First part is out of 30 pts.
Second part is out of 75 pts.
Total points possible is 105 pts.
→ Grade is out of 100 pts.

Quadratic Functions

In Chapter 3, we will discuss polynomial functions

Def. A polynomial is a function that adds integer powers of *x*, each of which has a constant coefficient. The highest power of *x* is called the degree of the polynomial.

$$f(x) = 5x^9 - \tfrac{1}{3}x^4 + x^3 - 7x + 2$$

There are no roots and we don't divide by *x*.

A polynomial of degree 2 is called a quadratic function.

$$g(x) = x^2 + 6x + 2$$

$$h(x) = \tfrac{1}{4}x^2 - 9$$

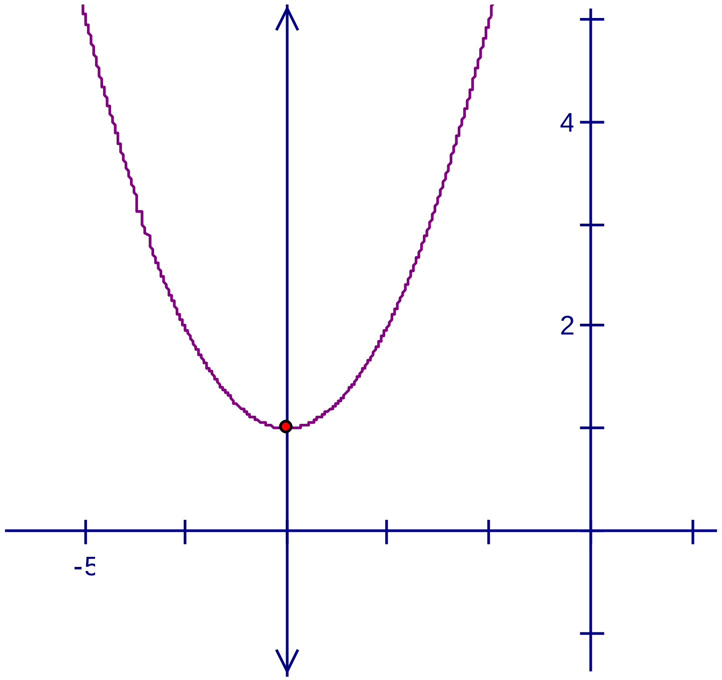
$$j(x) = (x-2)(x-1)$$

The general form of a quadratic function is

$$f(x) = ax^2 + bx + c$$

The graph is called a parabola.

$$f(x) = ax^2 + bx + c$$



Parabolas are symmetric with respect to a vertical line, called the axis of symmetry

This line has the equation

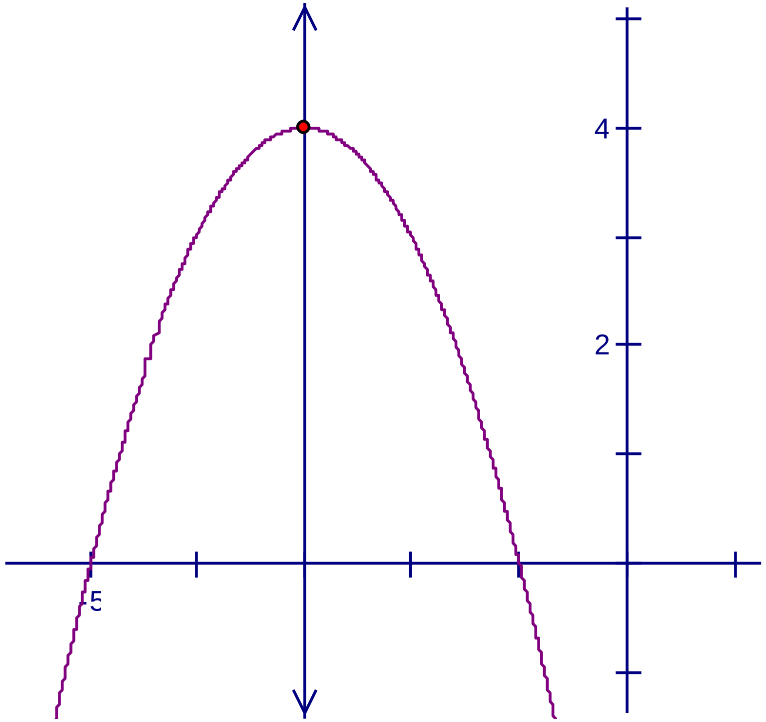
$$x = -\frac{b}{2a}$$

The turning point of a parabola is called the vertex.

