Exponential and Logarithmic Equations

Ex. Solve for x

a) $2^x = 32$

$2^x = 2^5 \rightarrow x = 5$

b) $\ln x - \ln 3 = 0$

$\ln x = \ln 3 \rightarrow x = 3$

c) $\left(\frac{1}{3}\right)^x = 9$

$(3^{-1})^x = 3^2 \rightarrow 3^{-x} = 3^2$

$-x = 2$

$x = -2$

Ex. Solve for x

d) $\cancel{\ln}\ x = -3$ → $x = e^{-3}$

e
e) $\cancel{\log}\ x = -1$ → $x = 10^{-1}$

10

f) $e^{x^2} = e^{-3x+4}$

$x^2 = -3x+4$

$x^2+3x-4=0$

$(x+4)(x-1)=0$

$\boxed{x=-4}$ $\boxed{x=1}$

Ex. Solve for x

g) $\frac{\cancel{3}(2^x)}{\cancel{3}} = \frac{42}{3}$

$\ln(2^x) = \ln(14)$

$\ln(2^x) = \ln 14$

$\frac{x \cancel{\ln 2}}{\cancel{\ln 2}} = \frac{\ln 14}{\ln 2}$

$\boxed{x = \frac{\ln 14}{\ln 2}}$

h) $e^x + 5 = 60$

$\quad -5 \quad -5$

$\ln e^x = \ln 55$

$x = \ln 55$

Ex. Solve for x

i) $2(3^{2x-5})-\cancel{4}=11$

$\quad\quad\quad\quad\quad\quad +4 \quad\ \ +4$

$$\frac{2(3^{2x-5})}{2}=\frac{15}{2}$$

$$\ln(3^{2x-5})=\ln\left(\frac{15}{2}\right)$$

$$\ln(3^{2x-5})=\ln\left(\frac{15}{2}\right)$$

$$\frac{(2x-5)\cancel{\ln 3}}{\cancel{\ln 3}}=\frac{\ln\left(\frac{15}{2}\right)}{\ln 3}$$

