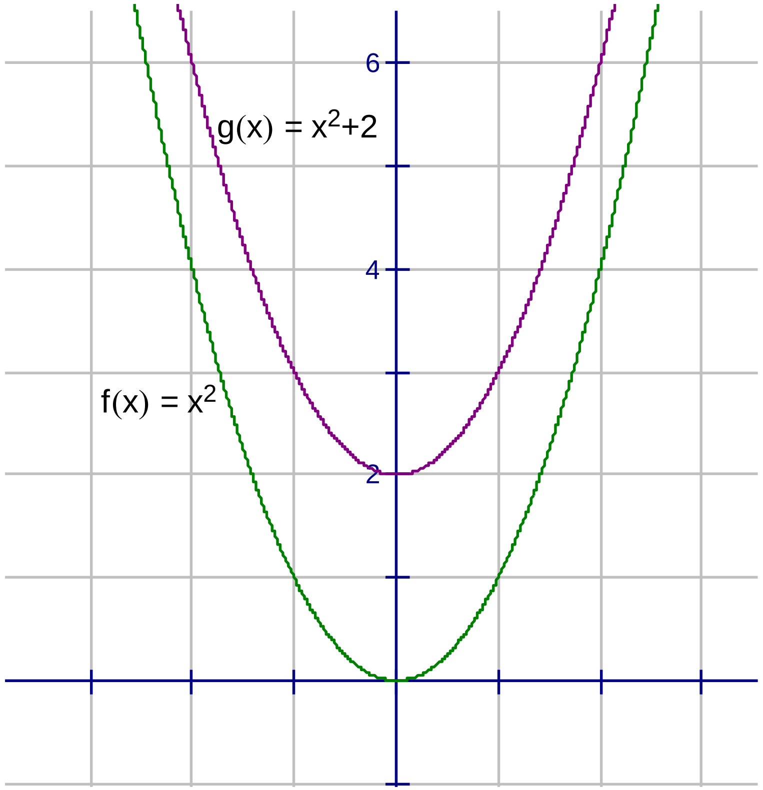
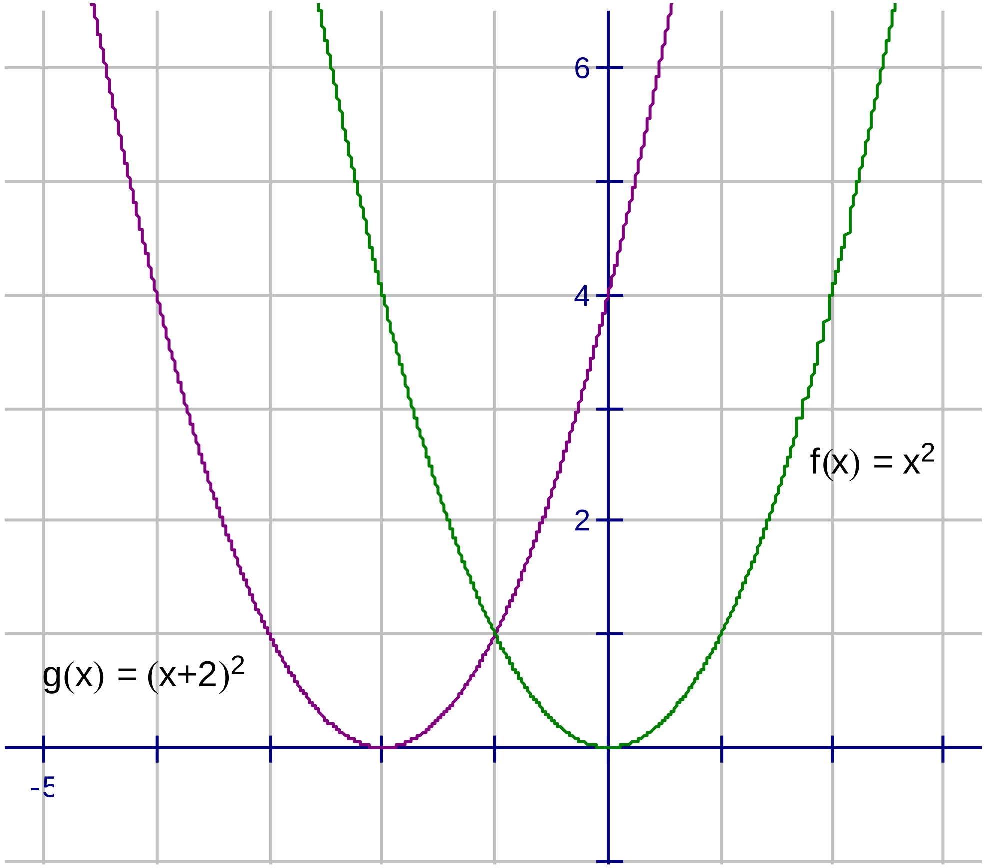
Transformations