Notice that the *x*-coordinate of the vertex is also $x = -\frac{b}{2a}$

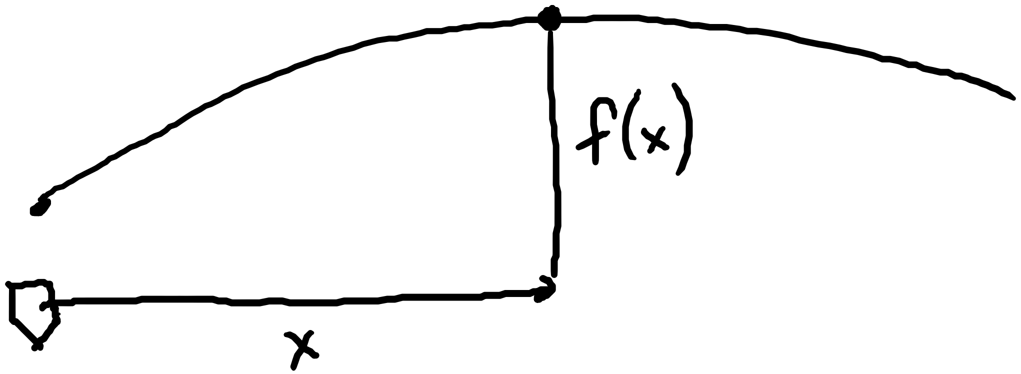
If the lead coefficient, *a*, is positive, the parabola opens upward.

$$f(x) = ax^2 + bx + c$$



If the lead coefficient, a, is negative, the parabola opens downward.

Ex. A baseball is hit so that the path of the ball is given by the function $f(x) = -0.0032x^2 + x + 3$, where $f(x)$ is the height of the ball (in ft.) and x is the horizontal distance from home plate (in ft.). What is the maximum height reached by the baseball?



vertex

$$x = -\frac{b}{2a} = \frac{-1}{2(-.0032)}$$

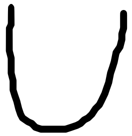
$$x = 156.25$$

$$\text{height} = f(156.25)$$

$$= -.0032(156.25)^2 + 156.25 + 3$$

$$= 81.125 \text{ ft.}$$

Ex. ~~Convert to standard form~~ Find the vertex of $f(x) = 2x^2 + 8x + 7$, and then sketch a graph.

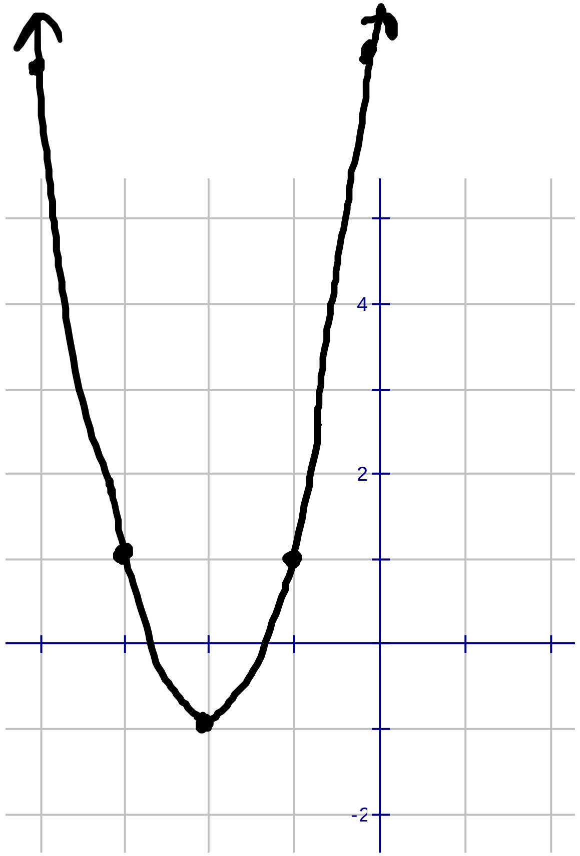


$(-2, -1)$ vertex:

