Graph of a Function

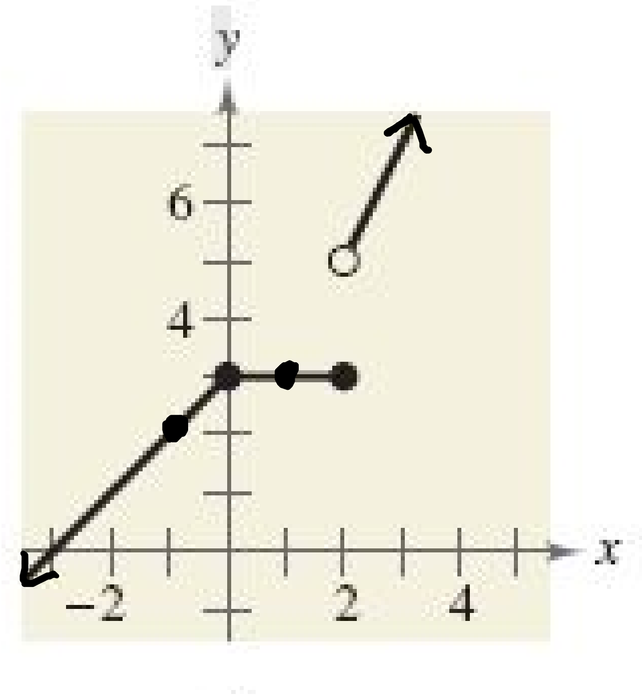
<u>Ex.</u> Using the graph, find:

a) domain all reals

b) range $(-\infty, 3] \cup (5, \infty)$

c) $f(-1)$, $f(1)$, and $f(2)$

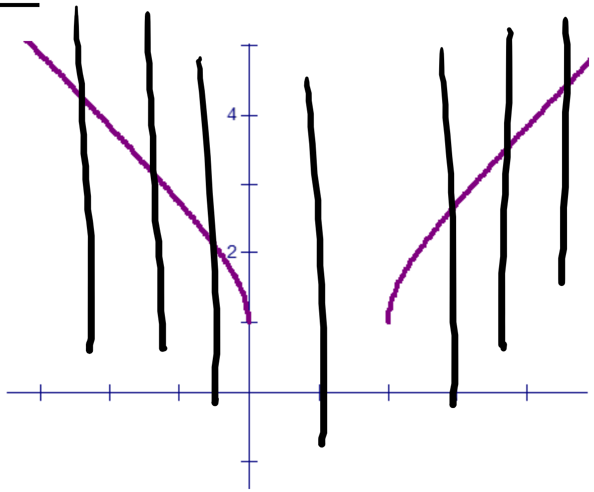
2 3 3



The graph of a function will pass the <u>vertical line test</u> – all vertical lines will pass through the graph at most once.

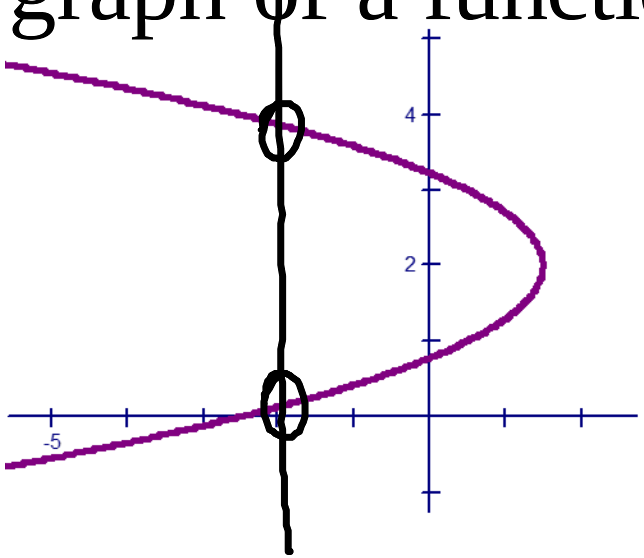
<u>Ex.</u> Determine if this is the graph of a function

a)



yes

b)



no

<u>Def.</u> The <u>zeroes</u> of a function f are the x-values for which $f(x) = 0$.