We're going to take the functions from last class and alter them to get new ones

Ex. Compare $y = x^2$ and $y = x^2 + 2$



Ex. Compare $y = x^2$ and $y = (x + 2)^2$



Vertical and Horizontal Shifts

Let c be a positive real number. **Vertical and horizontal shifts** in the graph of $y = f(x)$ are represented as follows.

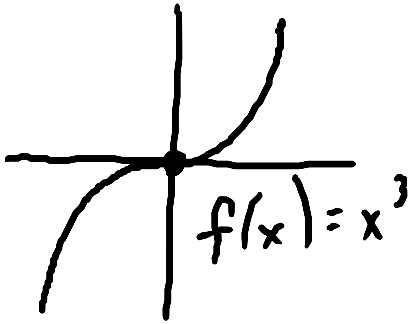
1. Vertical shift c units *upward:* $h(x) = f(x) + c$
2. Vertical shift c units *downward:* $h(x) = f(x) - c$
3. Horizontal shift c units to the *right:* $h(x) = f(x - c)$
4. Horizontal shift c units to the *left:* $h(x) = f(x + c)$

outside = vertical

inside = horizontal

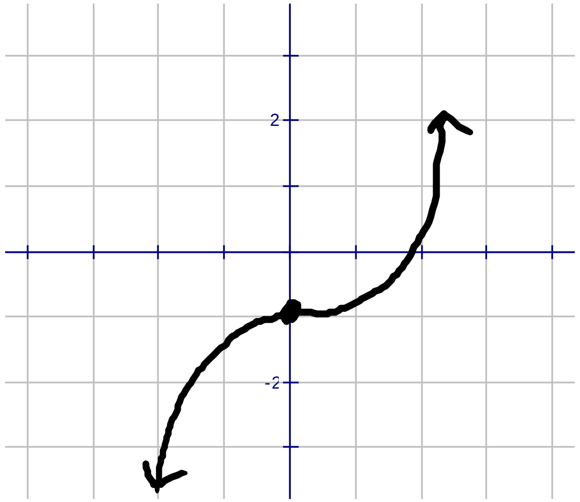
opposite direction

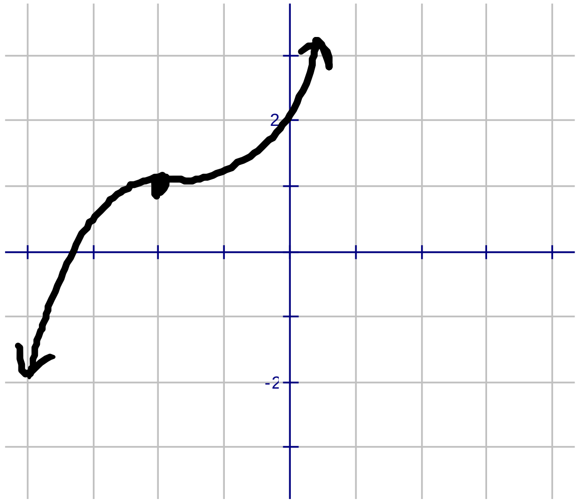
Ex. Use the graph of $f(x) = x^3$ to sketch:



a. $g(x) = x^3 \overbrace{-\,1}^{\text{down 1}}$

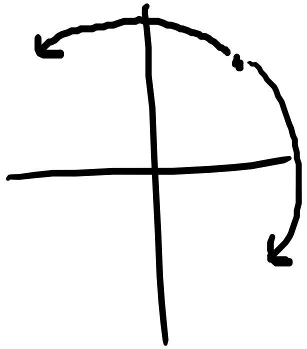
b. $g(x) = (x \overbrace{+\,2}^{\text{left 2}})^3 \overbrace{+\,1}^{\text{up 1}}$





Ex. For part b, use function notation to write $g(x)$ in terms of $f(x)$.

