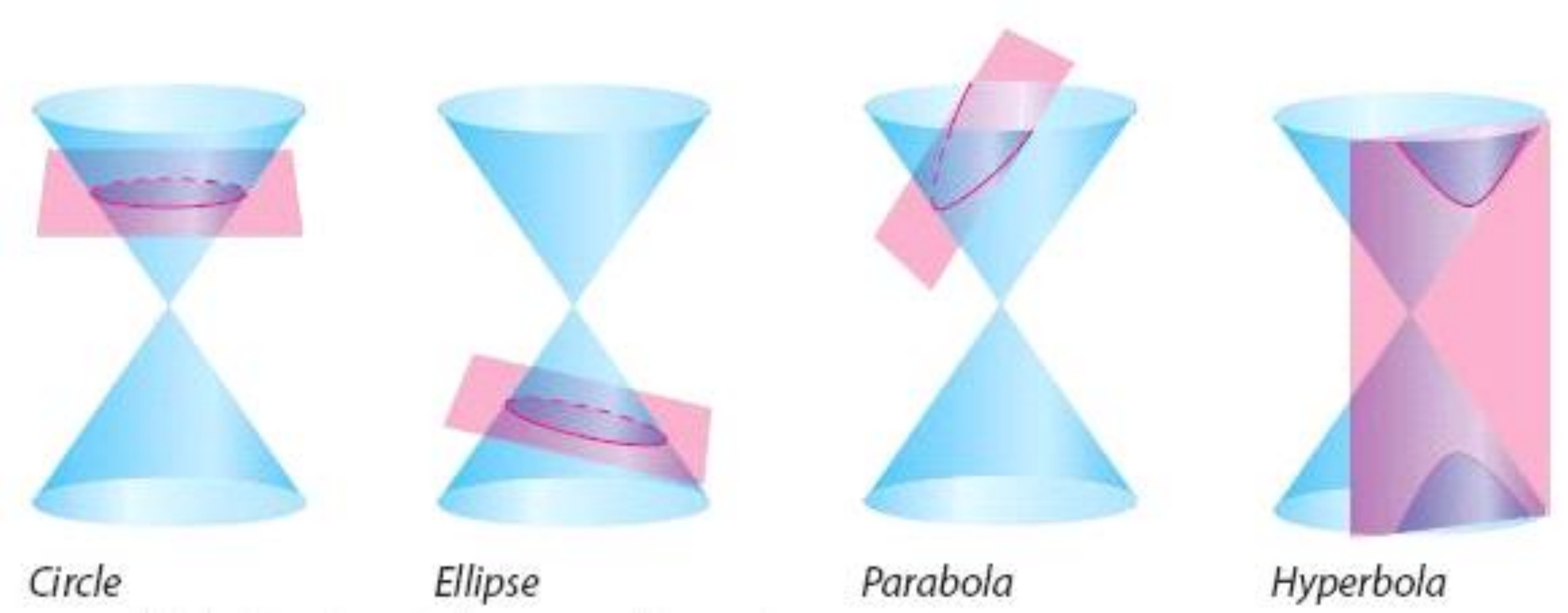
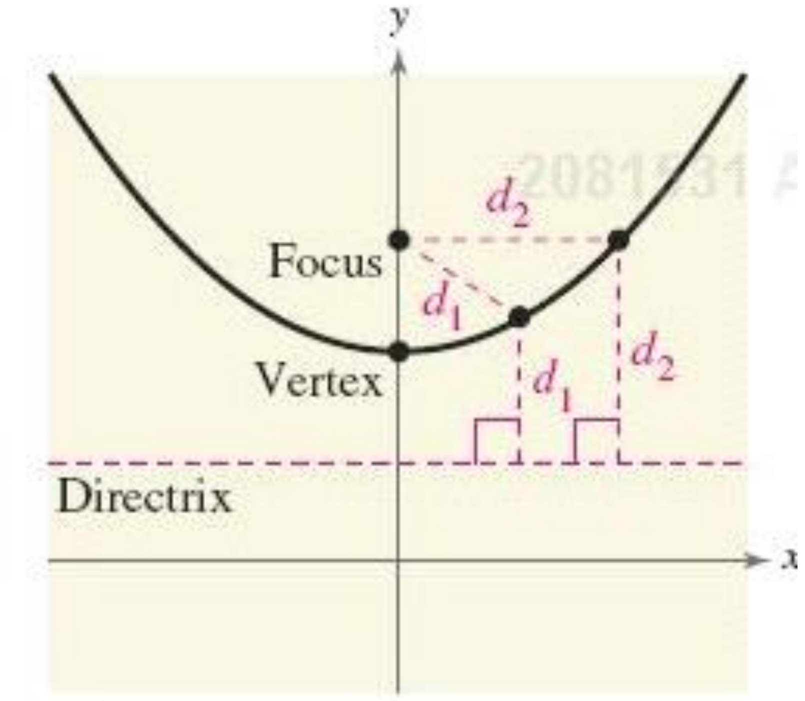
Conics

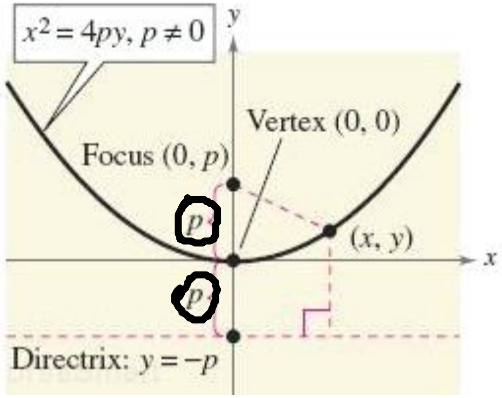
A conic section is a graph that results from the intersection of a plane and a double cone.