<u>Ex.</u> Find the zeroes of the function

a) $f(x) = 3x^2 + 1x - 10 = 0$

$$(3x + -5)(1x + 2) = 0$$

$3x - 5 = 0$

$3x = 5$

$x = \frac{5}{3}$

$x + 2 = 0$

$x = -2$

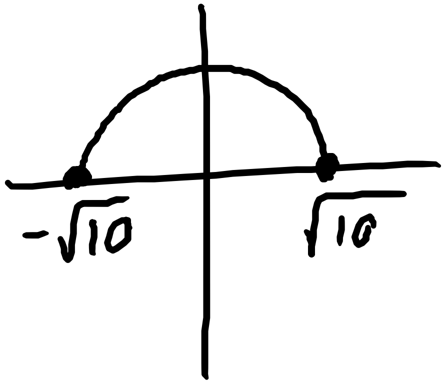
Ex. Find the zeroes of the function

b) $g(x) = \sqrt{10 - x^2} = 0$

$10 - x^2 = 0$

$\sqrt{10} = \sqrt{x^2}$

$x = \pm\sqrt{10}$



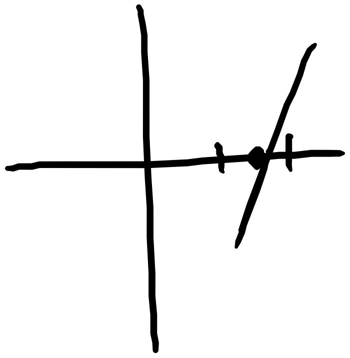
c) $h(t) = \dfrac{2t - 3}{t + 5} = 0$

$\dfrac{(t+5)}{1} \cdot \dfrac{2t-3}{t+5} = 0 \cdot (t+5)$

$2t - 3 = 0$

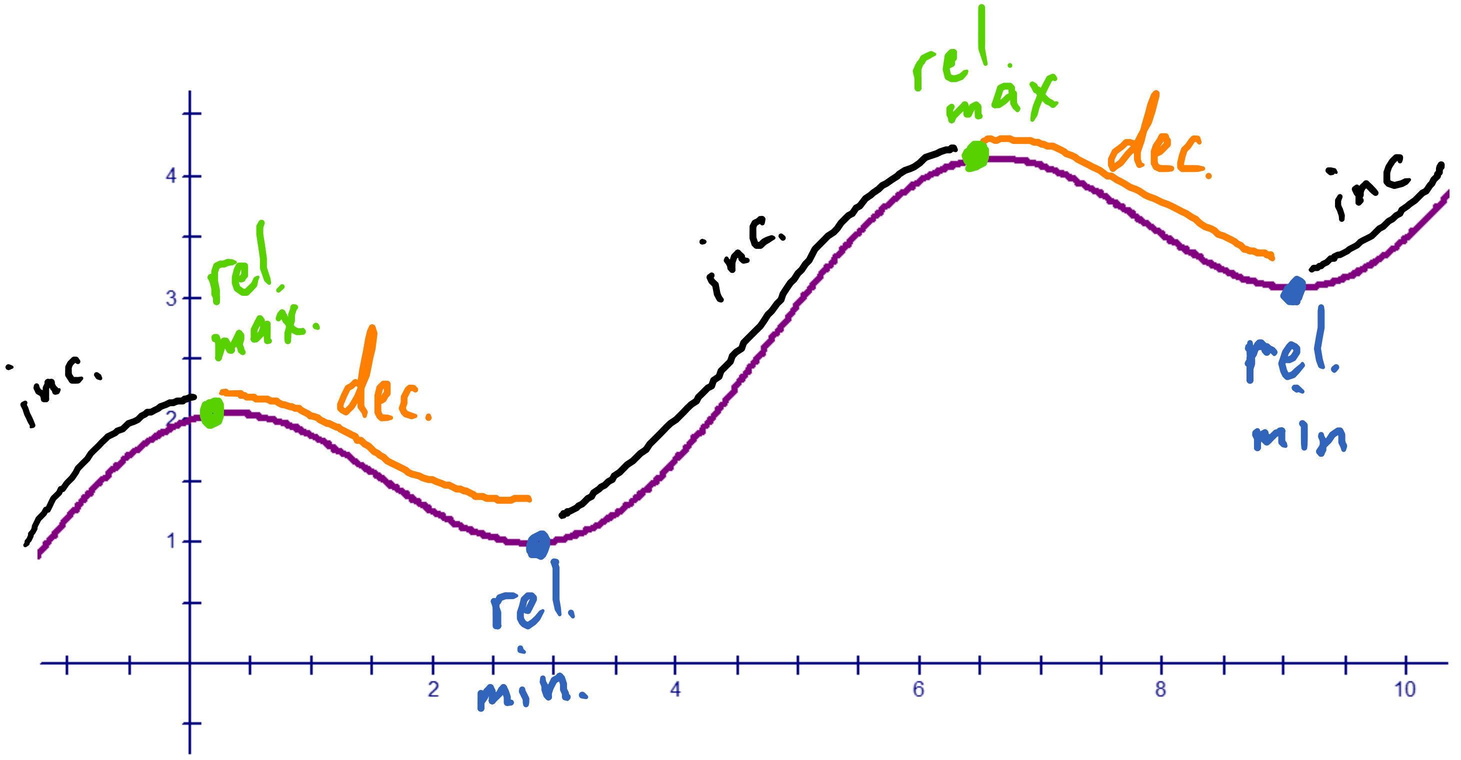
$2t = 3$

$t = \frac{3}{2}$



This is where the graph crosses the *x*-axis.

