Polynomial Division

Ex. $$\begin{array}{r} 754+\frac{2}{7} \\ 7\overline{)5280} \\ \underline{-49} \\ 38 \\ \underline{-35} \\ 30 \\ \underline{-28} \\ 2 \end{array}$$

Ex. Divide $6x^3 - 19x^2 + 16x - 4$ by $x - 2$, and use the results to factor the polynomial completely.

$$\begin{array}{r} 6x^2 - 7x + 2 \\ x-2 \overline{) 6x^3 - 19x^2 + 16x - 4} \\ -6x^3 + 12x^2 \qquad\qquad\quad \\ \hline -7x^2 + 16x \qquad \\ +7x^2 - 14x \qquad \\ \hline 2x - 4 \\ -2x + 4 \\ \hline 0 \end{array}$$

$f(x) = 6x^3 - 19x^2 + 16x - 4$

$= (x-2)(6x^2 - 7x + 2)$

$= \boxed{(x-2)(3x-2)(2x-1)}$

<u>Ex.</u> Divide $x^2 + 3x + 5$ by $x + 1$.

$$\boxed{x + 2 + \frac{3}{x+1}}$$

$$x+1 \overline{\big) \, x^2 + 3x + 5}$$

$$-x^2 - x$$

$$2x + 5$$

$$-2x - 2$$

$$3$$

Ex. Divide $x^3 - 1$ by $x - 1$.

$$
\begin{array}{r r}
 & \boxed{x^2 + x + 1} \\
x - 1 \,\big) & \overline{x^3 + 0x^2 + 0x - 1} \\
 & -x^2 + x^2 \qquad\qquad\quad \\
\hline
 & x^2 + 0x \qquad\; \\
 & -x^2 + x \qquad\; \\
\hline
 & x - 1 \\
 & -x + 1 \\
\hline
 & 0
\end{array}
$$

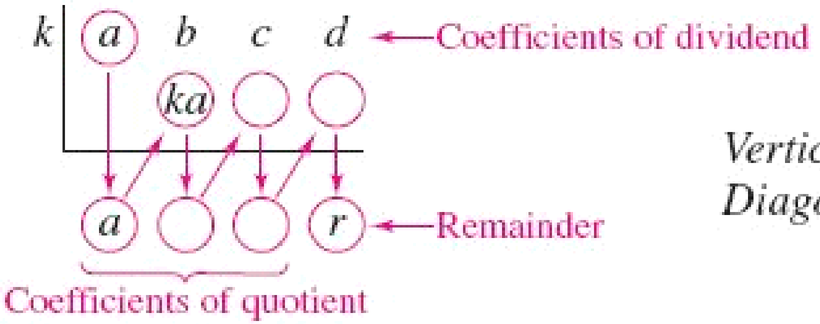
Ex. Divide $2x^4 + 4x^3 - 5x^2 + 3x - 2$ by $x^2 + 2x - 3$.

$$
\begin{array}{r l}
 & \quad 2x^2 \qquad\qquad + 1 \\
x^2 + 2x - 3 \,\big) & 2x^4 + 4x^3 - 5x^2 + 3x - 2 \\
 & -2x^4 - 4x^3 + 6x^2 \\
\hline
 & \qquad\qquad\quad x^2 + 3x - 2 \\
 & \qquad\qquad -x^2 - 2x + 3 \\
\hline
 & \qquad\qquad\qquad\quad x + 1
\end{array}
$$

$$\boxed{2x^2 + 1 + \frac{x+1}{x^2+2x-3}}$$

Synthetic Division (for a Cubic Polynomial)

To divide $ax^3 + bx^2 + cx + d$ by $x - k$, use the following pattern.



Vertical pattern: Add terms.
Diagonal pattern: Multiply by k.

<u>Ex.</u> Divide $x^4 - 10x^2 - 2x + 4$ by $x + 3$.

add

mult.

$$\begin{array}{r|rrrrr}
-3 & 1 & 0 & -10 & -2 & 4 \\
 & & -3 & 9 & 3 & -3 \\
\hline
 & 1 & -3 & -1 & 1 & \boxed{1} \\
 & x^3 & x^2 & x & \text{const.} & \text{remaind.}
\end{array}$$

$$\boxed{1x^3 - 3x^2 - 1x + 1 + \frac{1}{x+3}}$$

Ex. Show that $\boxed{(x - 2)}$ and $(x + 3)$ are factors of $f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$, then find the remaining factors.