A parabola is the set of all points that are equidistant from a line (the directrix) and a point (the focus). The vertex is the midpoint between the focus and directrix, and the axis is the line through focus and vertex.



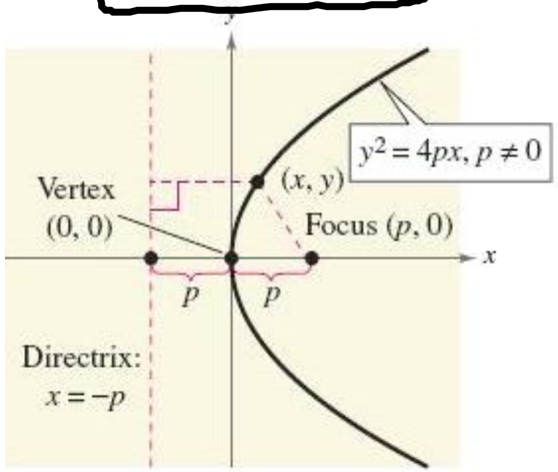
A parabola with vertex (0,0) and directrix $y = -p$ has the equation $\boxed{x^2 = 4py}$



1) p is dist. from vertex to focus/directrix.

2) variable that's not squared is direction parabola opens

A parabola with vertex (0,0) and directrix $x = -p$ has the equation $\boxed{y^2 = 4px}$



point line

Ex. Find the focus and directrix of the parabola $y = -2x^2$.

up/down → y not squared

down → coeff. is neg.

$x^2 = 4py$

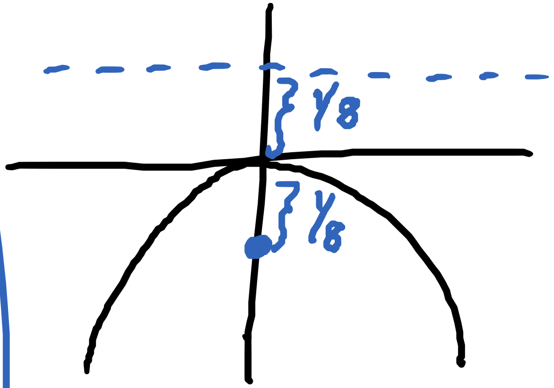
$x^2 = -\frac{1}{2}y$

$4p\cancel{y} = -\frac{1}{2}\cancel{y}$

$\frac{4p}{4} = \frac{-\frac{1}{2}}{4}$

$p = -\frac{1}{8}$

$\boxed{\begin{array}{c}\underline{\text{direct.}} \\ y = \frac{1}{8} \\ \\ \underline{\text{focus}} \\ (0, -\frac{1}{8})\end{array}}$

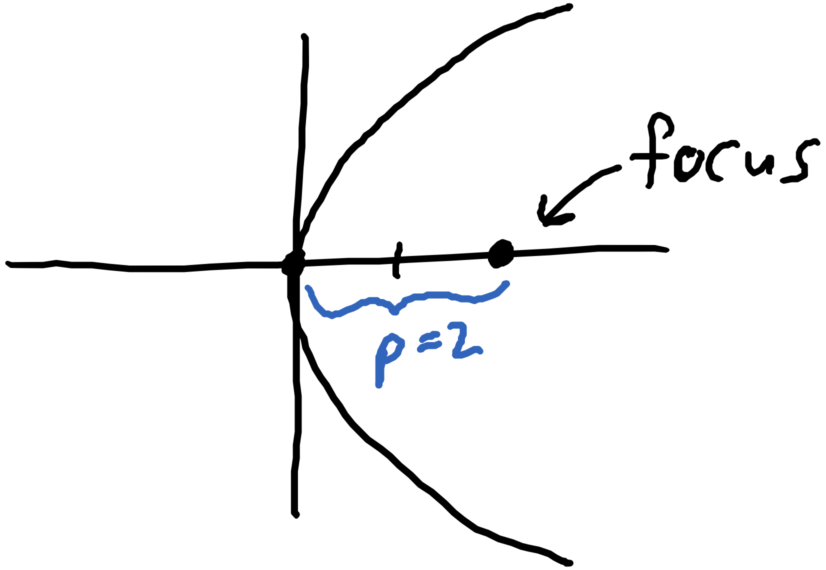


Ex. Find the equation of the parabola with vertex at the origin and focus at (2,0).