Let discuss increasing, decreasing, relative minimum, and relative maximum

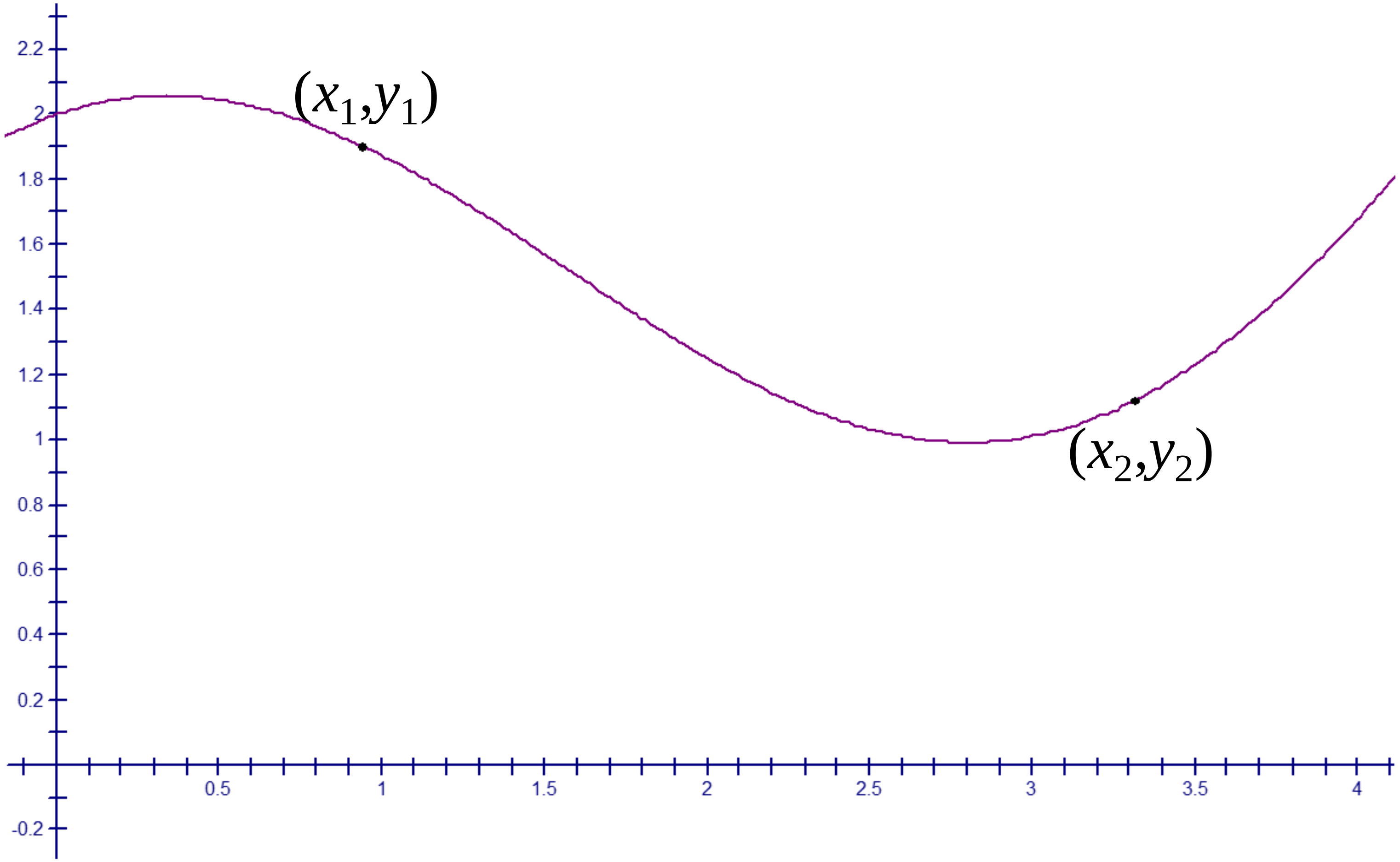


Ex. Use a calculator to approximate the relative minimum of the function $f(x) = 3x^2 - 4x - 2$.

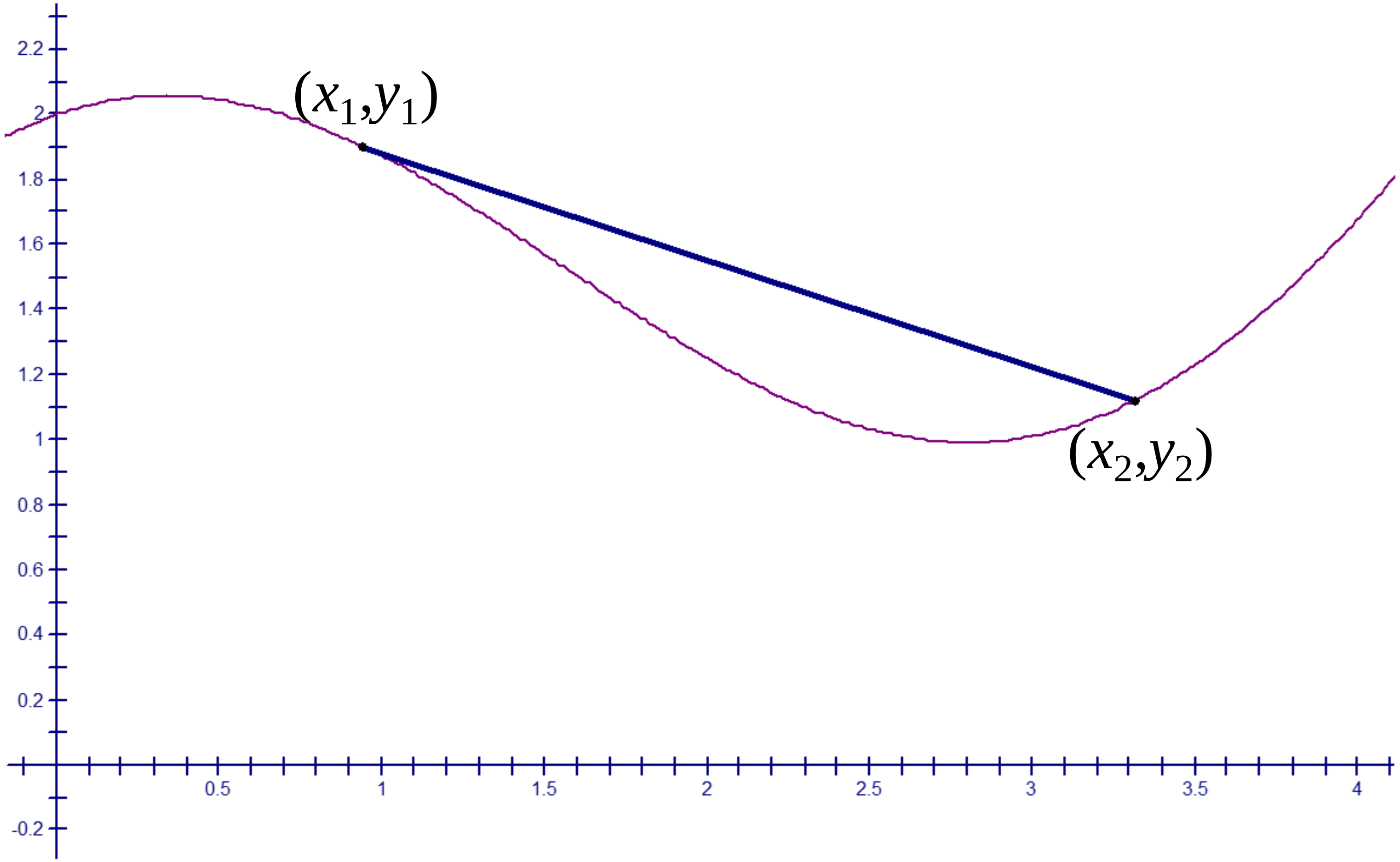
Earlier, we worked with slope as the rate of change of a line