$x = \frac{-b}{2a} = \frac{-8}{2(2)} = \frac{-8}{4} = -2$

$y = 2(-2)^2 + 8(-2) + 7 = 2(4) - 16 + 7 = 8 - 9 = -1$

x	$y = 2x^2 + 8x + 7$
0	$2(0)^2 + 8(0) + 7 = 0 + 0 + 7 = 7$
-1	$2(-1)^2 + 8(-1) + 7 = 2 - 8 + 7 = 1$
-2	-1
-3	1
-4	7



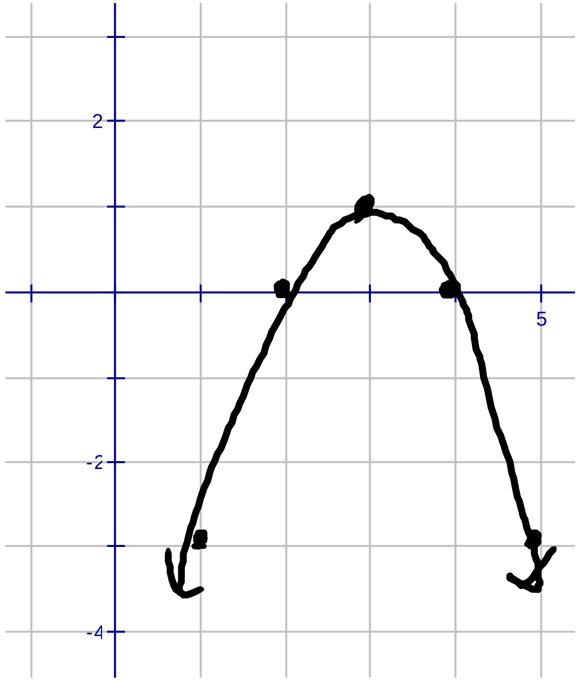
Pract. Find the coordinates of the vertex of $f(x) = -x^2 + 6x - 8$, then sketch the graph.

(3, 1)

vertex: $x = \frac{-b}{2a} = \frac{-6}{2(-1)} = \frac{-6}{-2} = 3$

$y = -(3)^2 + 6(3) - 8 = -9 + 18 - 8 = 1$

x	$y = -x^2 + 6x - 8$
1	$-(1)^2 + 6(1) - 8 = -1 + 6 - 8 = -3$
2	$-(2)^2 + 6(2) - 8 = -4 + 12 - 8 = 0$
3	1
4	0
5	-3



The standard form of a quadratic function is

$$f(x) = a(x-h)^2 + k$$

The vertex of the parabola is the point (h,k).

If $a > 0$, the parabola opens upward.

If $a < 0$, the parabola opens downward.

Ex. Write the equation of the parabola whose vertex is (1,2) and that contains the point (3,5).

h, k

$$y = a(x-h)^2 + k$$

$$y = a(x-1)^2 + 2$$

$$5 = a(3-1)^2 + 2$$

$$5 = a(2)^2 + 2$$

$$-2 \qquad -2$$

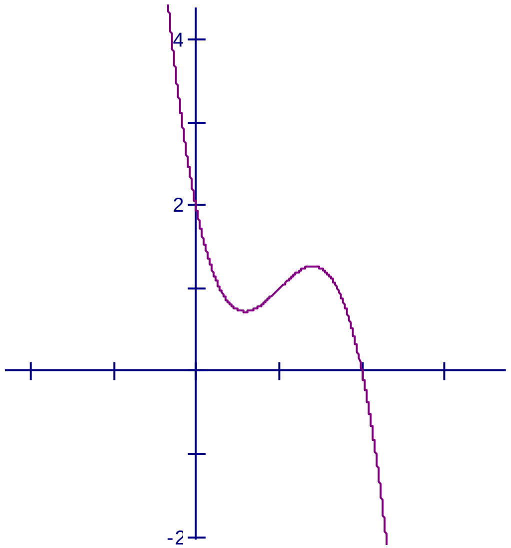
$$\frac{3}{4} = \frac{4a}{4} \rightarrow a = \frac{3}{4}$$