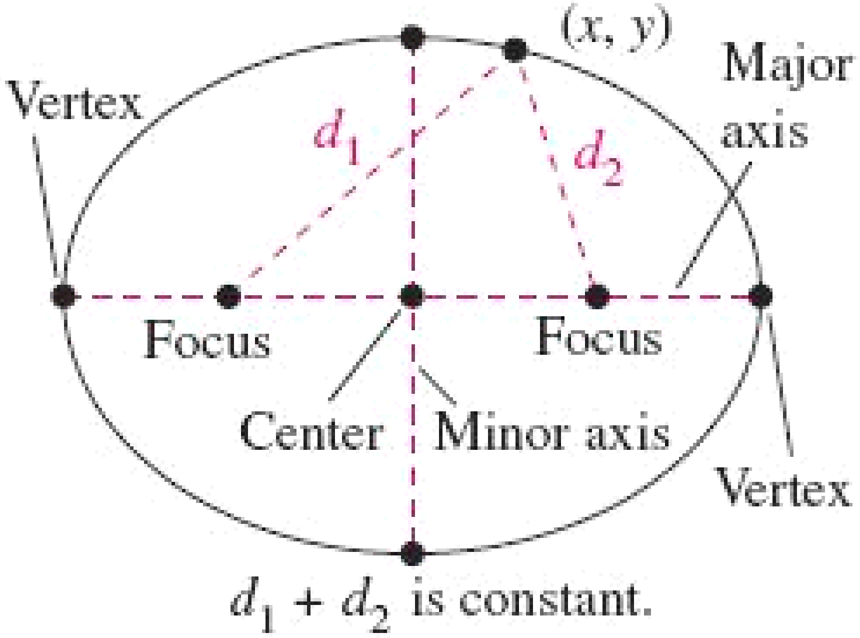
$$y^2 = 4px$$

$$y^2 = 4(2)x$$

$$\boxed{y^2 = 8x}$$



An ellipse is the set of all points, the sum of whose distances from two points (foci) is constant. The major axis goes through the foci, the minor axis is perpendicular to the major axis at the center.



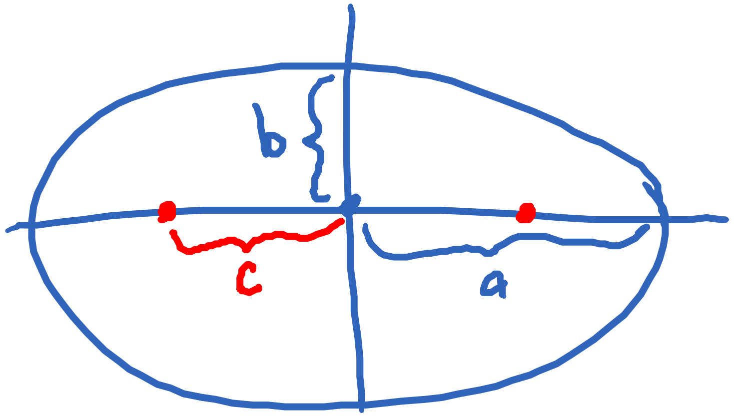
An ellipse centered at (0,0) with horizontal axis length 2a and vertical axis length 2b has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

a: horiz. "radius"
b = vert. "radius"

The vertices and foci lie on the major (longer) axis. The foci lie c units from the center, where $c^2 = a^2 - b^2$.

you will need to switch these if b > a.



Ex. Sketch a graph of the ellipse $4x^2 + y^2 = 36$, and identify the vertices and foci.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{4x^2}{36} + \frac{y^2}{36} = \frac{36}{36}$$