$g(x) = f(x+2) + 1$



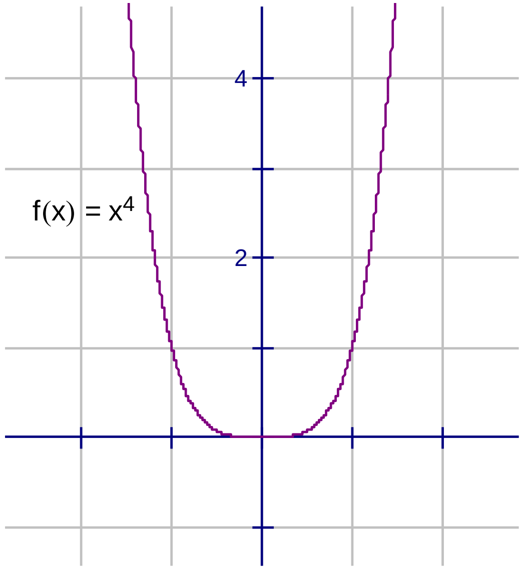
Reflections in the Coordinate Axes

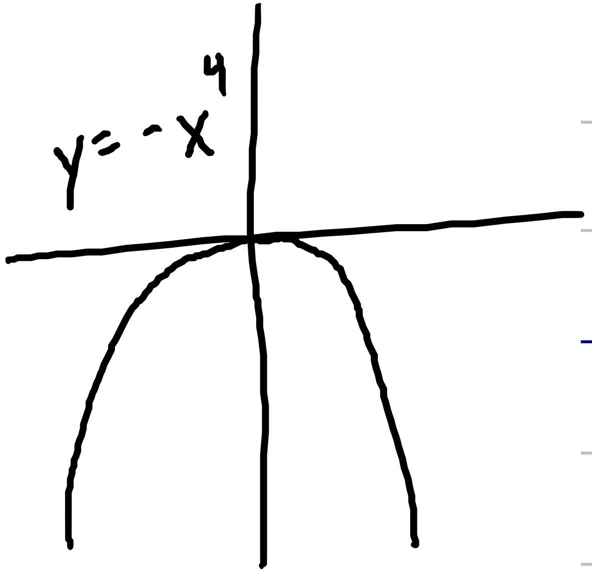
Reflections in the coordinate axes of the graph of $y = f(x)$ are represented as follows.

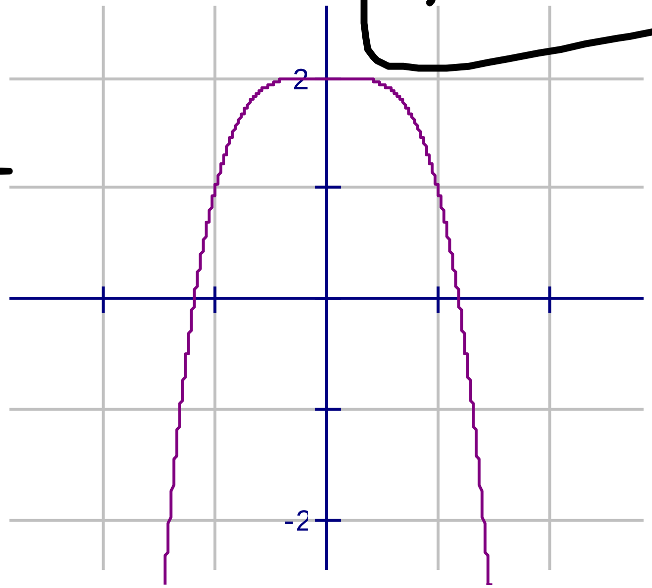
1. Reflection in the x-axis: $h(x) = -f(x)$ → outside = vertical
2. Reflection in the y-axis: $h(x) = f(-x)$ → inside = horizontal

<u>Ex.</u> Given the graph of $y = x^4$ below, identify the equation of the second graph.

