## Multivariable Systems

These systems will have three equations and three variables  $\sum x = 2y + 3z = 9$   $\sum x = 15$  2 = 2 $(1) \times -2(9) + 3(2) = 9 \\ \times -18 + 6 = 9 \\ \times -12 = 9$ (2)  $\gamma + 3(2) = 15$  $\gamma + 6 = 15$ x=2 (21,9,2)

When working with systems of equations, we are allowed to perform the following <u>row operations</u> to get an equivalent system:

- Interchange two equations
- Multiply an equation by a nonzero constant
- Add a multiple of one equation to another

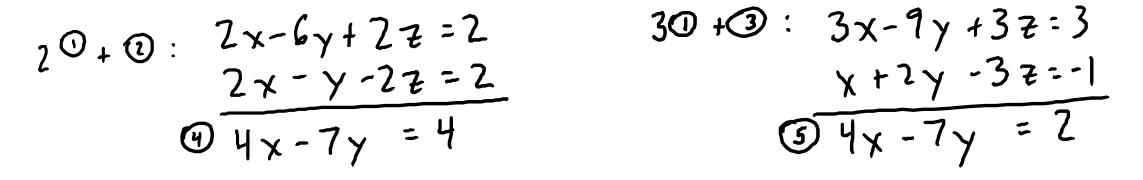
The previous example was in row-echelon form (triangular)

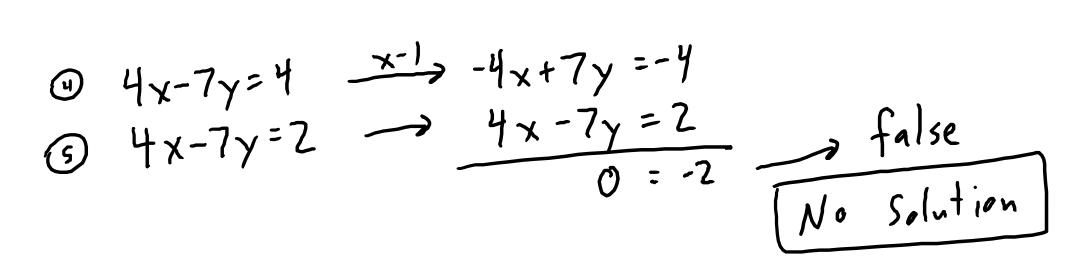
- You can use it if you'd like, but it's not necessary
- We'll work with it more in the next chapter

Ex. Solve the system
$$\begin{cases}
3x - 2y = -1 \rightarrow 3x - 2y = -1 \\
x - y = 0 \xrightarrow{x \cdot 2} -2x + 2y = 0 \\
\hline x = -1 \\
3(-1) - 2y = -1 \\
-3 - 2y = -1 \\
-2y = 2 \\
y = -1
\end{cases}$$

$$\underbrace{Ex. \text{ Solve the system}^{\textcircled{0}}}_{(3)} \begin{bmatrix} x - 2y + 3z = 9 \\ -x + 3y &= -4 & \xrightarrow{*^2} & -2x + 6y &= -8 \\ 2x - 5y + 5z = 17 & \xrightarrow{*} & 2x - 5y & +5z = 17 \end{bmatrix}$$