$$y = \frac{3}{4}(x-1)^2 + 2$$

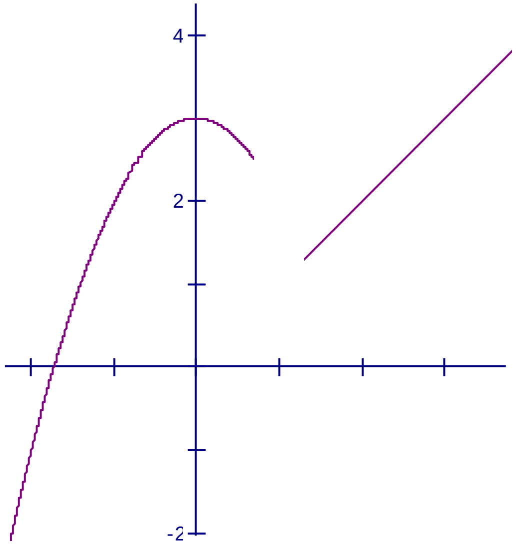
Polynomial Functions

While graphing higher order polynomials will be difficult, there are some things that will help.

All polynomials are continuous (no gaps or jumps).

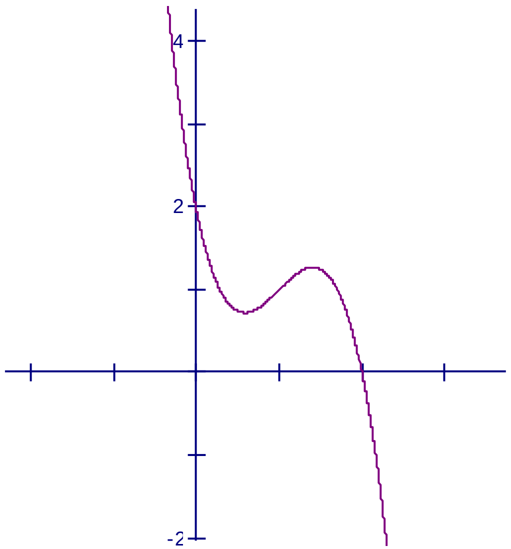


polynomial

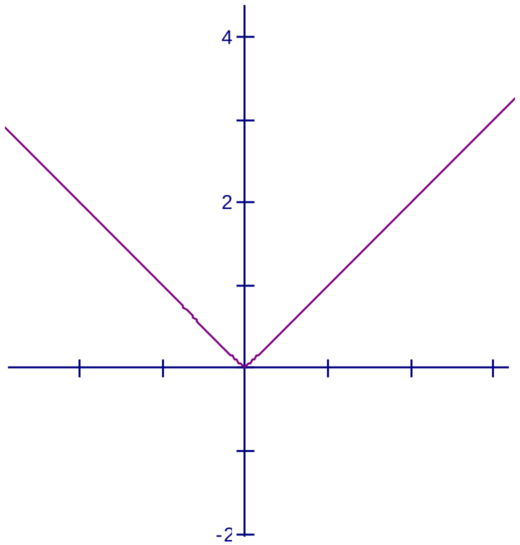


not a polynomial

All polynomials are smooth (no sharp turns).