$$\frac{x^2}{9} + \frac{y^2}{36} = 1$$

a^2 b^2

$a = 3$

$b = 6$

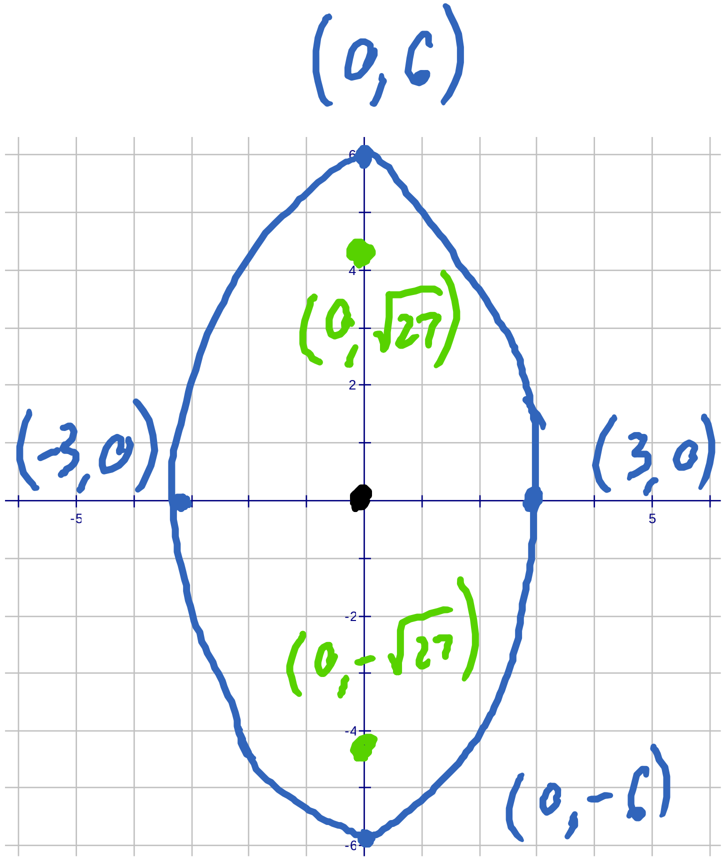
$c^2 = b^2 - a^2$

$c^2 = 6^2 - 3^2$

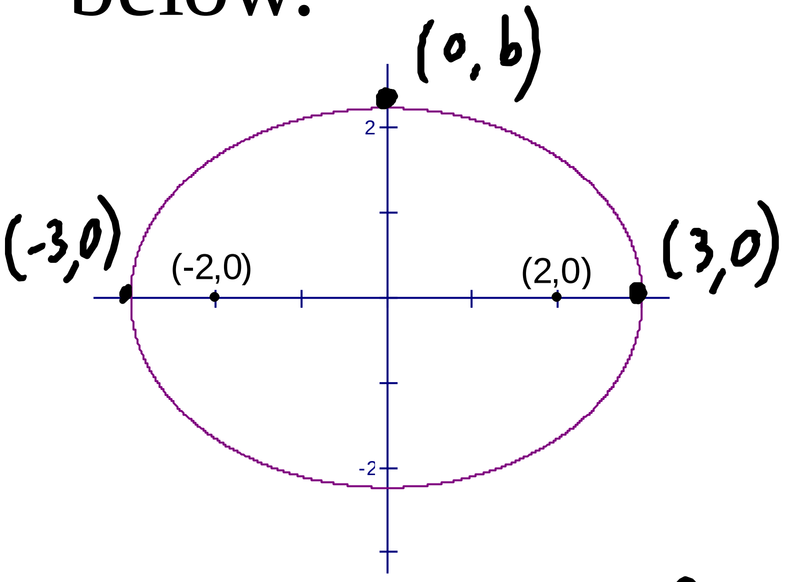
$c^2 = 36 - 9$

$c^2 = 27$

$c = \sqrt{27}$



Ex. Find the equation of the ellipse shown below.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a = 3$

$c = 2$

$b = ?$

$c^2 = a^2 - b^2$

$2^2 = 3^2 - b^2$

$4 = 9 - b^2$

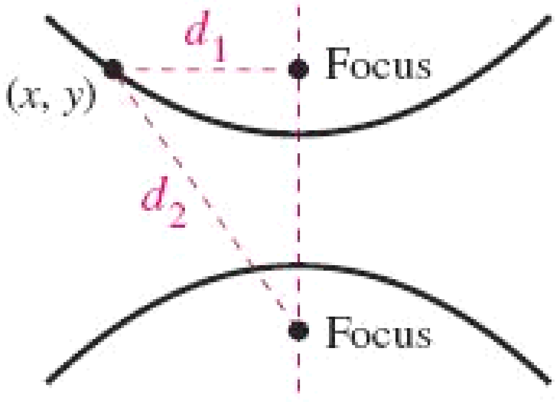
$-5 = -b^2$

$\sqrt{5} = \sqrt{b^2} \longrightarrow b = \sqrt{5}$

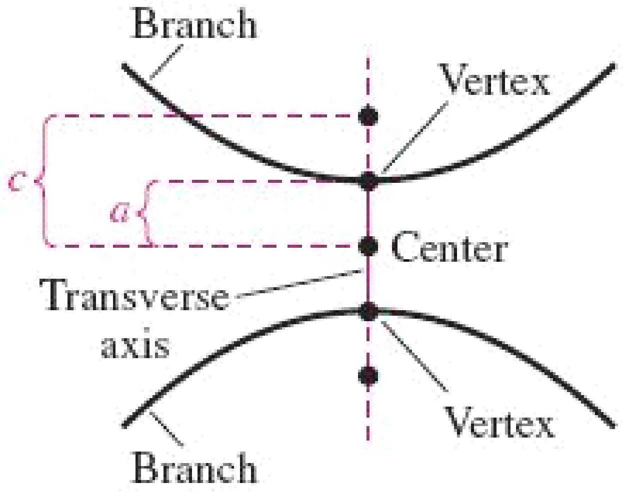
$$\boxed{\frac{x^2}{3^2} + \frac{y^2}{(\sqrt{5})^2} = 1}$$

$$\boxed{\frac{x^2}{9} + \frac{y^2}{5} = 1}$$

A hyperbola is the set of all points, the difference of whose distances from two points (foci) is constant. The transverse axis is the line connecting the vertices, and the midpoint of the transverse axis is the center.



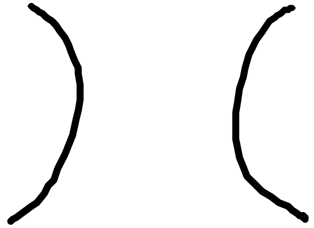
$d_2 - d_1$ is a positive constant.



A hyperbola centered at (0,0) with a horizontal transverse axis has equation

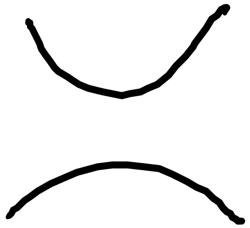
opens in direction of pos. variable

$$\boxed{\frac{x^2}{a^2}-\frac{y^2}{b^2}=1}$$



A hyperbola centered at (0,0) with a vertical transverse axis has equation

$$\boxed{\frac{y^2}{b^2}-\frac{x^2}{a^2}=1}$$



The foci lie c units from the center, where $c^2 = a^2 + b^2$.

Ex. Sketch a graph of the hyperbola $4x^2 - y^2 = 16$, and identify the foci.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{4x^2}{16} - \frac{y^2}{16} = \frac{16}{16}$$

