Properties of Logarithms

If we want to evaluate a logarithm on the calculator, we may need to change the base

$$\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$$

Ex. Evaluate $f(x) = \log_4 x$ at $x = 25$.

$$f(25) = \log_4 25 = \frac{\ln 25}{\ln 4} = 2.3$$

Properties of Logarithms

$$\log(AB) = \log A + \log B$$ → log of product = sum of logs

$$\log\left(\frac{A}{B}\right) = \log A - \log B$$ → log of quotient = diff. of logs

$$\log(A^n) = n \log A$$ → power inside becomes coeff. outside

These apply to all logarithms, not just the common log

<u>Ex.</u> Find the exact value without a calculator

a) $\log_5 \sqrt[3]{5} = \log_5\left(5^{1/3}\right) = \frac{1}{3}$

b) $\ln e^6 - \ln e^2 = \ln\left(\frac{e^6}{e^2}\right) = \ln\left(e^4\right) = 4$

$\ln e^6 - \ln e^2 = 6 - 2 = 4$

<u>Ex.</u> Expand each logarithmic expression

a) $\log_4(5 \cdot x^3 \cdot y) = \log_4 5 + \log_4(x^3) + \log_4 y$

$= \log_4 5 + 3\log_4 x + \log_4 y$

b) $\ln\left(\frac{\sqrt{3x-5}}{7}\right) = \ln(3x-5)^{1/2} - \ln 7$

$= \frac{1}{2}\ln(3x-5) - \ln 7$

Ex. Condense each logarithmic expression

a) $2\ln(x+2)-\ln x$ $= \ln(x+2)^2 - \ln x = \ln\left(\frac{(x+2)^2}{x}\right)$

b) $\frac{1}{3}[\log_2 x + \log_2(x+1)]$ $= \frac{1}{3}\log_2[x(x+1)] = \log_2[x(x+1)]^{1/3}$

c) $5\ln x - 3\ln y + \ln z$ $= \ln(x^5) - \ln(y^3) + \ln z = \ln\left(\frac{x^5 z}{y^3}\right)$