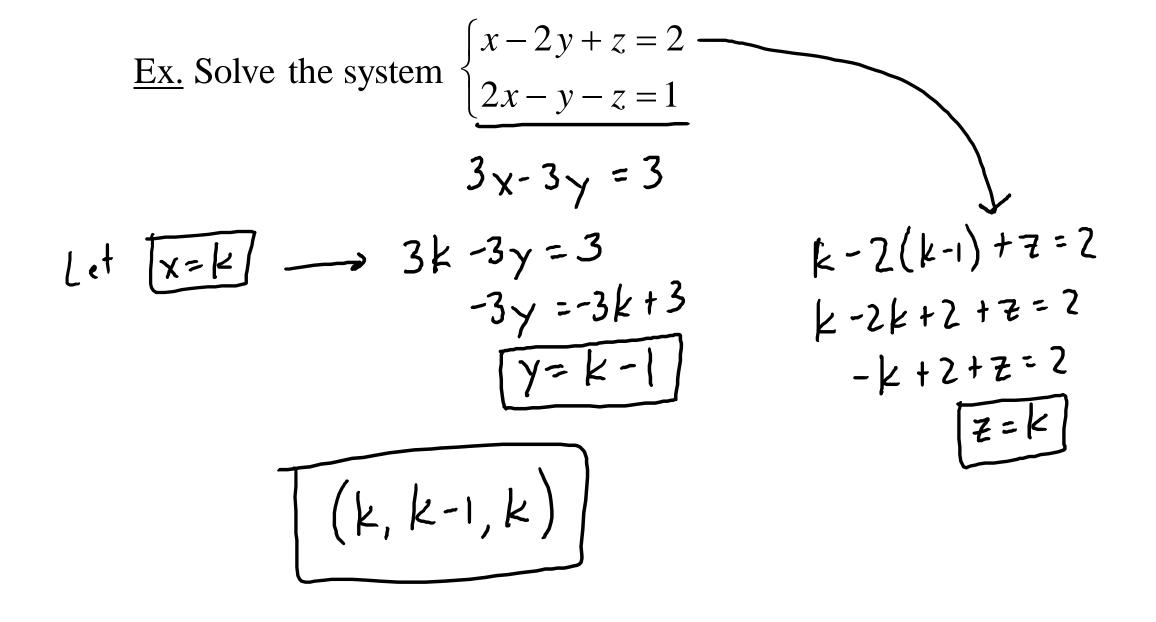
$$\underbrace{Ex. \text{ Solve the system}^{\bigcirc}}_{x+2y-3z=-1} x - 3y + z = 1$$





These systems have been square (same number of equations and variables)

- If there are less equations than variables, then there will not be a unique solution
- We can still describe the solutions, as we did in the last example



Ex. Find a quadratic equation 
$$\underline{y} = ax^2 + bx + c$$
 that passes  
through the points (-1,3), (1,1), and (2,6).  
(-1,3)  $\vdots$   $3 = a(-1)^{1+}b(-1) + c \longrightarrow @a - b + c = 3 \longrightarrow 2a - 2b + 2c = 6$   
(1,1)  $\vdots$   $| = a(1)^{1+}b(1) + c \longrightarrow @a + b + c = 1$   
(1,1)  $\vdots$   $| = a(1)^{1+}b(1) + c \longrightarrow @a + b + c = 1$   
(2,6)  $\vdots$   $6 = a(2)^{1+}b(2) + c \longrightarrow @a + 2b + c = 6 \longrightarrow @a + 2b + c = 6$   
 $0 + 0 : 2a + 2c = 4 \xrightarrow{x^{-3}} - 6a - 6c = -12$   
 $20 + 0 : 2a + 2c = 4 \xrightarrow{x^{-3}} - 6a - 6c = -12$   
 $0 + 3 : 6a + 3c = 12 \longrightarrow 6a + 3c = 12$   
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 $0 + 3 : 7 = 12 \longrightarrow 6a + 3c = 12 \longrightarrow 6a + 3c = 12 \longrightarrow 6a + 3c = 12 \longrightarrow 7 = 12 \longrightarrow 7$ 

<u>Ex.</u> A total of \$12,000 is invested in three funds that pay money market (5%), municipal (6%), and mutual (12%). The amount in <u>mutual funds</u> is \$4000 more than the amount in <u>municipal</u>. If the total interest is \$1120, how much was invested in each fund?

x = ant in money market .05x = int from money market y = ant in municipal .06y = int from municipal z = ant in mutual .12z = int from mutual

x + y + z = 12000  $\longrightarrow$  x + y + z = 12000.05x+.06y+.12z=1120  $\times 100$  5x + 6y + 12z = 112000z = 4000 + y  $\longrightarrow$  -y + z = 4000

$$-50 + 2 : y + 7z = 52000$$

$$3 -y + z = 4000 - y + 7000 = 4000$$

$$y = 3000$$

$$y = 3000$$

$$z = 56000$$

$$z = 7000$$

$$(1) \quad \begin{array}{c} \chi + 3000 + 7000 = 12000 \\ \chi + 10000 = 12000 \\ \chi = $^{2}2000 \end{array} \\ \end{array}$$