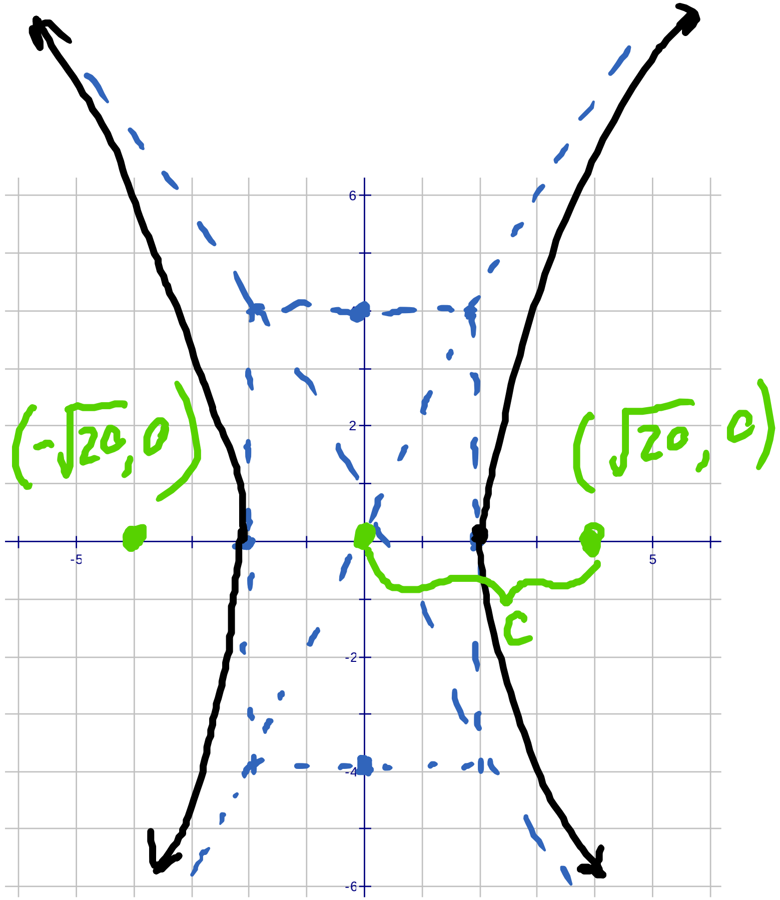
$$\frac{x^2}{4} - \frac{y^2}{16} = 1$$

$a^2 \nearrow 4$, $b^2 \nearrow 16$

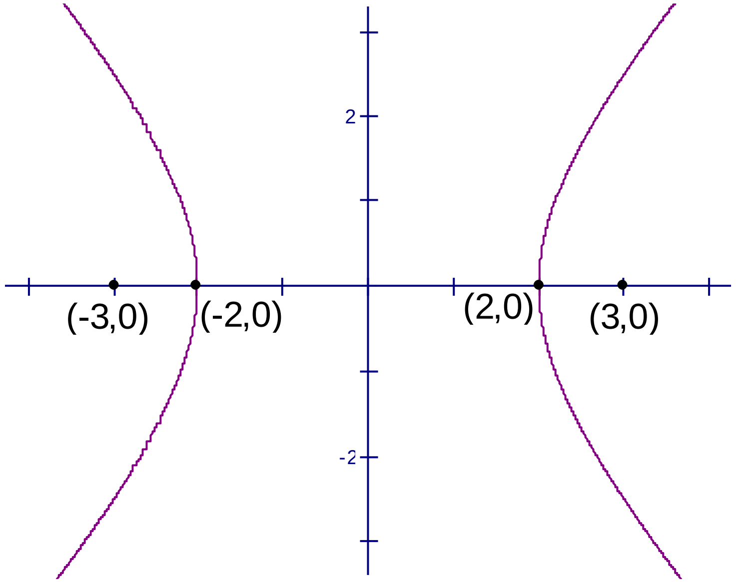
$a = 2$

$b = 4$

$$c^2 = a^2 + b^2$$
$$c^2 = 2^2 + 4^2$$
$$c^2 = 4 + 16$$
$$c^2 = 20$$
$$c = \sqrt{20}$$



Ex. Find the equation of the hyperbola shown below.



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$a = 2$

$c = 3$

$b = ?$

$c^2 = a^2 + b^2$

$3^2 = 2^2 + b^2$

$9 = 4 + b^2$

$5 = b^2$

$b = \sqrt{5}$

$$\boxed{\frac{x^2}{2^2} - \frac{y^2}{(\sqrt{5})^2} = 1}$$

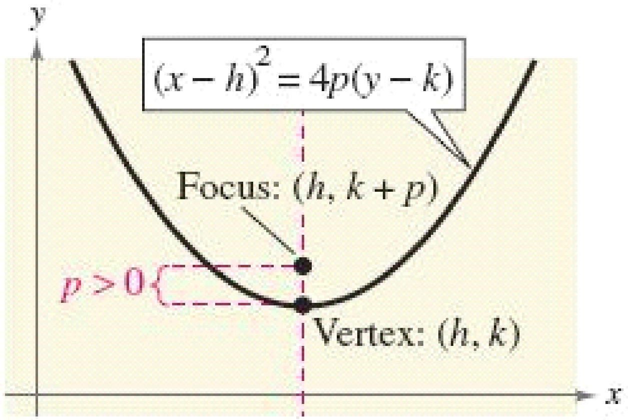
$$\boxed{\frac{x^2}{4} - \frac{y^2}{5} = 1}$$

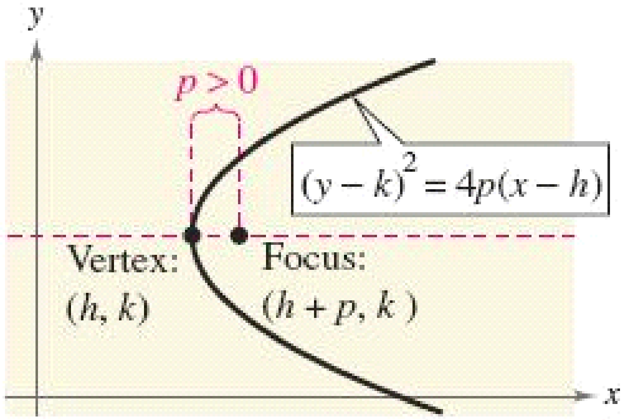
Shifted Conics

In the last section, all conics were centered at the origin.