→If the graph is nonlinear, we still want to talk about rate of change, but this slope is different at every point.

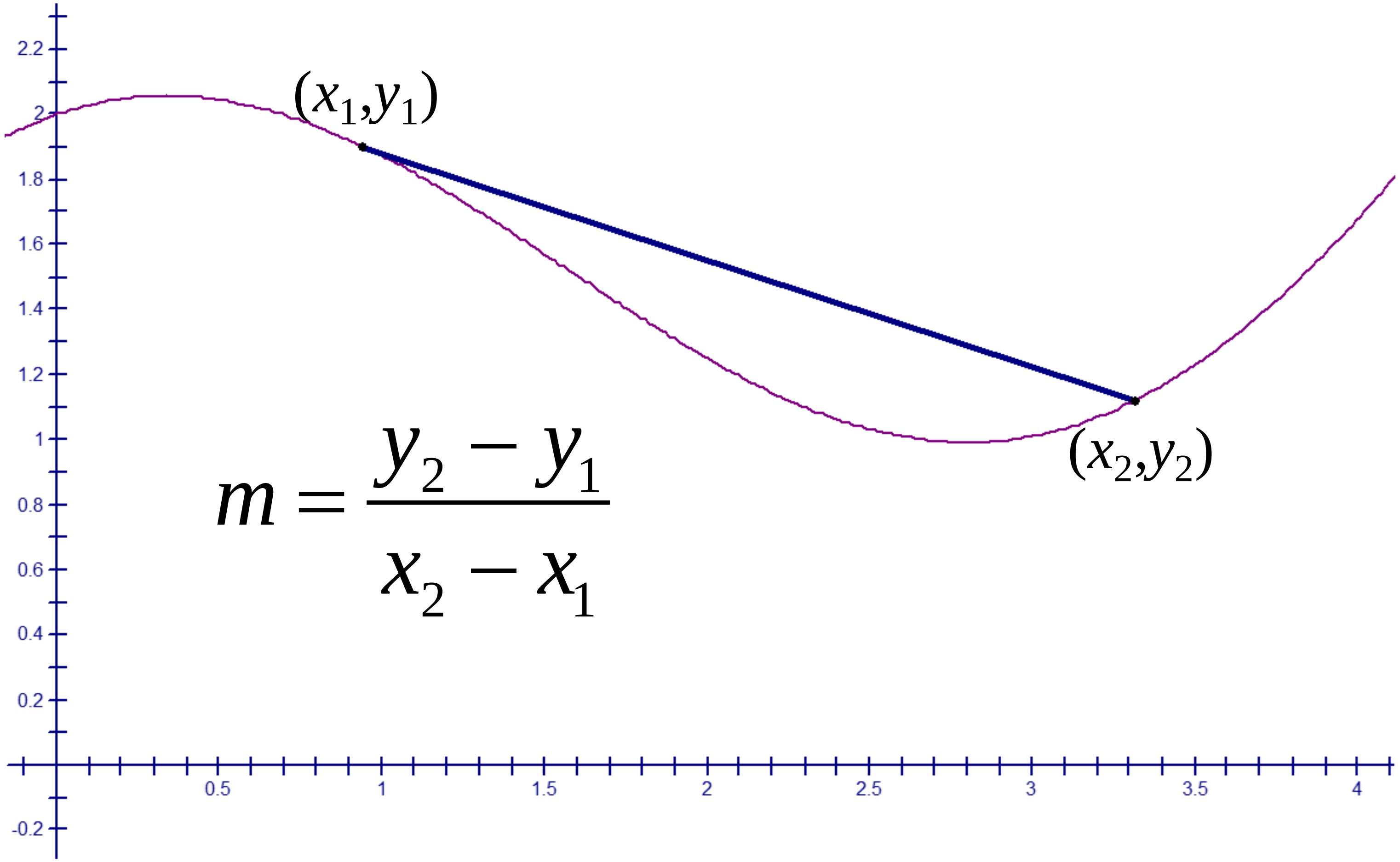
We can discuss the <u>average rate of change</u> between two points.



The points can be connected using a <u>secant line</u>



The average rate of change is the slope between the points



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Ex. Find the average rate of change of $f(x) = x^3 - 3x$ from $x_1 = -2$ to $x_2 = 1$.

$(-2, -2)$ $(1, -2)$

$f(-2) = (-2)^3 - 3(-2)$
$= -8 + 6 = -2$

$f(1) = 1^3 - 3(1)$
$= 1 - 3 = -2$

$\begin{pmatrix}\text{ave. rate}\\ \text{of change}\end{pmatrix} = \dfrac{-2-(-2)}{-2-1} = 0$

Ex. The distance s (in feet) a moving car has traveled is given by the function $s(t) = 20t^{3/2}$, where t is time (in seconds). Find the average speed from $t_1 = 4$ to $t_2 = 9$.