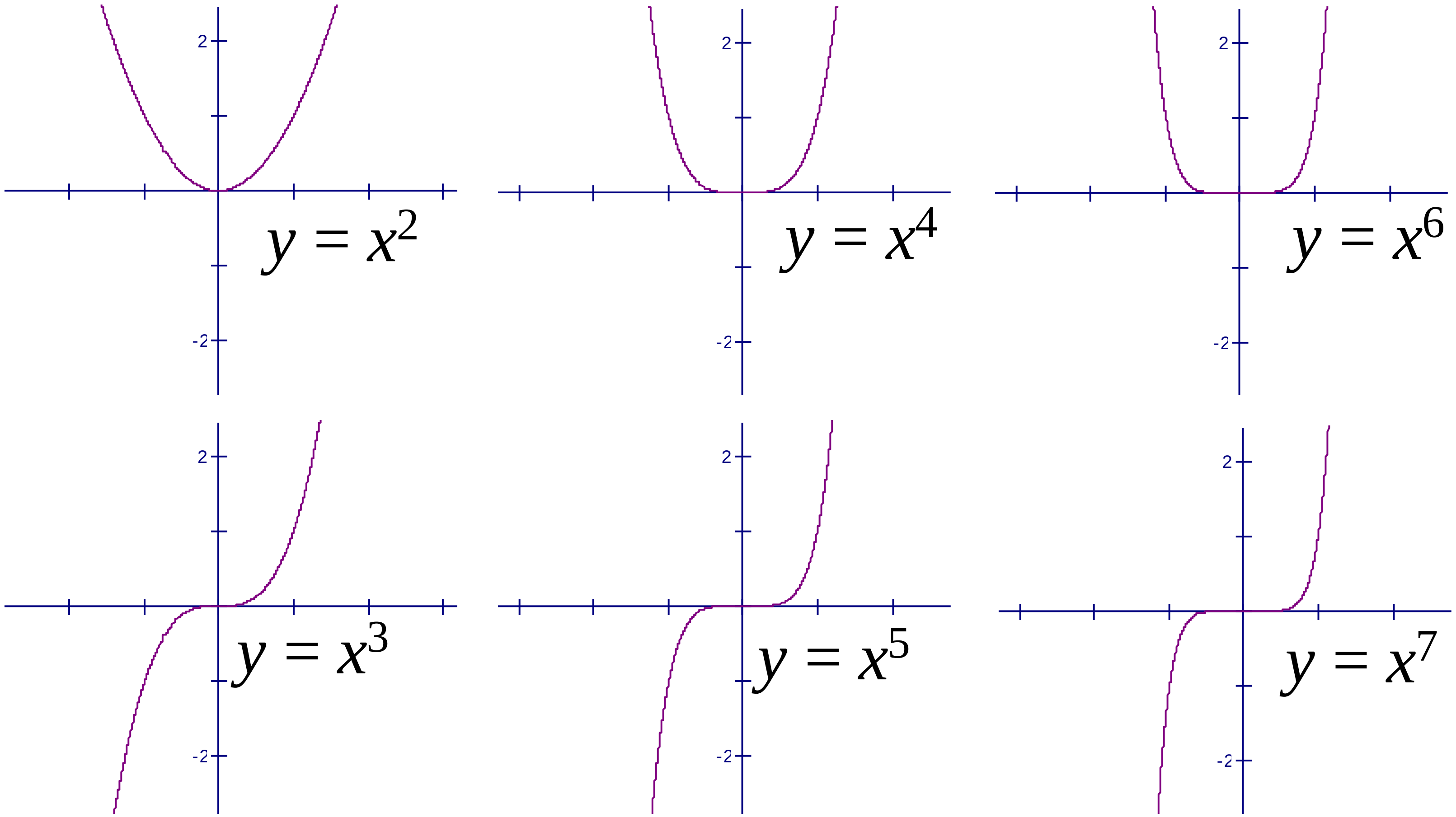


polynomial



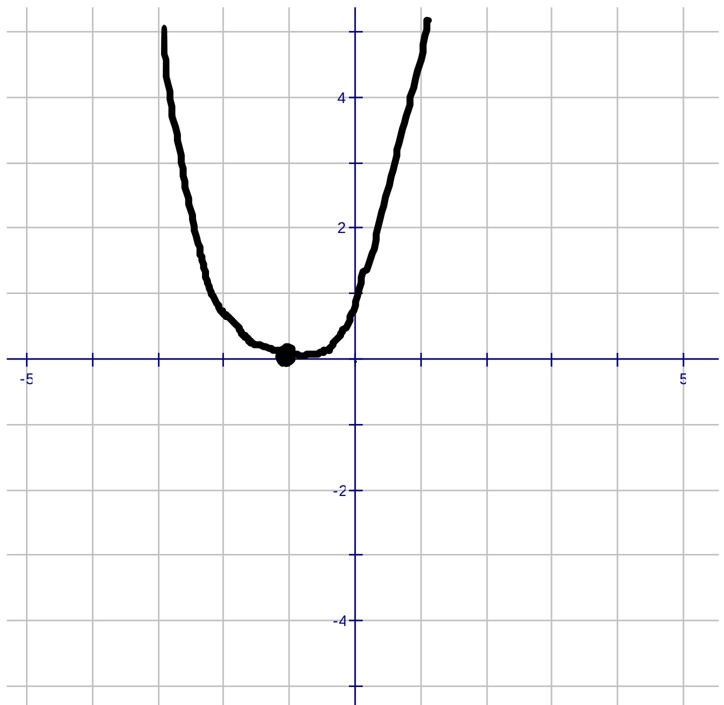
not a polynomial

Some simple polynomials to graph look like $f(x) = x^n$, where n is a positive integer.
These are called <u>power functions</u>.



