Systems of Equations: Substitution

A system of equations refers to multiple equations involving multiple variables

$$\begin{cases} 2x + y = 5 \\ 3x - 2y = 4 \end{cases}$$

A solution is an ordered pair that satisfies both equations.

(2,1) is the solution to the system

One method for solving a system of equations is substitution

- Solve for a variable in one equation, and then plug into the other equation and solve
- Find the value of the other variable by using one of the original equations

Ex. Solve the system $\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$

Ex. A total of $12,000 is invested in two funds paying 5% and 3% simple interest [$I = prt$]. If the yearly interest is $500, how much was invested at each rate?

Ex. Solve the system $\begin{cases} x^2 + 4x - y = 7 \\ 2x - y = -1 \end{cases}$

Ex. Solve the system $\begin{cases} -x + y = 4 \\ x^2 + y = 3 \end{cases}$

One solution: graphs intersect once

Two solutions: graphs intersect twice

No solutions: graphs don't intersect

Ex. Solve the system $\begin{cases} x^2 - x - y = 1 \\ x - y = 1 \end{cases}$ by graphing

Ex. A shoe company invests $300,000 in equipment to produce a line of shoes. Each pair costs $5 to produce and is sold for $60. How many pairs of shoes must be sold before the business breaks even?

Ex. A store sells books and movies. Each book costs $5 and each movie costs $12. One day, a total of 16 items were sold. If the total amount received was $150, how many of each item was sold?

Systems of Equations: Elimination

We are allowed to add the two equations in a system

By manipulating the coefficients, we can eliminate a variable and make the system easier to solve

Ex. Solve the system $\begin{cases} 3x + 2y = 4 \\ 5x - 2y = 8 \end{cases}$

Method of Elimination

- Using multiplication, get the coefficients of a variable to be opposites
- Add the equations to eliminate a variable
- Solve for one variable, then use one of the original equations to find the other variable.

Ex. Solve the system $\begin{cases} 2x - 3y = -7 \\ 3x + y = -5 \end{cases}$

Ex. Solve the system $\begin{cases} 5x + 3y = 9 \\ 2x - 4y = 14 \end{cases}$

Ex. Solve the system $\begin{cases} 2x - y = 1 \\ 4x - 2y = 2 \end{cases}$

These graphs coincide

Ex. Solve the system $\begin{cases} x - 2y = 3 \\ -2x + 4y = 1 \end{cases}$

These graphs are parallel

Ex. Solve the system $\begin{cases} 0.02x - 0.05y = -0.38 \\ 0.03x + 0.04y = 1.04 \end{cases}$