$\boxed{y = -x^4 + 2}$

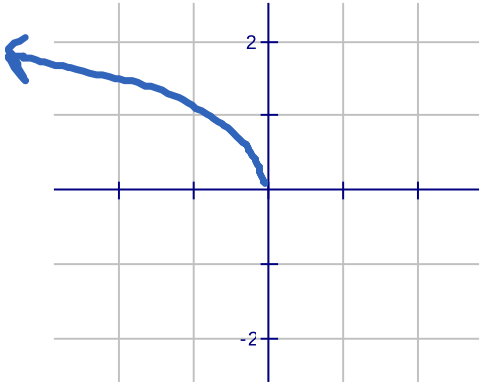


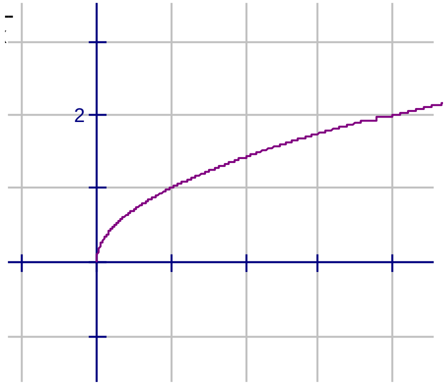




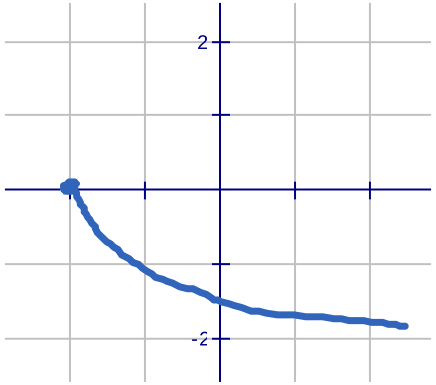
<u>Ex.</u> Use the graph of $f(x) = \sqrt{x}$ below to describe and draw the graph of:

a) $f(x) = \sqrt{\underbrace{-x}_{\text{horiz. flip}}}$





b) $f(x) = \underbrace{-}_{\text{vert. flip}}\sqrt{\underbrace{x+2}_{\text{shift left 2}}}$

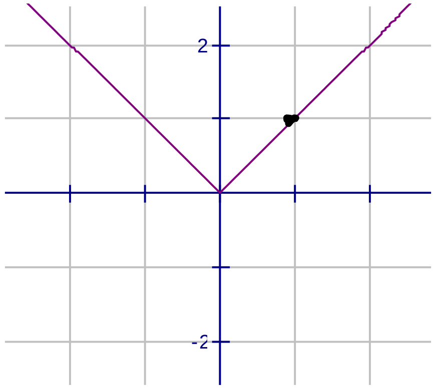


Multiplying by a number will cause a stretch or shrink

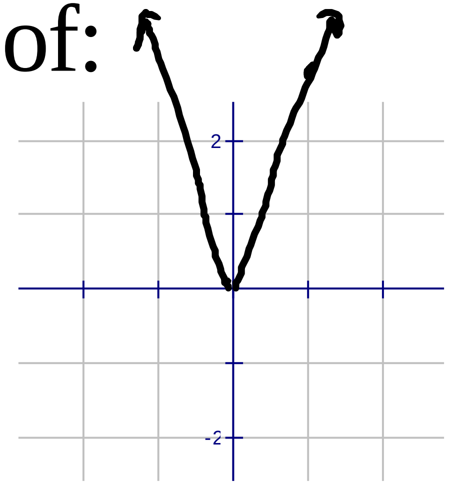
This is called a nonrigid transformation

You could memorize what each does, but it's easier to figure it out by plugging in numbers.