$s(4) = 20(4)^{3/2} = 20(\sqrt{4})^3$
$= 20(2)^3 = 20(8) = 160$

$s(9) = 20(9)^{3/2} = 20(\sqrt{9})^3$
$= 20(3)^3 = 20(27) = 540$

$(4, 160)$ — s, ft. $(9, 540)$ — s, ft.

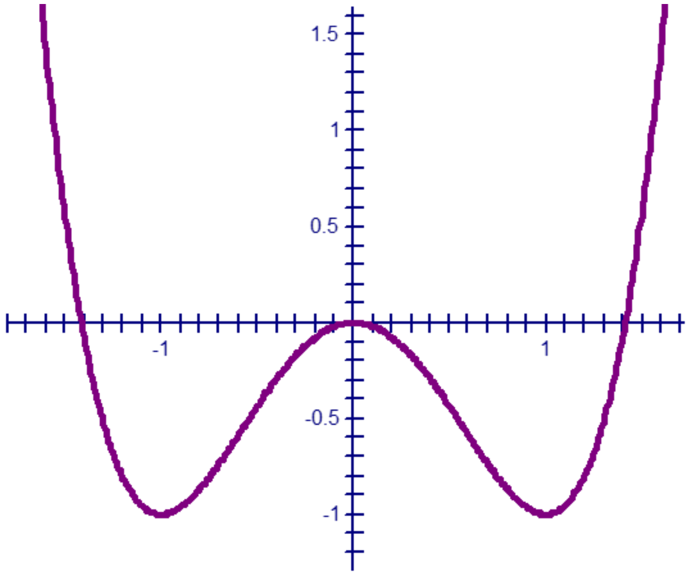
(ave. rate of change) $= \dfrac{540-160}{9-4}$ $\dfrac{\text{ft.}}{\text{sec.}}$

$= \dfrac{380}{5}$

$= 76$ ft./sec.

Def. A function $f(x)$ is even if $f(-x) = f(x)$.

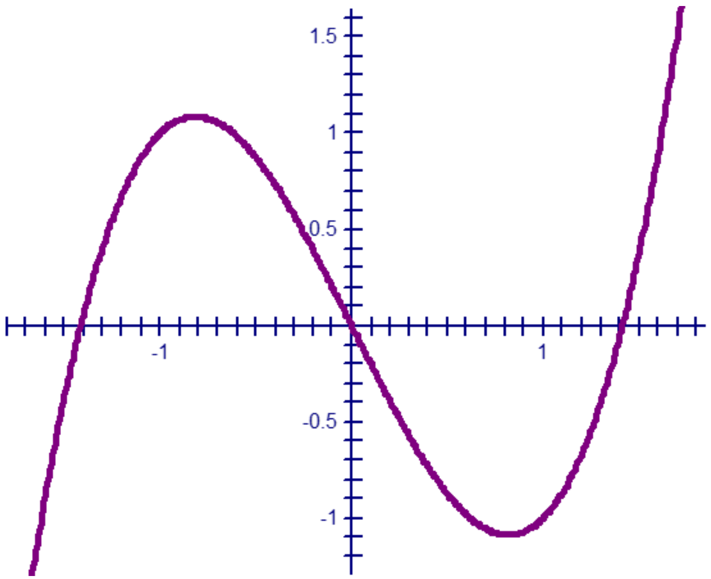
→ The graph will have y-axis symmetry



$$y = x^4 - 2x^2$$

Def. A function $f(x)$ is odd if $f(-x) = -f(x)$.

→ The graph will have origin symmetry



$$y = x^3 - 2x$$

Ex. Determine if the function is even, odd, or neither:

a) $g(x) = 3x^3 - 2x$

odd odd

odd

b) $h(x) = x^2 + 1$

even even

even

<u>Ex.</u> Determine if the function is even, odd, or neither:

c) $f(x) = x^{3} - 4x + 8$

Handwritten annotations: exponents circled as 3, 1, 0 (written $8x^{0}$); under the terms: odd, odd, even; answer: neither

Parents Functions

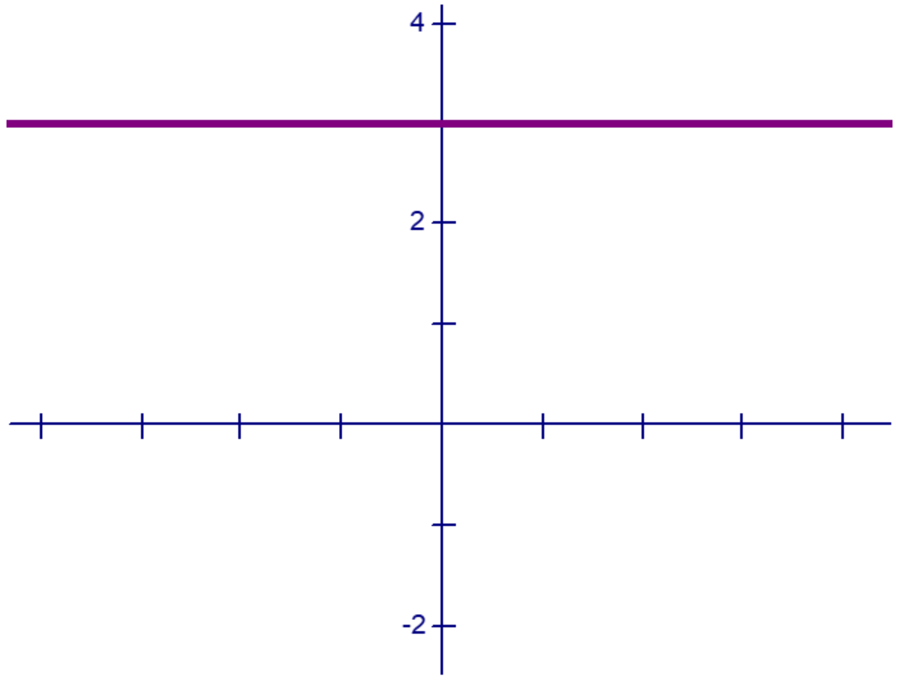
We are going to talk about some basic functions, and next class we will expand upon them.