2	2	7	−4	−27	−18
		4	22	36	18
	2	11	18	9	0

−3	2	11	18	9
		−6	−15	−9
	2	5	3	0

$$f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$$
$$= (x-2)(2x^3 + 11x^2 + 18x + 9)$$
$$= (x-2)(x+3)(2x^2 + 5x + 3)$$
$$= (x-2)(x+3)(2x+3)(x+1)$$

Zeroes of a Polynomial

Fundamental Theorem of Algebra

If a polynomial has degree n, then it has exactly n zeroes.

- This includes repeating zeroes

- Some of the zeroes may be complex numbers

- If k is a zero, then the polynomial can be divided by $(x - k)$

- We know that the zeroes exist, though we may not be able to find them

a) $f(x) = x - 2$ has one zero, $x = 2$

b) $f(x) = x^2 - 6x + 9$ has two zeroes, $x = 3$ and $x = 3$

c) $f(x) = x^3 + 4x$ has three zeroes, $x = 0$, $x = 2i$, and $x = -2i$

d) $f(x) = x^4 - 1$ has four zeroes, $x = 1$, $x = -1$, $x = i$, and $x = -i$

To help us find zeroes, we can use:

Rational Zero Test

If a polynomial has integer coefficients, every rational zero has the form

$$\frac{p}{q}$$

where p = factor of constant coefficient
q = factor of lead coefficient

Ex. Factor $f(x) = x^4 - x^3 + x^2 - 3x - 6.$

$\frac{3}{1} \quad \frac{2}{1} \quad \frac{6}{1} \quad \frac{1}{1}$

$\boxed{\cancel{1}, 2, 3, 6, -1, -2, -3, -6}$

$f(1) = 1^4 - 1^3 + 1^2 - 3(1) - 6 = 1 - 1 + 1 - 3 - 6 = -\cancel{8}$

$f(-1) = (-1)^4 - (-1)^3 + (-1)^2 - 3(-1) - 6 = 1 + 1 + 1 + 3 - 6 = 0$ ✓

$\to x = -1$ is a zero

$\to (x+1)$ is a factor

-1	1	-1	1	-3	-6
		-1	2	-3	6
	1	-2	3	-6	0

$f(2) = 2^4 - 2^3 + 2^2 - 3(2) - 6 = 16 - 8 + 4 - 6 - 6 = 0$ ✓

$\to x = 2$ is a zero

$\to (x-2)$ is a factor

2	1	-2	3	-6
		2	0	6
	1	0	3	0

$f(x) = (x+1)(x^3 - 2x^2 + 3x - 6)$

$= (x+1)(x-2)(x^2+3)$

Ex. Find all real solutions of $f(x) = \boxed{2x^3 + 3x^2 - 8x + 3} = 0$

$$\frac{3}{\;} \quad \frac{1}{\;} \quad \frac{\;}{2} \quad \frac{\;}{1}$$

$$\boxed{\frac{3}{2},\ 3,\ \frac{1}{2},\ 1,\ -\frac{3}{2},\ -3,\ -\frac{1}{2},\ -1}$$

$f(1) = 2(1)^3 + 3(1)^2 - 8(1) + 3 = 2 + 3 - 8 + 3 = 0$

→ $x = 1$ is a zero

→ $(x-1)$ is a factor

1	2	3	-8	3
		2	5	-3
	2	5	-3	$\boxed{0}$

$2x^3 + 3x^2 - 8x + 3 = 0$

$(x-1)(2x^2 + 5x - 3) = 0$

$(x-1)(2x-1)(x+3) = 0$

$\boxed{x = 1}$ $\quad$ $\boxed{x = \frac{1}{2}}$ $\quad$ $\boxed{x = -3}$

If a complex number is a zero of a polynomial, then so is the conjugate.

Ex. Find a fourth-degree polynomial that has 1, -1, and 3*i* $\longrightarrow -3i$ as zeroes.

zeroes		factors
1	$\longrightarrow$	$x-1$
-1	$\longrightarrow$	$x+1$
$3i$	$\longrightarrow$	$x-3i$
$-3i$	$\longrightarrow$	$x+3i$

$$f(x)=\underbrace{(x-1)(x+1)}\underbrace{(x-3i)(x+3i)}$$

$$=(x^2-1)\quad(x^2-9i^2)$$

$$=(x^2-1)(x^2+9)$$

$$=\boxed{x^4+8x^2-9}$$

Ex. Find all zeroes of $f(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$ given that $1 + 3i$ is a zero.