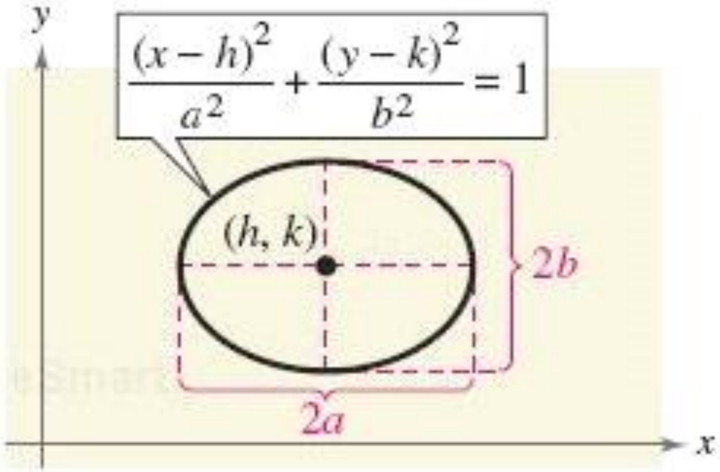
To move the center to the point (h,k), we replace x with $(x - h)$ and y with $(y - k)$ in the equations

Parabola: Vertex = (h, k)

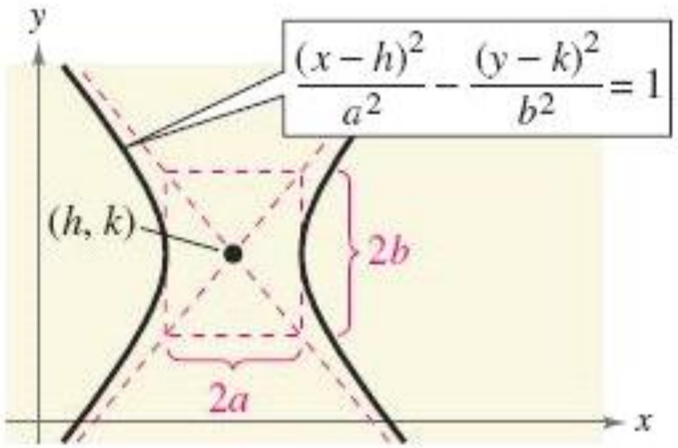




Ellipse: Center = (h, k)



Hyperbola: Center = (h, k)



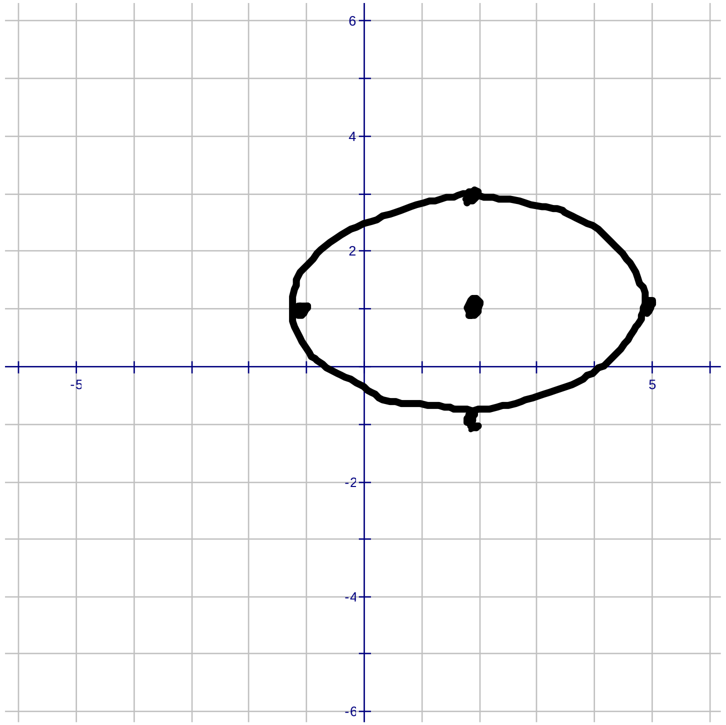
Ex. Identify and graph $\frac{(x-2)^2}{9}+\frac{(y-1)^2}{4}=1$ ellipse