Ex. Sketch $f(x) = (x + 1)^4$

left 1



The lead coefficient can tell you about the “end behavior” of the graph

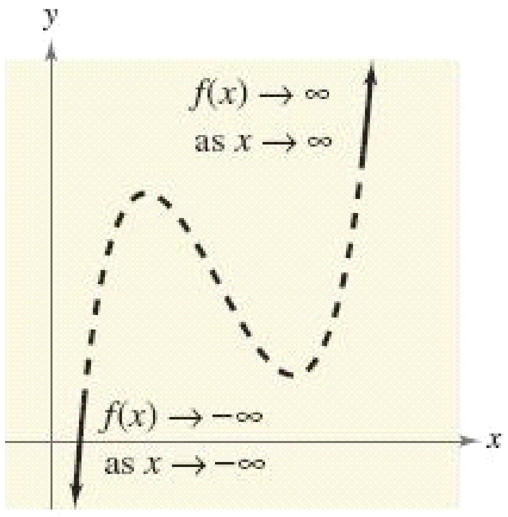
- What happens if the graph keeps going left or right

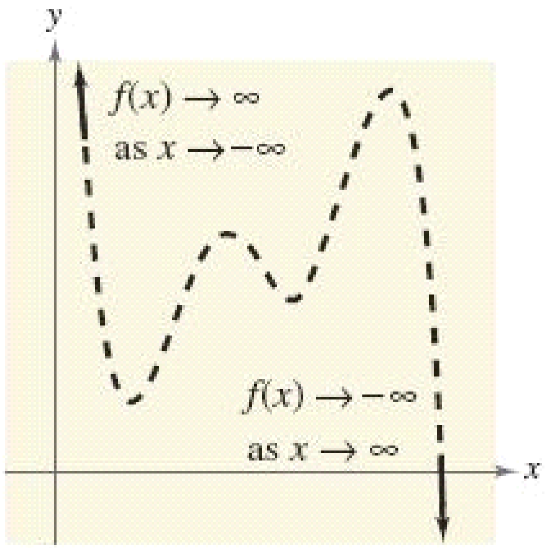
- The degree of the polynomial affects the results

If the degree is odd:

Positive lead coeff.

Negative lead coeff.





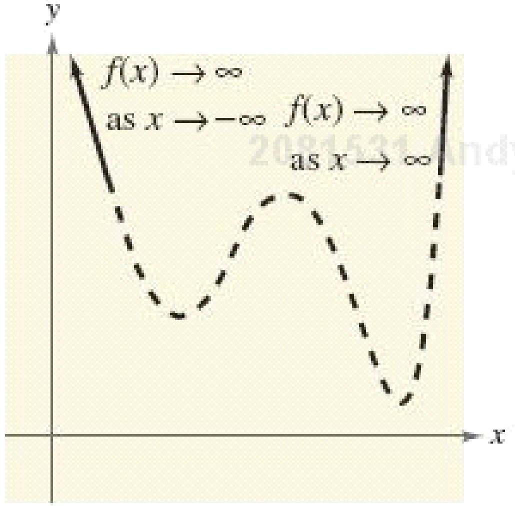
$$f(x) = 2x^3 - 3x^2 + 2x + 2$$

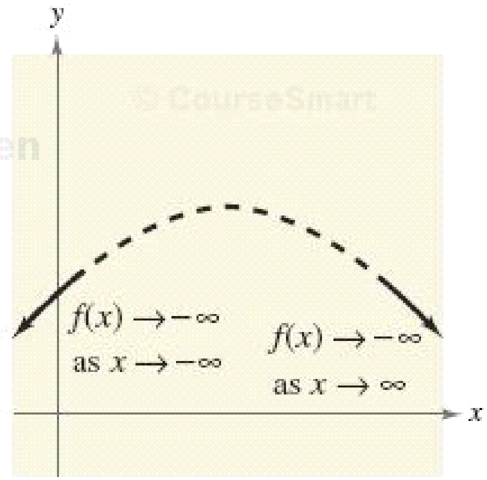
$$f(x) = -3x^5 + 4x^2 + 8$$

If the degree is even:

Positive lead coeff.