Ex. Use the graph of $f(x) = |x|$ below to draw the graph of:

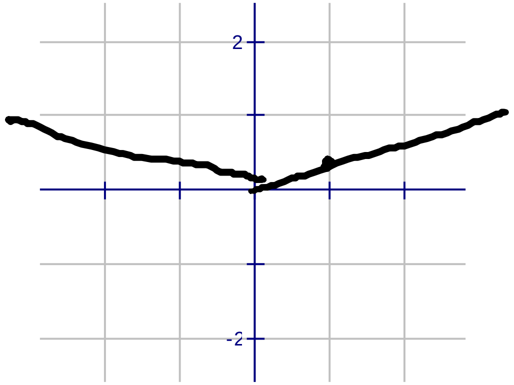


a) $f(x) = 3|x|$



$f(1) = 3|1| = 3$

b) $f(x) = \frac{1}{3}|x|$



$f(1) = \frac{1}{3}|1| = \frac{1}{3}$

Composite Functions

This means combining two functions to get a new function.

Ex. Let $f(x) = 2x - 3$ and $g(x) = x^2 - 1$, find

a) $(f + g)(x) = 2x - 3 + x^2 - 1 = x^2 + 2x - 4$

b) $(fg)(x) = (2x-3)(x^2-1) = 2x^3 - 2x - 3x^2 + 3$

c) $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x-3}{x^2-1}$

<u>Ex.</u> Let $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 1$,

find $(f - g)(2)$. $= f(2) - g(2)$

$= 5 - 7 = \boxed{-2}$

$f(2) = 2(2) + 1 = 5$

$g(2) = 2^2 + 2(2) - 1 = 7$

$$(f \circ g)(x) \quad \text{means} \quad f(g(x))$$

Ex. Given $f(x) = x + 2$ and $g(x) = 4 - x^2$

a) $(f \circ g)(x) = f(\underline{g(x)}) = f(\underline{4 - x^2}) = (4 - x^2) + 2$

$= 6 - x^2$

b) $(g \circ f)(x) = g(f(x)) = g(x + 2) = 4 - (x+2)^2$

<u>Ex.</u> Write $h(x) = \dfrac{1}{(x-2)^2}$ as the composition of two functions.

$$f(g(x)) = \frac{1}{(x-2)^2}$$

$$f(x) = \frac{1}{x^2}$$

$$g(x) = x-2$$

Inverse Functions

Ex. Make t-charts for $f(x) = x + 4$ and $g(x) = x - 4$.

x	$f(x)=x+4$
1	5
2	6
3	7
4	8
5	9

x	$g(x)=x-4$
5	1
6	2
7	3
8	4
9	5

Two functions are <u>inverses</u> if the roles of x and y are switched.

<u>Ex.</u> If $\overset{y}{\cancel{f(x)}} = \sqrt[3]{x+1}$, find $f^{-1}(x)$. f - inverse

$$\left(x\right)^{3} = \left(\sqrt[3]{y+1}\right)^{3}$$

$$x^{3} = y + 1$$

$$-1 \qquad\qquad -1$$

$$x^{3} - 1 = y$$

$$\boxed{f^{-1}(x) = x^{3} - 1}$$

An inverse function doesn't always exist, and you won't always be able to solve for y

Ex. For the previous example, find

$(f^{-1} \circ f)(x)$.

$f(x) = \sqrt[3]{x+1}$

$f^{-1}(x) = x^3 - 1$

$= f^{-1}(f(x))$

$= f^{-1}\left(\sqrt[3]{x+1}\right)$

$= \left(\sqrt[3]{x+1}\right)^3 - 1$

$= x + 1 - 1$

$= x$

For any function, $(f^{-1} \circ f)(x) = x$

<u>Ex.</u> Show that $f(x)=\dfrac{5}{x-2}$ and $g(x)=\dfrac{5}{x}+2$ are inverses.