Mathematical Modeling

y is directly proportional to x if $y = kx$ for some constant k

k is called the constant of proportionality

We also say varies directly

Ex. The state income tax is directly proportional to gross income. If the tax is \$46.05 for an income of \$1500, write a mathematical model for income tax.

i : income

t : tax

$t = 46.05 \rightarrow i = 1500$

$t = k\,i$

$$\frac{46.05}{1500} = \frac{k(\cancel{1500})}{\cancel{1500}}$$

$k = .0307$

$$\boxed{t = .0307\,i}$$

y is directly proportional to the n^{th} power to x if $y = kx^n$ for some constant k

Ex. The distance a ball rolls down a hill is directly proportional to the square of the time it rolls. During the first second, the ball rolls 8 ft. Write an equation for this model. How long does it take for the ball to roll 72 ft?

$$\begin{cases} t = 1 \\ d = 8 \end{cases}$$

d = distance

t = time

$$d = kt^2$$

$$8 = k(1)^2$$

$$8 = k$$

$$\boxed{d = 8t^2}$$

$$\frac{72}{8} = \frac{8t^2}{8}$$

$$\sqrt{9} = \sqrt{t^2}$$

$$\boxed{t = 3}$$

y is <u>inversely proportional</u> to x if $y = \frac{k}{x}$ for some constant k

We also say <u>varies inversely</u>

z is <u>jointly proportional</u> to x and y if $z = kxy$ for some constant k

We also say <u>varies jointly</u>

Ex. The simple interest for a savings account is jointly proportional to the time and the principal. After one quarter, the interest on a principal of \$5000 is \$43.75. Write an equation for this model.

$i = ktp$

$43.75 = k\left(\frac{1}{4}\right)(5000)$

$\frac{43.75}{1250} = \frac{k(\cancel{1250})}{\cancel{1250}}$

$k = .035$

$$\boxed{i = .035tp}$$

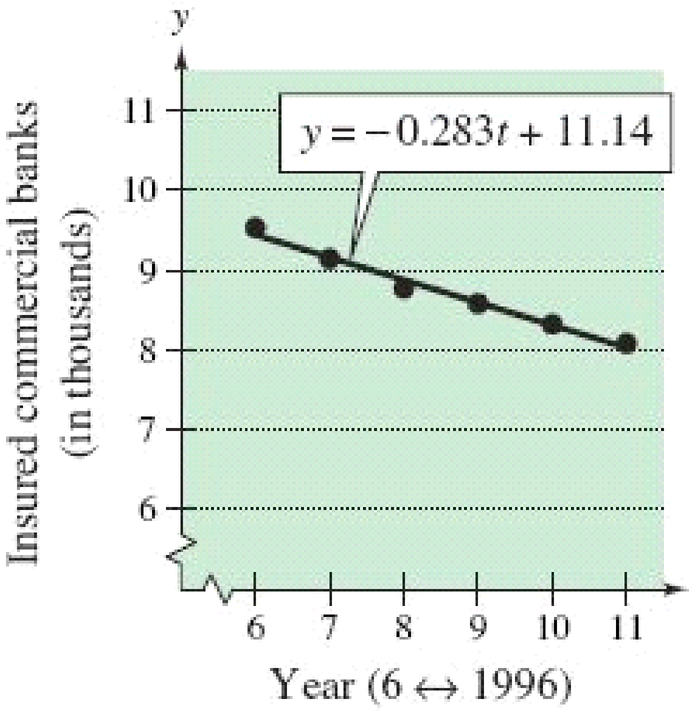
$t = \frac{1}{4}$

$p = 5000$

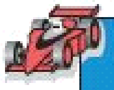
$i = 43.75$

In the real world, you will have data points and will want to find a function to describe the situation.

Year	Insured commercial banks, y
1996	9.53
1997	9.14
1998	8.77
1999	8.58
2000	8.32
2001	8.08



Ex. Use your calculator to find a linear regression for the data below.



Year	Prize money, p
1995	8.06
1996	8.11
1997	8.61
1998	8.72
1999	9.05
2000	9.48
2001	9.61
2002	10.03
2003	10.15
2004	10.25