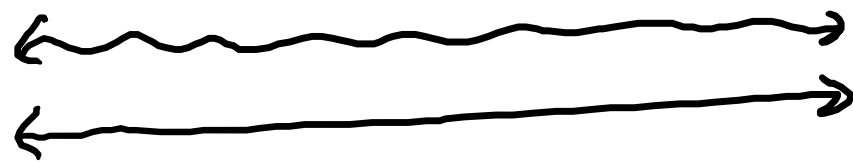
$$\boxed{x = \frac{3}{2}}$$

$$x^2-16=0$$

$$\sqrt{x^2} = \sqrt{16}$$

$$\boxed{x = \pm 4}$$

Ex. Solve $x^2 + 2x + 4 > 0$



↓ $x=0$

$$0 + 0 + 4 > 0 \quad \checkmark$$

$$x^2 + 2x + 4 = 0$$
$$x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} = \frac{-2 \pm \sqrt{4 - 16}}{2}$$

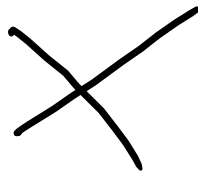
all reals

Ex. Solve $x^2 + 2x + 4 < 0$



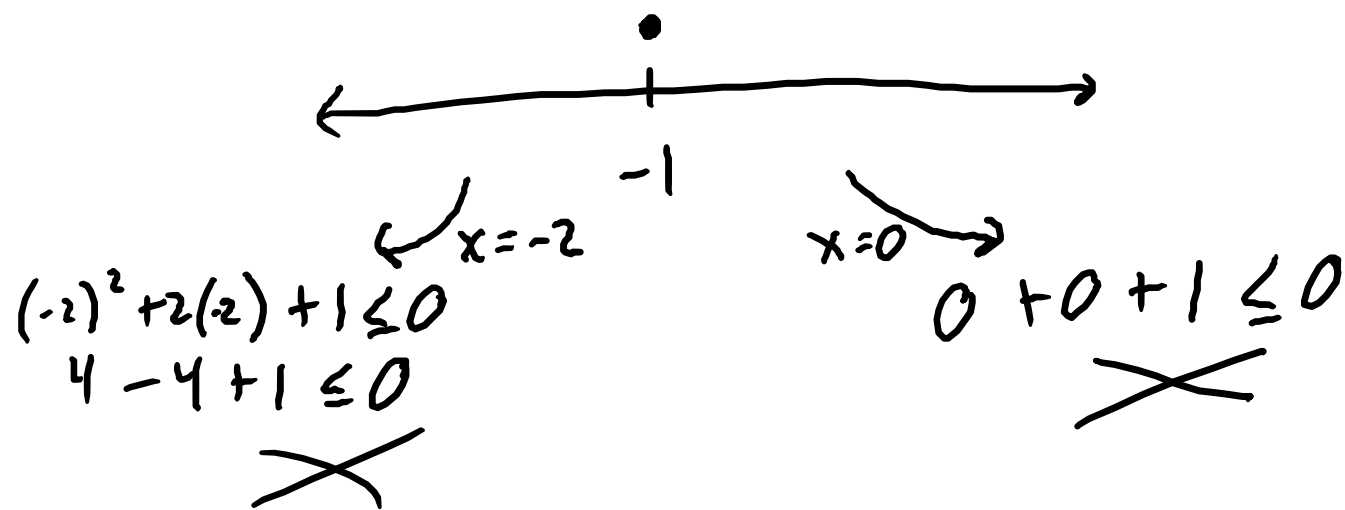
↓ $x=0$

$$0 + 0 + 4 < 0$$



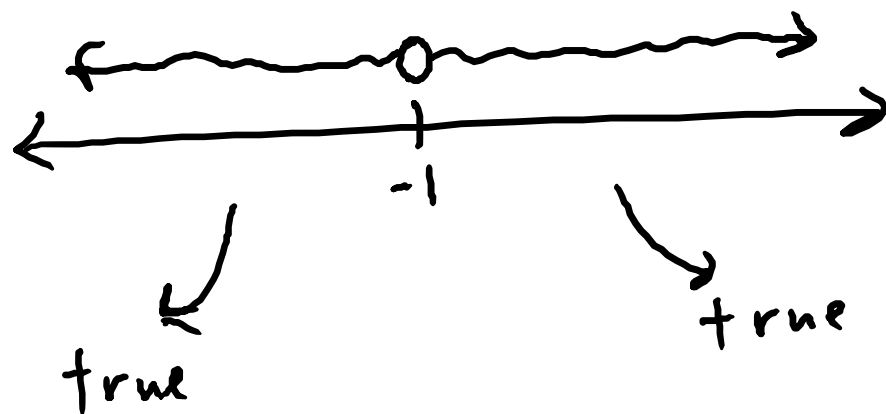
no solution

Ex. Solve $x^2 + 2x + 1 \leq 0$



$$x^2 + 2x + 1 = 0$$
$$(x+1)(x+1) = 0$$
$$x+1 = 0$$
$$x = -1$$

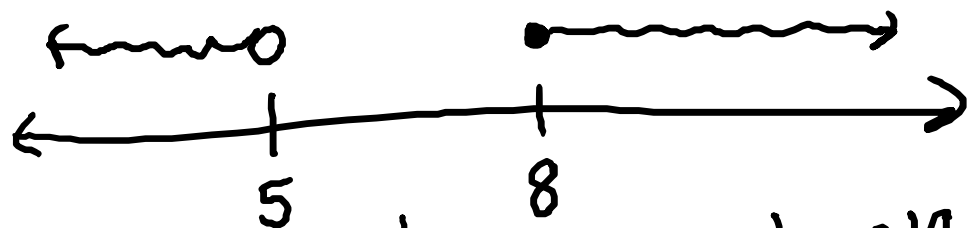
Ex. Solve $x^2 + 2x + 1 > 0$



$$(-\infty, -1) \cup (-1, \infty)$$

$$x \neq -1$$

Ex. Solve $\frac{2x-7}{x-5} \leq 3$



$x=0$

$$\frac{0-7}{0-5} \leq 3$$

$$\frac{7}{5} \leq 3$$

✓

$x=7$

$$\frac{2(7)-7}{7-5} \leq 3$$

$$\frac{14-7}{2} \leq 3$$

$$\frac{7}{2} \leq 3$$

✗

$x=10$

$$\frac{2(10)-7}{10-5} \leq 3$$

$$\frac{20-7}{5} \leq 3$$

$$\frac{13}{5} \leq 3$$

✓

~~$$\frac{2x-7}{x-5} \leq \frac{3}{1}$$~~

$$2x-7 = 3(x-5)$$

$$2x-7 = 3x-15$$

$$8 = x$$

$$x-5=0$$

$$x=5$$

$$(-\infty, 5) \cup [8, \infty)$$

Ex. A projectile is fired straight ^{v_0} upward from the ground $s_0 = 0$
with an initial velocity of 384 ft/sec. During what time
period will its height exceed 2000 ft?

$$[s = -16t^2 + v_0t + s_0]$$

$$-16t^2 + 384t + 0 > 2000$$