$a^2 \nearrow 9$ $\qquad$ $b^2 \nearrow 4$

center: $(2, 1)$

$a = 3$

$b = 2$

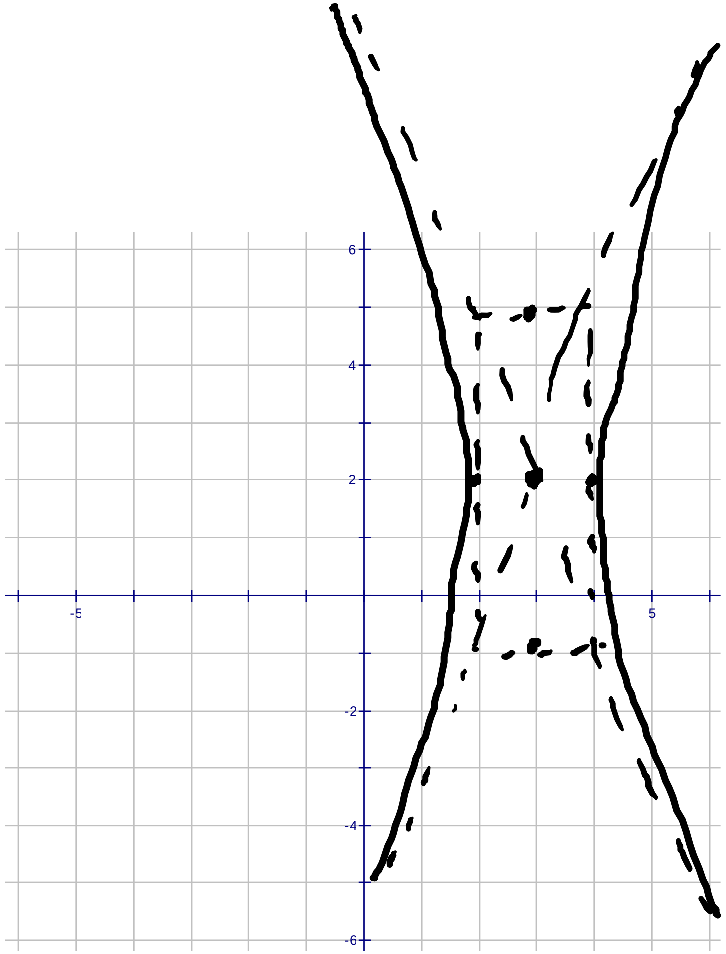


Ex. Identify and graph $\frac{(x-3)^2}{1} - \frac{(y-2)^2}{9} = 1$ hyperbola

center: $(3, 2)$

$a = 1$

$b = 3$



Ex. Identify and graph $(x + 3)^2 + (y - 2)^2 = 16$

$$\frac{(x+3)^2}{16} + \frac{(y-2)^2}{16} = \frac{16}{16}$$

$$\frac{(x+3)^2}{16} + \frac{(y-2)^2}{16} = 1$$

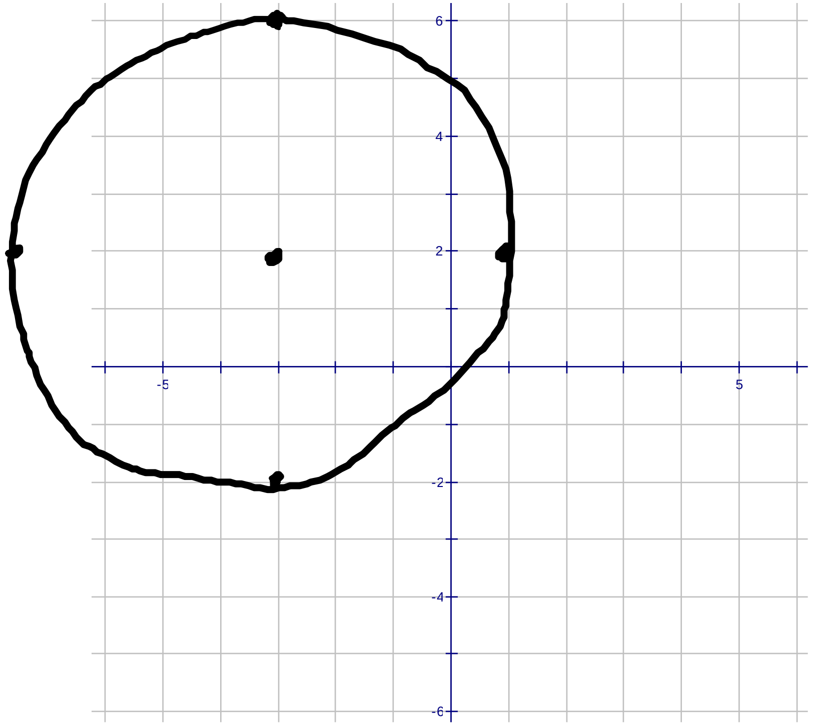
~~ellipse~~

circle

center: $(-3, 2)$

$a = 4$

$b = 4$



Ex. Find the vertex and focus of the parabola

$x^2 - 2x + 4y - 3 = 0$

$-4y \quad +3 \quad -4y \quad +3$

$(x-h)^2 = 4p(y-k)$

$x^2 - 2x + \underline{1} = -4y + 3 + \underline{1}$

$(x-1)^2 = -4y + 4$

$(x-1)^2 = -4(y-1)$

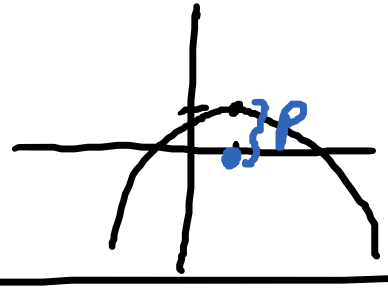
$(x-h)^2 = 4p(y-k)^2$

$4p = -4$

$p = -1$

vertex: $(1, 1)$

focus: $(1, 0)$



in a parabola, only 1 variable is squared

~~parab.~~ ← both var. squ.

ellipse ← both squ. same sign

hyperbola

Ex. Sketch $x^2 + 4y^2 + 6x - 8y + 9 = 0$

$\qquad\qquad\qquad\qquad\qquad\qquad -9 \quad -9$

$$x^2 + 6x + 4y^2 - 8y = -9$$

$$(x^2 + 6x + \underline{9}) + 4(y^2 - 2y + \underline{1}) = -9 + \underline{9} + \underline{4}$$