$\Rightarrow$ Show $f(g(x))=x$

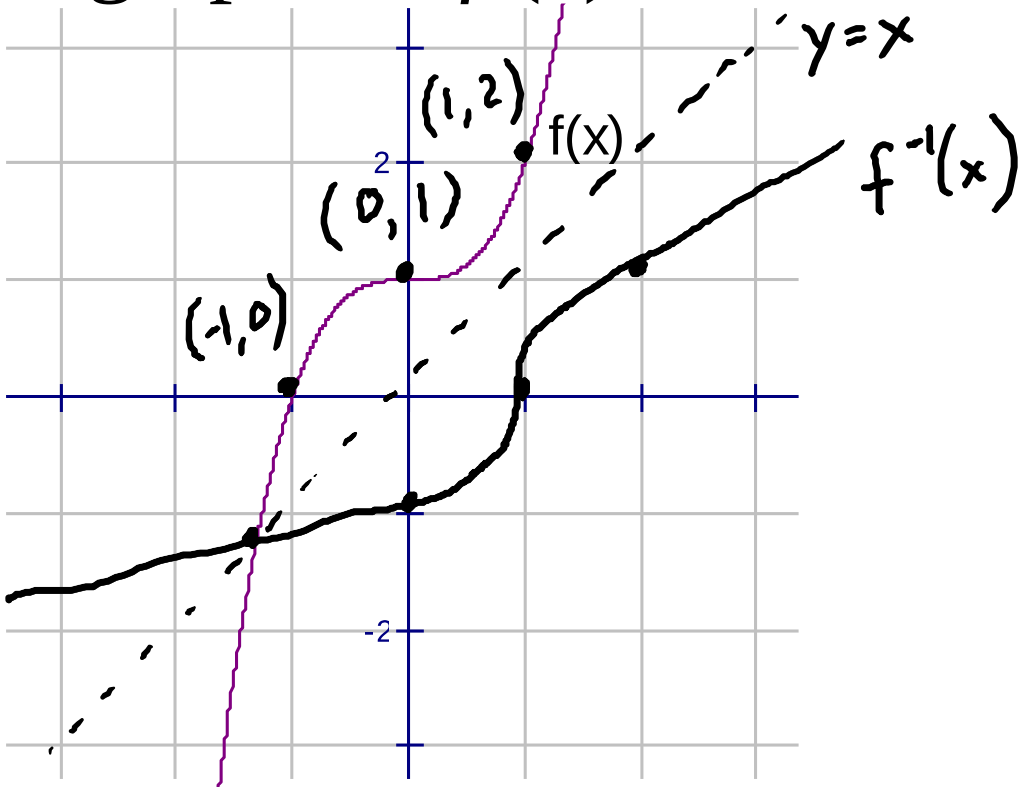
$$f(g(x)) = f\left(\frac{5}{x}+2\right) = \frac{5}{\left(\frac{5}{x}+2\right)-2} = \frac{5}{5/x} = \cancel{5}\cdot\frac{x}{\cancel{5}}$$

$$= x$$

✓

Because the x 's and y 's are switched, the graphs of f and f^{-1} are reflected over the line $y = x$.