Negative lead coeff.





$$f(x) = x^4 + 5x^2 + 3x + 8$$

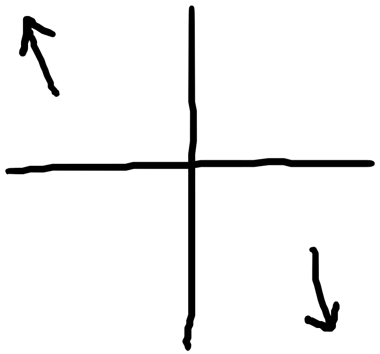
$$f(x) = -2x^2 + 12x - 15$$

Ex. Describe the end behavior of the graph.

a) $f(x) = -x^3 + 4x$

right: As $x \to \infty$, $y \to -(\infty)^3 = -\infty$ down

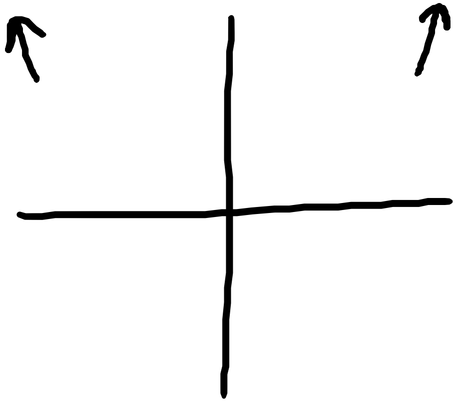
left: As $x \to -\infty$, $y \to -(-\infty)^3 = +\infty$ up



b) $f(x) = -5x + x^4 + 4$

right: As $x \to \infty$, $y \to (\infty)^4 = +\infty$ up

left: As $x \to -\infty$, $y \to (-\infty)^4 = +\infty$ up



- Remember, the zeroes of a function are the values of x that make $f(x) = 0$
- If the degree of a polynomial is n, then there will be at most n zeroes on the graph

Ex. Find the zeroes of $f(x) = -2x^4 + 2x^2 = 0$

$$2x^2(-x^2+1) = 0$$

$$\frac{2x^2}{2} = \frac{0}{2} \qquad -x^2 + 1 = 0$$

$$\qquad\qquad +x^2 \quad +x^2$$

$$\sqrt{x^2} = \sqrt{0} \qquad \sqrt{1} = \sqrt{x^2}$$

$$\boxed{x = 0} \qquad \boxed{x = \pm 1}$$

Let's look at the graph of the function.

Pract. Consider the function $f(x) = 3x^4 - 4x^3$

a) Find the zeroes.

b) Describe the end behavior.

c) Sketch a graph.