$$2x-5=\frac{\ln\left(\frac{15}{2}\right)}{\ln 3}$$

$\quad +5 \quad\quad\quad\quad +5$

$$2x=\frac{\ln\left(\frac{15}{2}\right)}{\ln 3}+5$$

$$\boxed{x=\frac{\frac{\ln\left(\frac{15}{2}\right)}{\ln 3}+5}{2}}$$

Ex. Solve for x

j) $\ln(5e^{2x}) = \ln(e^{x-3})$

$$\ln(5 \cdot e^{2x}) = \ln(e^{x-3})$$

$$\ln 5 + \ln(e^{2x}) = x - 3$$

$$\ln 5 + 2x = \cancel{x} - 3$$
$$-x \qquad \cancel{-x}$$

$$\cancel{\ln 5} + x = -3$$
$$\cancel{-\ln 5} \qquad -\ln 5$$

$$\boxed{x = -3 - \ln 5}$$

Ex. Solve for x

k) $e^{2x} - 3e^{x} + 2 = 0$

$(e^x)^2 - 3e^x + 2 = 0 \xrightarrow{y = e^x} y^2 - 3y + 2 = 0$

$(e^x - 2)(e^x - 1) = 0 \longleftarrow (y-2)(y-1) = 0$

$e^x - \cancel{2} = 0 \qquad e^x - \cancel{1} = 0$

$\cancel{+2} \quad +2 \qquad \cancel{+1} \quad +1$

$\ln(e^x) = \ln 2 \qquad \ln(e^x) = \ln 1$

$\boxed{x = \ln 2} \qquad \boxed{x = 0}$

Ex. Solve for x

l) $\ln x = 2$ $\longrightarrow$ $x = e^2$

m) $\log_3(5x - 1) = \log_3(x + 7)$

$5x - 1 = x + 7$

$4x = 8$ $\longrightarrow$ $\boxed{x = 2}$

n) $\underbrace{\log_6(3x + 14) - \log_6 5} = \log_6 2x$

$\log_6\left(\frac{3x+14}{5}\right) = \log_6(2x)$

$\frac{3x+14}{5} = 2x$ $\longrightarrow$ $3x + 14 = 10x$

$14 = 7x$

$\boxed{x = 2}$

Ex. Solve for x

p) $\cancel{5} + 2\ln x = 4$

$-5 \qquad -5$

$$\frac{\cancel{2}\ln x}{\cancel{2}} = \frac{-1}{2}$$

$\ln x = -\frac{1}{2}$

$\boxed{x = e^{-1/2}} = \frac{1}{e^{1/2}} = \frac{1}{\sqrt{e}}$

q) $\frac{2\log_5(3x)}{2} = \frac{4}{2}$

$\log_5(3x) = 2$

$3x = 5^2$

$3x = 25$

$\boxed{x = \frac{25}{3}}$

Ex. You have deposited $500 in an account that pays 6.75% interest, compounded continuously. How long will it take your money to double?

P

$r = .0675$

$t = ?$

$A = 1000$

$$A = Pe^{rt}$$

$$\frac{1000}{500} = \frac{\cancel{500}e^{.0675t}}{\cancel{500}}$$

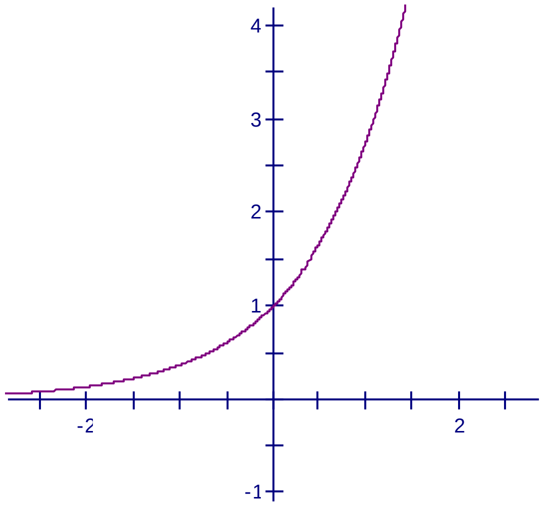
$$\ln(2) = \ln(e^{.0675t})$$

$$\frac{\ln 2}{.0675} = \frac{\cancel{.0675}t}{\cancel{.0675}}$$

$$t = \frac{\ln 2}{.0675} = 10.3$$

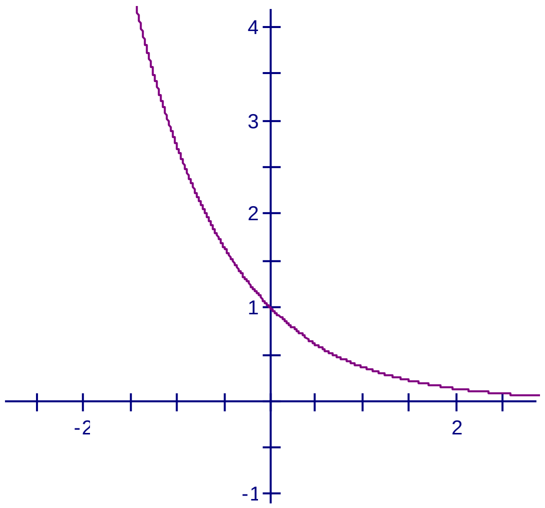
Exponential and Logarithmic Models

Exponential Growth



$$y = ae^{bx}$$

Exponential Decay



$$y = ae^{-bx}$$

Ex. The number of households, D, in millions, with HDTV is given by the exponential model $D = 30.92e^{0.1171t}$, where t is years since 2000. When will the number of households reach 100 million?

$$\frac{100}{30.92} = \frac{\cancel{30.92}e^{0.1171t}}{\cancel{30.92}}$$

$$\ln \frac{100}{30.92} = \ln e^{0.1171t}$$

$$\frac{\ln\left(\frac{100}{30.92}\right)}{.1171} = \frac{\cancel{0.1171}t}{\cancel{.1171}}$$

$$t = \frac{\ln\left(\frac{100}{30.92}\right)}{.1171} = 10.0$$

Ex. The number N of bacteria in a culture is modeled by $N = 450e^{kt}$, where t is time in hours. If $N = 600$ when $t = 3$, estimate the value of N when $t = 5$.