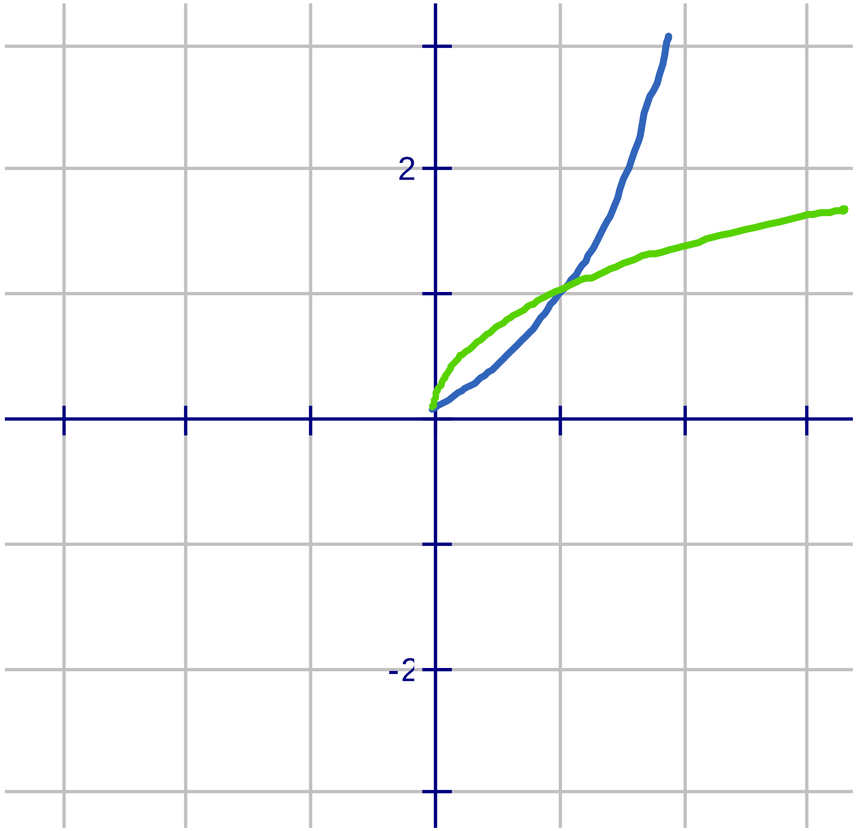
<u>Ex.</u> Given the graph of $f(x)$ below, sketch f^{-1}.



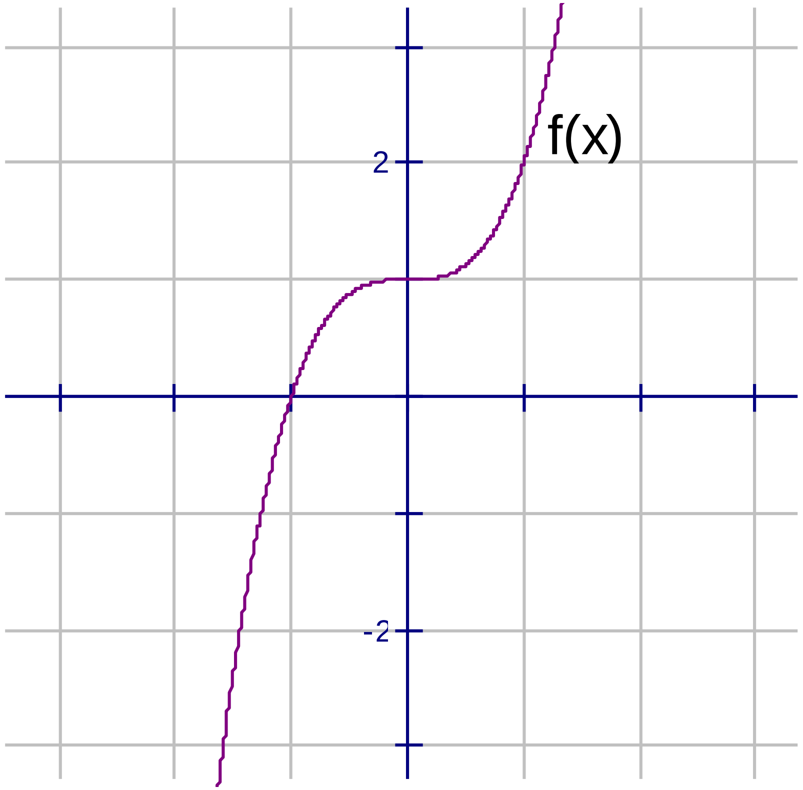
$y = \sqrt{x}$ and $y = x^2$ are not inverses.