Earlier, we saw that a function $f(x) = ax + b$ is linear

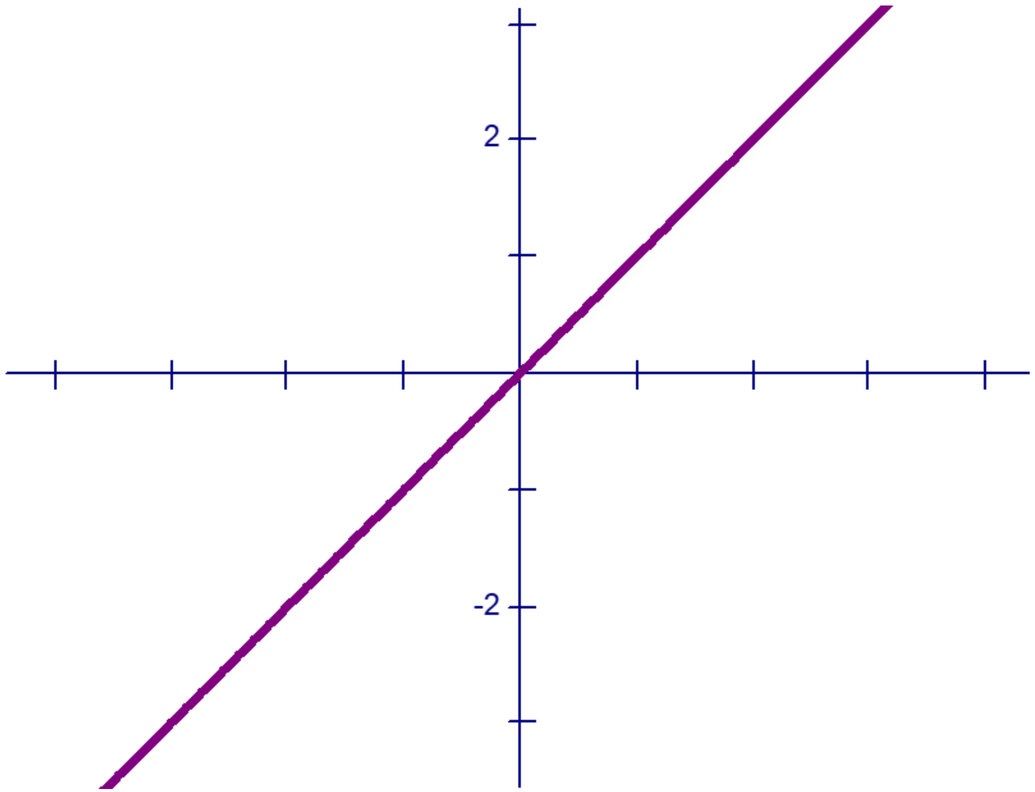
→ The domain of a linear function is all real numbers, and the range is all real numbers