$N = 600$
$t = 3$

$$N = 450e^{kt}$$

$$\frac{600}{450} = \frac{\cancel{450}e^{k(3)}}{\cancel{450}}$$

$$\ln \frac{600}{450} = \cancel{\ln} e^{3k}$$

$$\frac{\ln\left(\frac{600}{450}\right)}{3} = \frac{\cancel{3}k}{\cancel{3}}$$

$$k = \frac{\ln\left(\frac{600}{450}\right)}{3} = .096$$

$$N = 450e^{.096t}$$

$t = 5$: $N = 450e^{.096(5)}$

$$N = \boxed{726.8}$$

Ex. A population of fruit flies is increasing according to an exponential growth model. After 0 days, there are 100 flies, and after 4 days there are 300 flies. How many flies will there be after 5 days?

$t=0 \quad y=100$

$t=4 \quad y=300$

$t=5 \quad y=??$

$y=ae^{bt}$

$\left.\begin{matrix} t=0 \\ y=100 \end{matrix}\right\} \quad 100=ae^{b(0)}$

$a=100$

$\left.\begin{matrix} t=4 \\ y=300 \end{matrix}\right\} \quad y=100e^{bt}$

$\frac{300}{100}=\frac{\cancel{100}e^{b(4)}}{\cancel{100}}$

$\ln 3=\ln e^{4b}$

$\frac{\ln 3}{4}=\frac{\cancel{4}b}{\cancel{4}}$

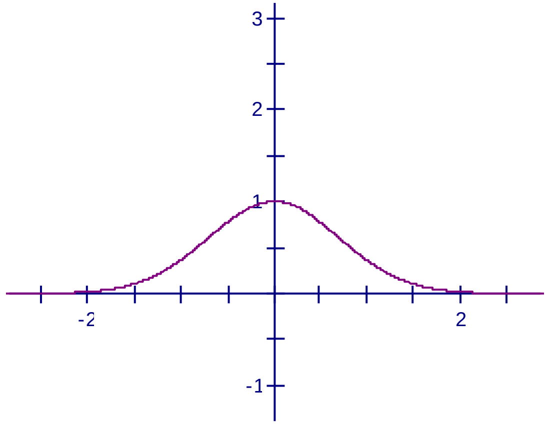
$b=\frac{\ln 3}{4}=.275$

$y=100e^{.275t}$

$t=5$: $y=100e^{.275(5)}$

$\boxed{y=394.8}$

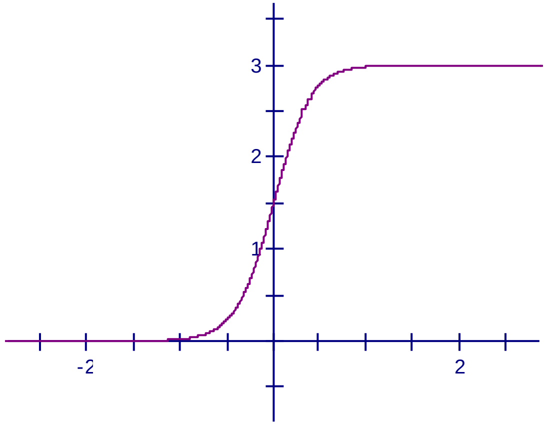
Gaussian Model



$$y = ae^{-(x-b)^2/c}$$

This is often used in probability and statistics to represent a normally distributed (bell-shaped) model

Logistic Growth Model



$$y = \frac{a}{1 + be^{-rx}}$$

This can be used to represent a model with a rapid increase followed by leveling off.

Ex. On a college campus of 5000 students, the spread of a virus is modeled by

$$y = \frac{5000}{1 + 4999e^{-0.8t}}$$

where y is the number of students infected after t days.

a) How many students are infected after 5 days?

b) If the college cancels classes when 40% of students are infected, after how many days will classes be cancelled?