$3x^4 - 4x^3 = 0$

$x^3(3x - 4) = 0$

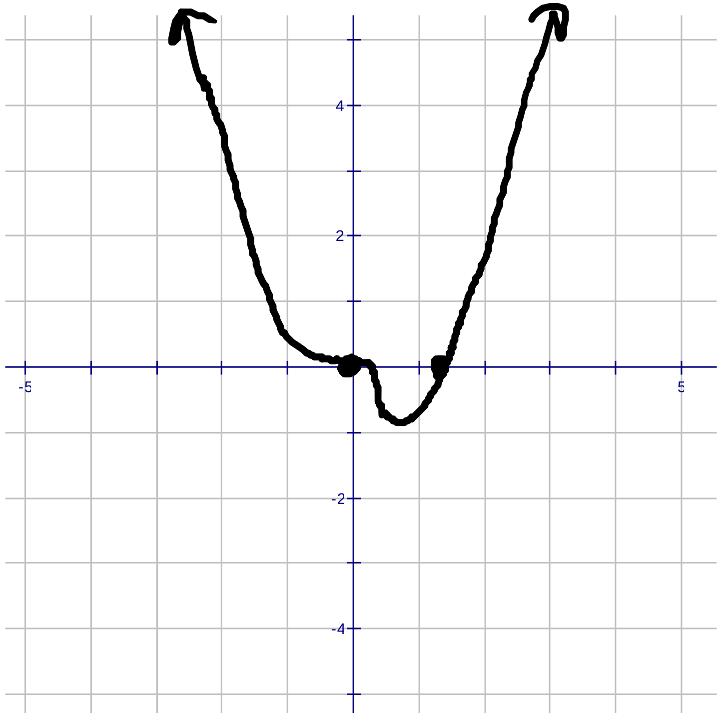
$x^3 = 0$ $3x - 4 = 0$

$\boxed{x = 0}$ $3x = 4$

$\boxed{x = \frac{4}{3}}$

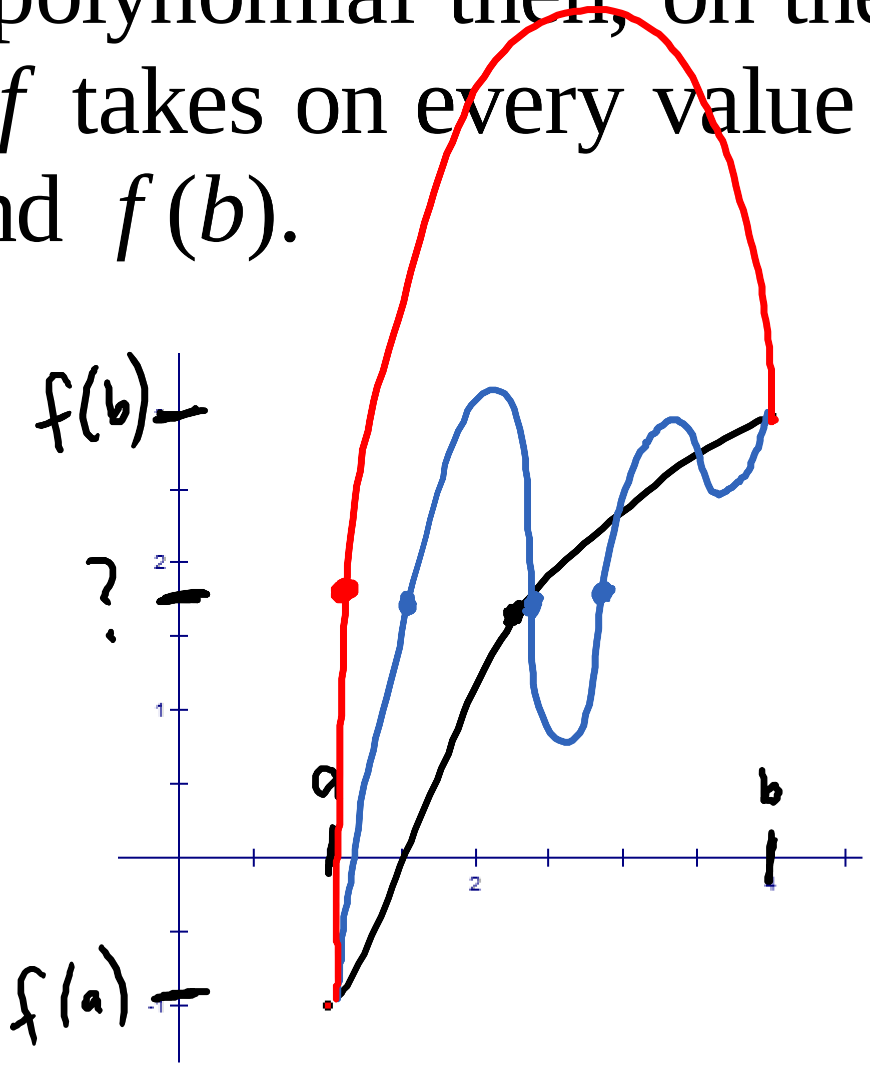
right: as $x \to \infty$, $y \to 3(\infty)^4 = +\infty$ up

left: as $x \to -\infty$, $y \to 3(-\infty)^4 = +\infty$ up



Intermediate Value Theorem

If f is a polynomial then, on the interval $[a,b]$, f takes on every value between $f(a)$ and $f(b)$.



Ex. Use the Intermediate Value Theorem to show that $f(x) = x^3 - x^2 + 1$ has a zero on the interval [-2,0].

$f(-2) = (-2)^3 - (-2)^2 + 1 = -8 - 4 + 1 = -11$

$f(0) = 0^3 - 0^2 + 1 = 1$

$f(-2) < 0$ and $f(0) > 0$,

so $f(x) = 0$ on the interval.