The <u>constant function</u> is $f(x) = c$

→ The graph is a horizontal line



The identify function is $f(x)=x$

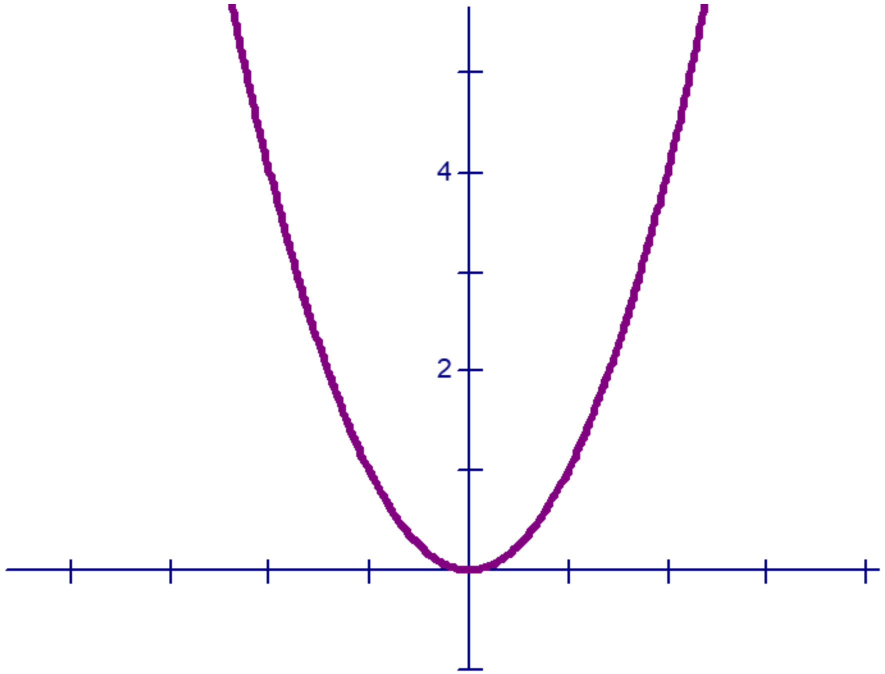


The <u>squaring function</u> is $f(x) = x^2$

→The domain is all real numbers

→The range is all nonnegative numbers

→The graph is even and has *y*-axis symmetry

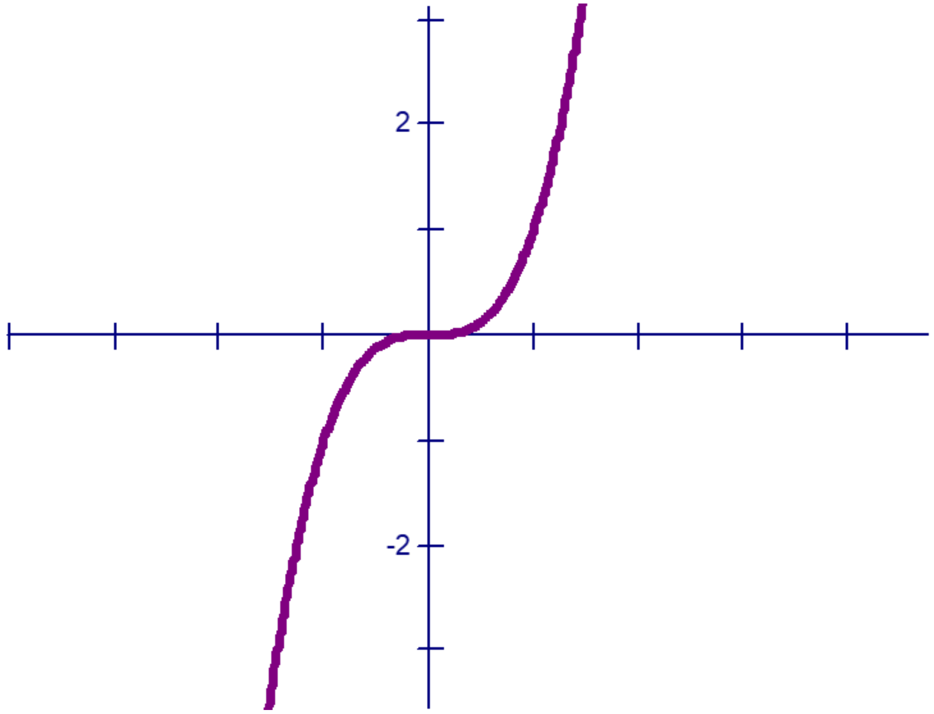


The <u>cubic function</u> is $f(x) = x^3$

→The domain is all real numbers

→The range is all real numbers

→The graph is odd and has origin symmetry

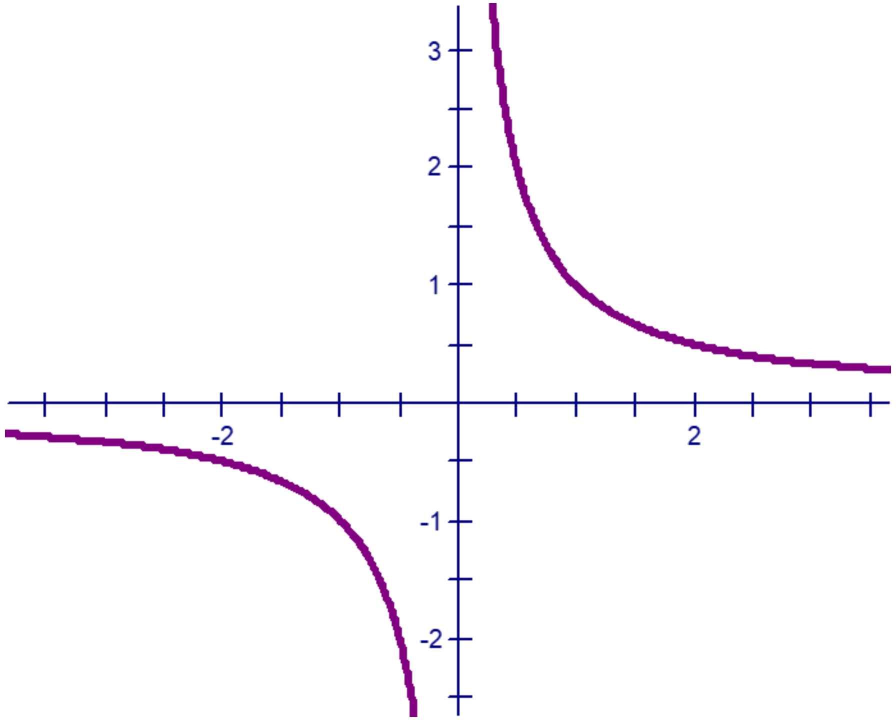


The <u>reciprocal function</u> is $f(x) = \dfrac{1}{x} = x^{-1}$

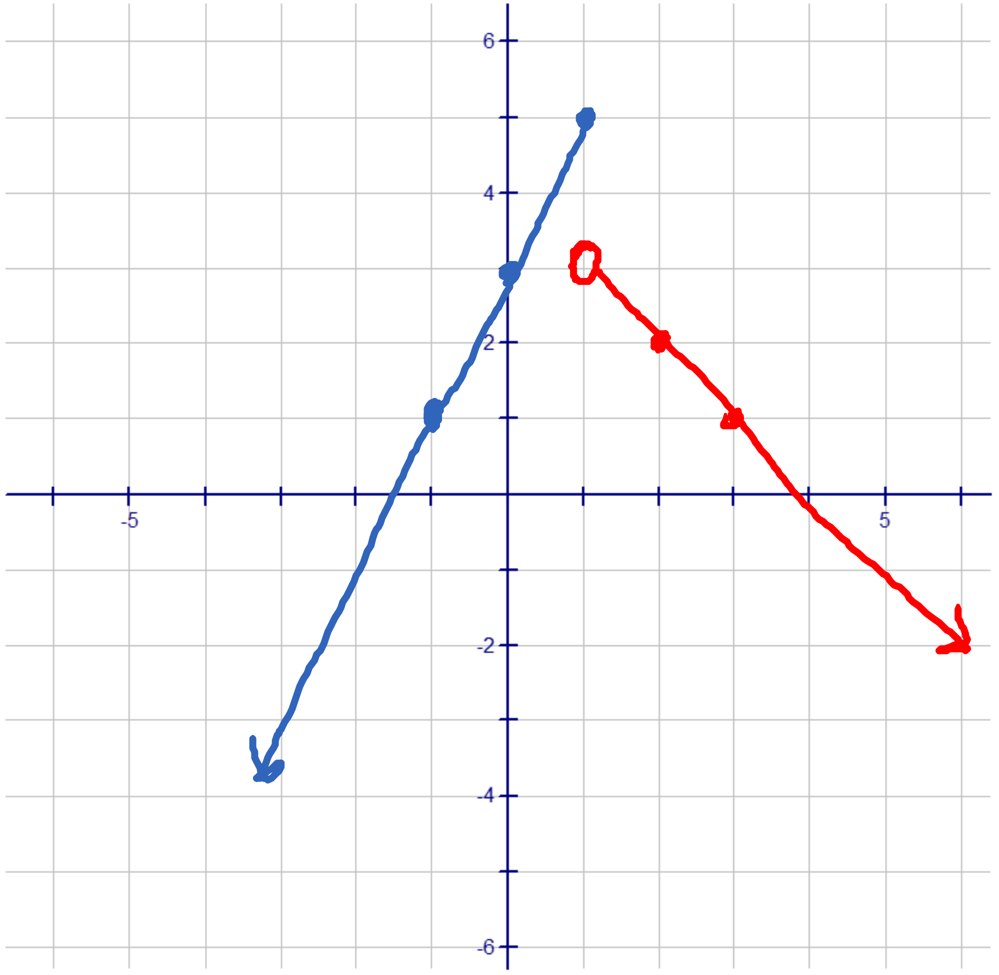
→The domain is all nonzero numbers