$x \geq 0$

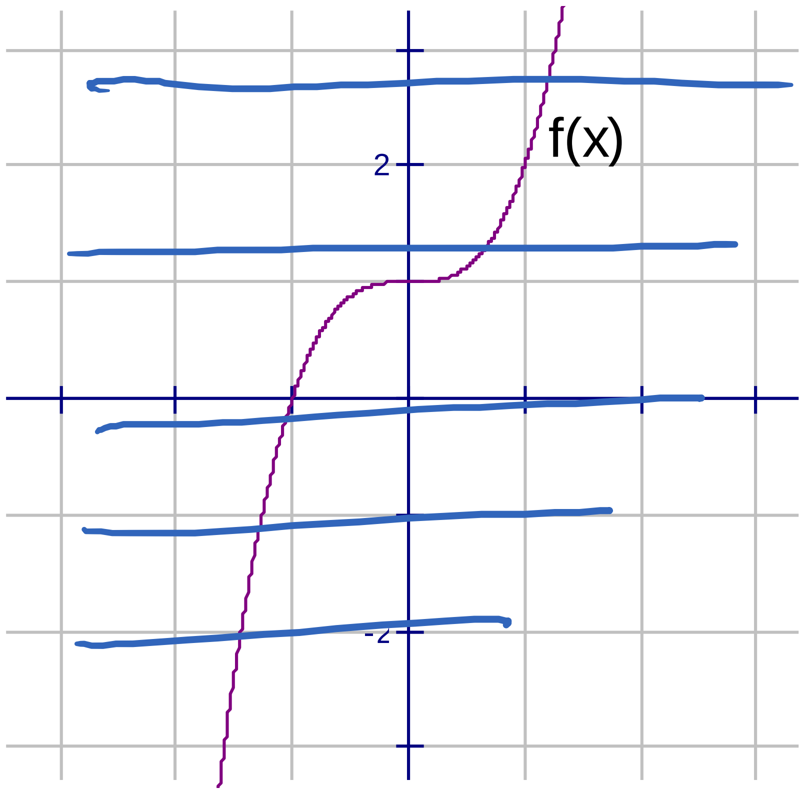
- Consider the graphs



A function is <u>one-to-one</u> if each *y*-coordinate corresponds to exactly one *x*-coordinate



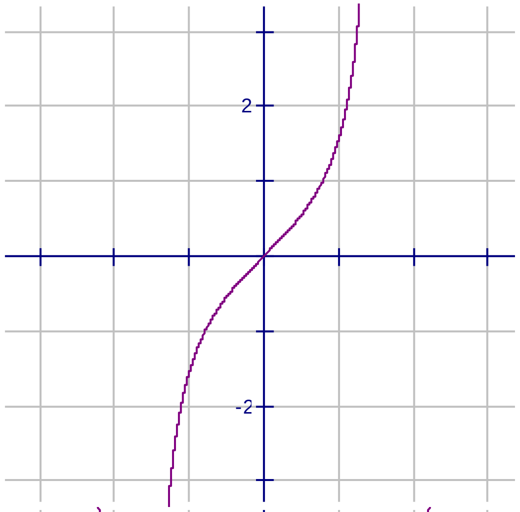
These graphs pass the horizontal line test



A function is invertible (it has an inverse) if it passes the horizontal line test

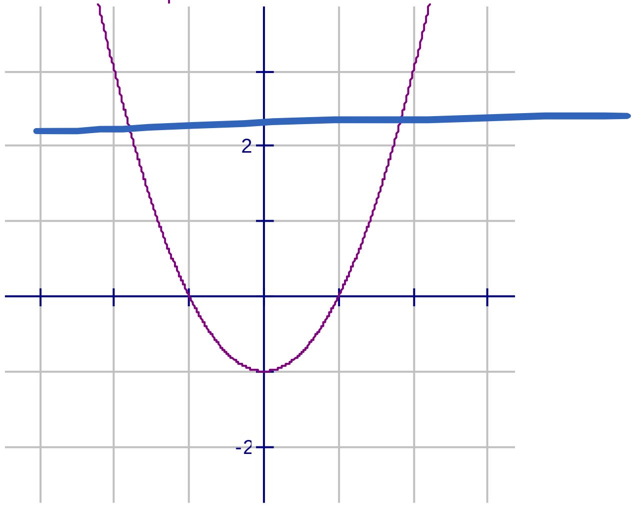
<u>Ex.</u> Are these functions invertible?

a)



yes

b)



no

Even if the function is invertible, you still may not be able to find an equation.