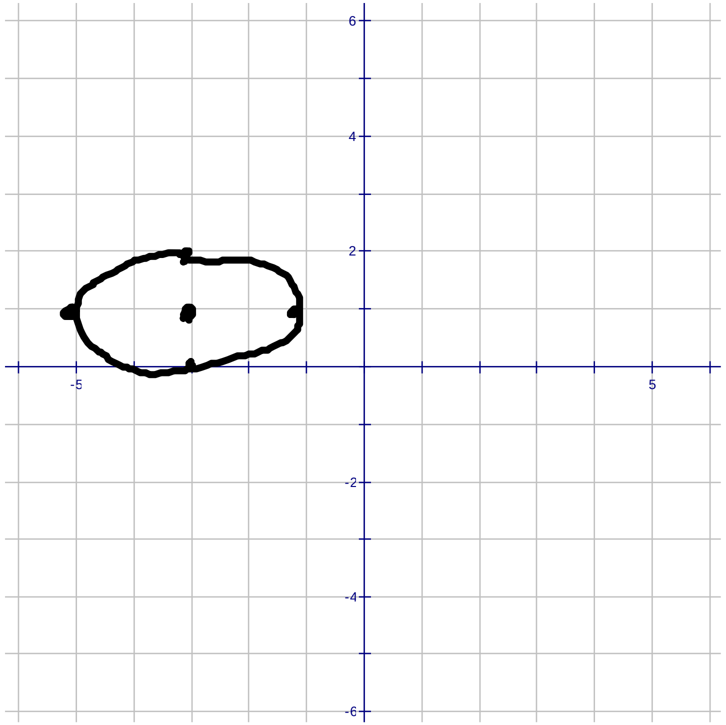
$$\frac{(x+3)^2}{4} + \frac{4(y-1)^2}{4} = \frac{4}{4}$$

$$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{1} = 1$$

center: $(-3, 1)$

$a = 2$

$b = 1$



Ex. Sketch $-4x^2 + y^2 + 24x + 4y - 41 = 0$

$ +41 \quad +41$

hyperbola

→ squ. are diff. signs

$-4x^2 + 24x + y^2 + 4y = 41$

$-4(x^2 - 6x + \underline{9}) + (y^2 + 4y + \underline{4}) = 41 + \underline{-36} + \underline{4}$

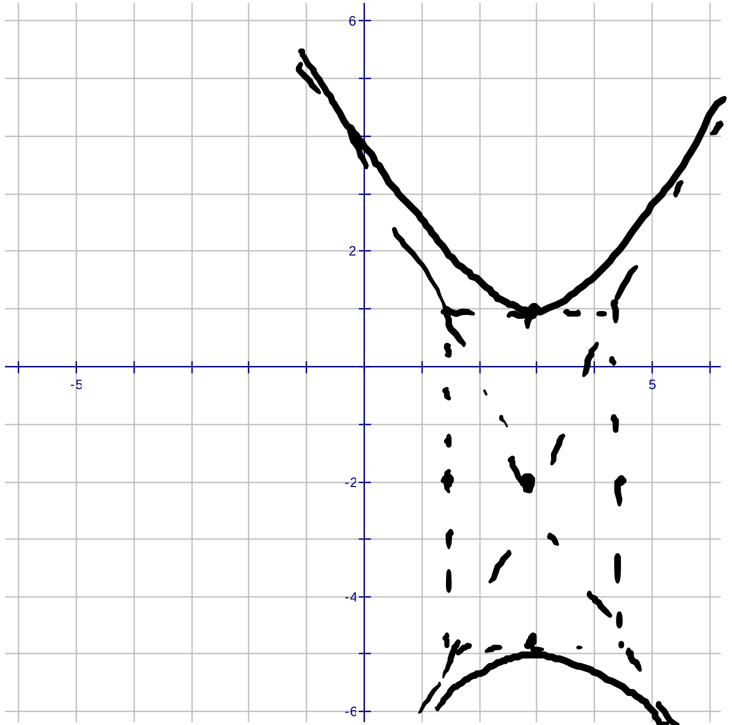
$\dfrac{-\cancel{4}(x-3)^2}{9/4} + \dfrac{(y+2)^2}{9} = \dfrac{9}{9}$

$\dfrac{(y+2)^2}{9} - \dfrac{(x-3)^2}{9/4} = 1$

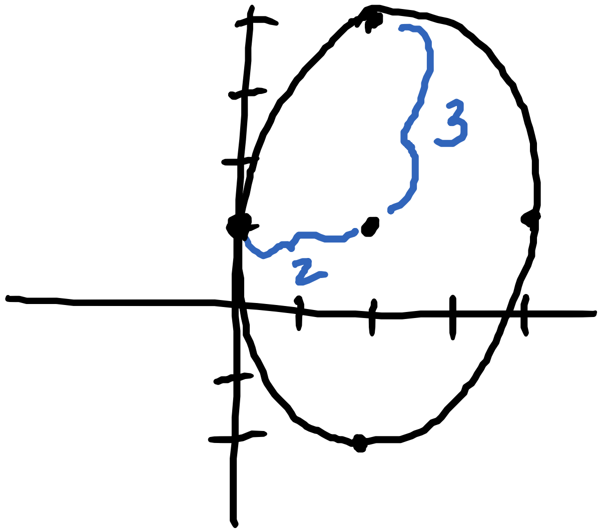
center: $(3, -2)$

$a = 3/2$

$b = 3$



Ex. The vertices of an ellipse are (2,-2) and (2,4), and the length of the minor axis is 4. Find the equation of the ellipse.



center: $(2,1)$

$a = 2$

$b = 3$

$$\boxed{\frac{(x-2)^2}{2^2} + \frac{(y-1)^2}{3^2} = 1}$$