Equation of a Line

Thm. A line has the equation $y = mx + b$, where m = slope and b = y-intercept.

→ This is called Slope-Intercept Form

Ex. Find the slope and y-intercept:

a) $y = 3x + 1$

slope = 3
y-int. = 1

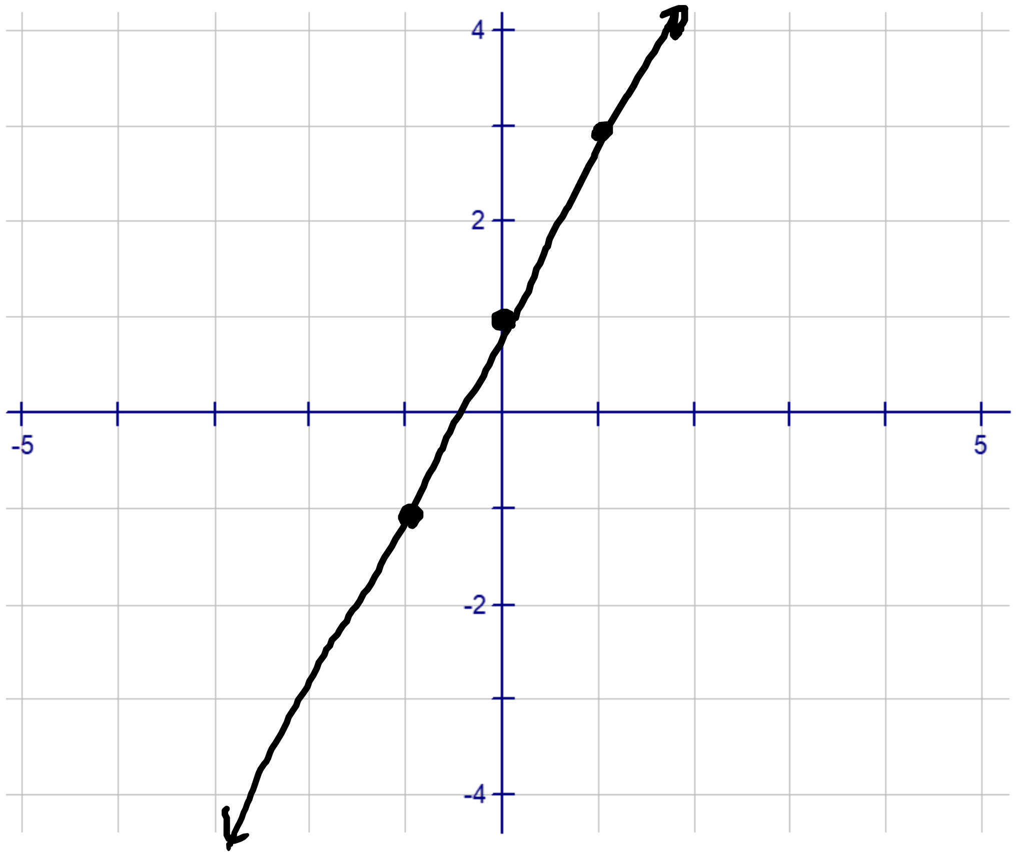
b) $\cancel{2x} + 3y = 1$

$-2x \qquad -2x$

$\frac{\cancel{3}y}{\cancel{3}} = \frac{-2x}{3} + \frac{1}{3} \longrightarrow y = \underbrace{-\frac{2}{3}}_{=\text{slope}}x + \overbrace{\frac{1}{3}}^{=\text{y-int.}}$

Ex. Graph:

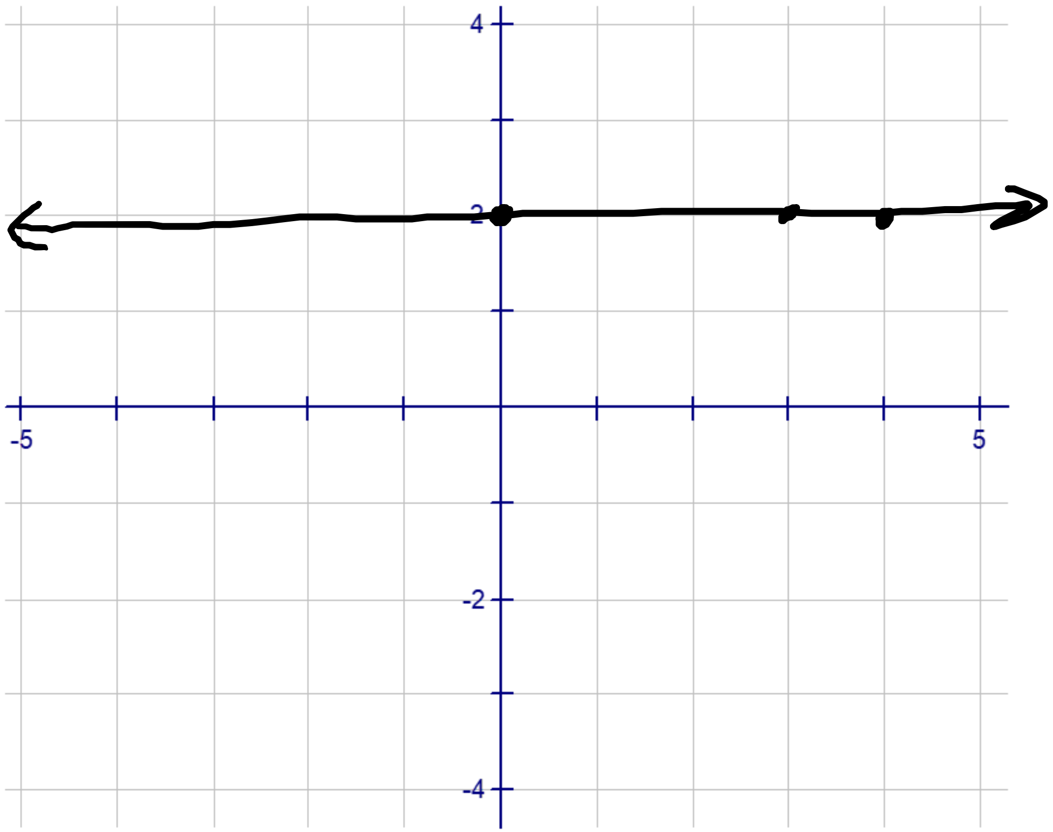
a) $y = 2x + 1$



x	$y = 2x + 1$
-1	$2(-1)+1 = -2+1 = -1$
0	$2(0)+1 = 0+1 = 1$
1	$2(1)+1 = 2+1 = 3$

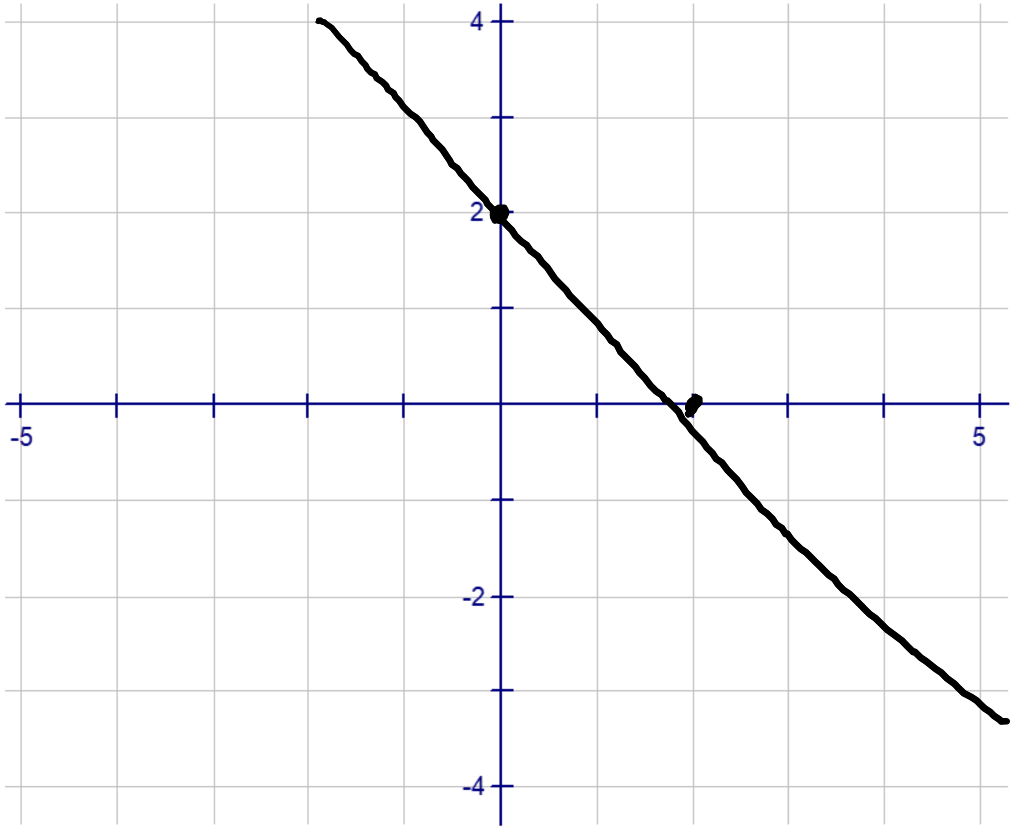
Ex. Graph:

b) $y = 2$



Ex. Graph:

c) $x + y = 2$



x-int. $\rightarrow$ $y = 0$

$x + 0 = 2$

$x = 2$

$(2, 0)$

y-int. $\rightarrow$ $x = 0$

$0 + y = 2$

$y = 2$

$(0, 2)$

Slope

Thm. The slope between (x_1,y_1) and (x_2,y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Ex. Find the slope between (-2,0) and (3,1).

$$m = \frac{1-0}{3-(-2)} = \frac{1}{5}$$

- Rising line has positive slope: $\frac{2}{5}, 4$

- Falling line has negative slope: $-8, -\frac{3}{7}$

- Horizontal line as slope of 0: $\frac{0}{5}, \frac{0}{-3}$

- Vertical line has undefined slope: $\frac{2}{0}, \frac{-8}{0}$

→A vertical line has an equation like $x = 3$, and can't be written as $y = mx + b$

Thm. Parallel lines have the same slope.

Thm. Perpendicular lines have slopes that are negative reciprocals.

$\frac{2}{5} \rightarrow -\frac{5}{2}$ $-\frac{6}{1} \rightarrow \frac{1}{6}$

Ex. Are the lines parallel, perpendicular, or neither?

(3,-1) to (-3,1) and (0,3) to (-1,0)

$m = \frac{-1-1}{3-(-3)} = \frac{-2}{6} = \frac{-1}{3}$

$m = \frac{3-0}{0-(-1)} = \frac{3}{1} = 3$

Thm. A line with m = slope that passes through the point (x_1, y_1) has the equation

$$y - y_1 = m(x - x_1)$$

→This is called Point-Slope Form

Ex. Write the equation in Slope-Intercept Form:

a) slope = 3, contains (1,-2)

$$y - \underline{y_1} = m(x - \underline{x_1})$$

$$y - (-2) = 3(x - 1)$$

$$y + 2 = 3x - 3$$

$$ -2 \qquad\quad -2$$

$$\boxed{y = 3x - 5}$$

Ex. Write the equation in Slope-Intercept Form:

b) contains (2,5) and (4,-1)

$$m = \frac{-1-5}{4-2} = \frac{-6}{2} = -3$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -3(x - 2)$$

$$y - \cancel{5} = -3x + 6$$

$$\cancel{+5} \qquad\qquad +5$$

$$y = -3x + 11$$

Ex. Find equations of the lines that pass through (2,-1) are:

a) parallel to, and

b) perpendicular to, the line $2x - 3y = 5$

$$-2x \qquad -2x$$

$$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{5}{-3}$$

$$y = \frac{2}{3}x - \frac{5}{3}$$

a) $m = \frac{2}{3}$, $(2,-1)$

$$\boxed{y - (-1) = \frac{2}{3}(x-2)}$$

b) $m = -\frac{3}{2}$, $(2,-1)$

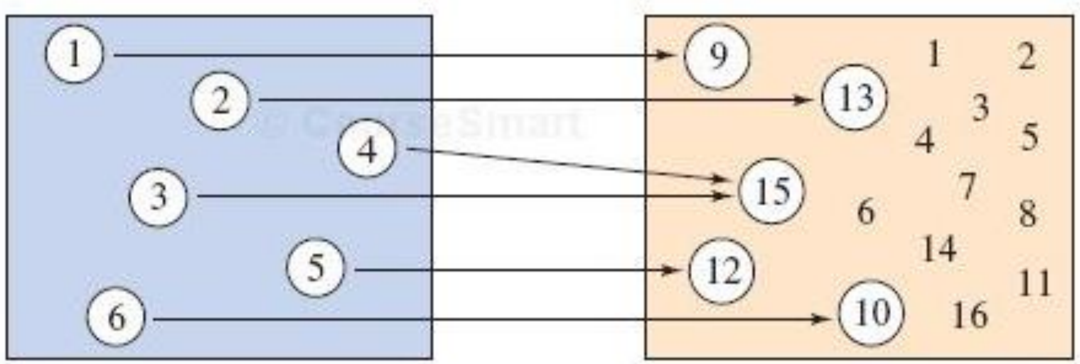
$$\boxed{y - (-1) = -\frac{3}{2}(x-2)}$$

$$\boxed{y - y_1 = m(x - x_1)}$$

Functions

Def. (formal) A function f from set A to set B is a relation that assigns to each element x in set A exactly one element y in set B.

Def. (informal) A set of ordered pairs is a function if no two points have the same x-coordinate



Set A
(the x's)
The input

Domain

Set B
(the y's)
The output

Range

x is called the independent variable

y is called the dependent variable

Rather than writing

$$y = 7x + 2$$

we can express a function as

"f of x"

$$f(x) = 7x + 2$$

→ This is called Function Notation

$$f(x) = 7x + 2$$

Ex. Let $g(x) = -x^2 + 4x + 1$, find

a) $g(2) = -2^2 + 4(2) + 1$

$= -4 + 8 + 1 = 5$

b) $g(t) = -t^2 + 4t + 1$

c) $g(t + 2) = -(t+2)^2 + 4(t+2) + 1$

$= -(t^2 + 4t + 4) + 4t + 8 + 1$

$= -t^2 - 4t - 4 + 4t + 9 = -t^2 + 5$

The next example is called a <u>piecewise function</u> because the equation depends on what we are plugging in.

<u>Ex.</u> Let $f(x) = \begin{cases} x^2 + 1 & x < 0 \\ x - 1 & x \geq 0 \end{cases}$

Find $f(-1)$, $f(0)$, and $f(1)$.

$f(-1) = (-1)^2 + 1 = 1 + 1 = 2$

$f(0) = 0 - 1 = -1$

$f(1) = 1 - 1 = 0$

<u>Ex.</u> Write the equation of a linear function $f(x)$, given that $f(1) = 4$ and $f(3) = 10$.

$(1, 4)$ $(3, 10)$

$m = \frac{10-4}{3-1} = \frac{6}{2} = 3$

$y - 4 = 3(x - 1)$

$y - \cancel{4} = 3x - 3$

$\quad \cancel{+4} \qquad +4$

$y = 3x + 1$

$\boxed{f(x) = 3x + 1}$

Often, finding the domain means finding the x's that can't be used in the function

<u>Ex.</u> Find the domain of the function

a) $g(x) = \dfrac{1}{x+5}$

$\boxed{x \neq -5}$

b) Volume of a sphere: $V = \frac{4}{3}\pi r^3$

$\boxed{r > 0}$

Ex. Find the domain of the function

c) $h(x) = \sqrt{4 - x^2}$

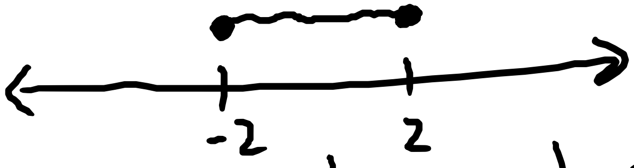
$\boxed{[-2, 2]}$

$4 - x^2 \geq 0$

$4 - x^2 = 0$

$\sqrt{4} = \sqrt{x^2}$

$x = \pm 2$



$x = -3$

$4 - (-3)^2 \geq 0$

$4 - 9 \geq 0$

✗

$x = 0$

$4 - 0^2 \geq 0$

✓

$x = 3$

$4 - 3^2 \geq 0$

$4 - 9 \geq 0$

✗

Ex. When a baseball is hit, the height of the baseball is given by the function $f(x) = -0.0032x^2 + x + 3$, where x is distance travelled (in ft) and $f(x)$ is height (in ft). Will the baseball clear a 10-foot fence that is 300 ft from home plate?