→The range is all nonzero numbers

→The graph is odd and has origin symmetry



Ex. Sketch a graph of $f(x)=\begin{cases} 2x+3 & x\le 1 \\ -x+4 & x>1 \end{cases}$



x	y=2x+3
1	2(1)+3 = 5
0	2(0)+3 = 3
-1	2(-1)+3 = 1

x	y=-x+4
1	-(1)+4=3
2	-(2)+4=2
3	-(3)+4=1

<u>Def.</u> The <u>greatest integer function</u>, $[\![x]\!]$, is defined as

[

$$f(x) = [\![x]\!] = \left(\begin{array}{c}\text{the greatest integer} \\ \text{less than or equal to } x\end{array}\right) = (\text{round down})$$

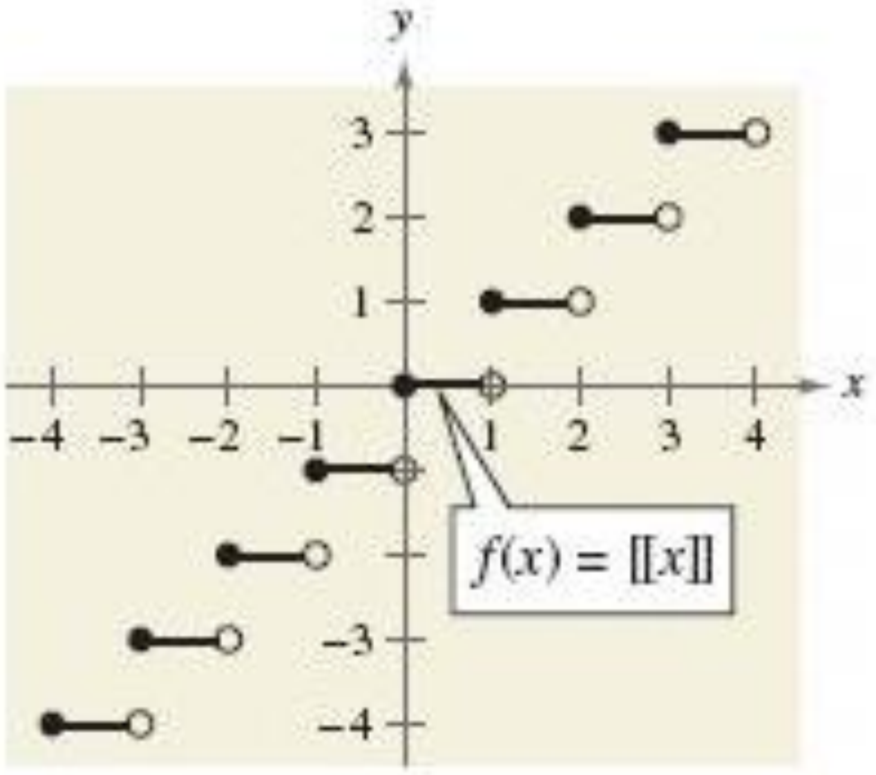
$$[\![1.5]\!] = 1$$

$$\left[\!\left[\tfrac{8}{5}\right]\!\right] = 1$$

$$[\![5]\!] = 5$$

$$[\![-3.7]\!] = -4$$

The graph looks like this:



This type of function is called a step function