Multivariable Systems

These systems will have three equations and three variables

Ex. Solve the system

$$\begin{cases} ① \; x - 2y + 3z = 9 \\ ② \; y + 3z = 15 \\ ③ \; \boxed{z = 2} \end{cases}$$

② $y + 3(2) = 15$

$y + 6 = 15$

$\boxed{y = 9}$

① $x - 2(9) + 3(2) = 9$

$x - 18 + 6 = 9$

$x - 12 = 9$

$\boxed{x = 21}$

$\boxed{(21, 9, 2)}$

When working with systems of equations, we are allowed to perform the following row operations to get an equivalent system:

- Interchange two equations
- Multiply an equation by a nonzero constant
- Add a multiple of one equation to another

The previous example was in row-echelon form (triangular)

- You can use it if you'd like, but it's not necessary
- We'll work with it more in the next chapter

<u>Ex.</u> Solve the system $\begin{cases} 3x - 2y = -1 \\ x - y = 0 \end{cases}$

$3x - 2y = -1 \longrightarrow 3x - 2y = -1$

$x - y = 0 \xrightarrow{x \cdot 2} -2x + 2y = 0$

$x = -1$

$3(-1) - 2y = -1$

$-3 - 2y = -1$

$-2y = 2$

$y = -1$

$\boxed{(-1, -1)}$

<u>Ex.</u> Solve the system

$$\begin{cases} x-2y+3z=9 & \text{①} \\ -x+3y \qquad\quad =-4 & \text{②} \xrightarrow{\times 2} -2x+6y=-8 \\ 2x-5y+5z=17 & \text{③} \longrightarrow 2x-5y+5z=17 \end{cases}$$

① + ② $y+3z=5 \xrightarrow{\times -1} -y-3z=-5$

2② + ③ $y+5z=9 \longrightarrow y+5z=9$

$$2z=4$$

$$\boxed{z=2}$$

$y+5(2)=9$

$\boxed{y=-1}$

① $x-2(-1)+3(2)=9$

$x+2+6=9$

$\boxed{x=1}$

$$\boxed{(1,-1,2)}$$

<u>Ex.</u> Solve the system
$$\begin{cases} x-3y+z=1 & (1)\\ 2x-y-2z=2 & (2)\\ x+2y-3z=-1 & (3)\end{cases}$$

$2(1)+(2)$:
$$\begin{array}{r} 2x-6y+2z=2\\ 2x-y-2z=2\\ \hline (4)\quad 4x-7y=4 \end{array}$$

$3(1)+(3)$:
$$\begin{array}{r} 3x-9y+3z=3\\ x+2y-3z=-1\\ \hline (5)\quad 4x-7y=2 \end{array}$$

$(4)\quad 4x-7y=4 \xrightarrow{\times -1} -4x+7y=-4$

$(5)\quad 4x-7y=2 \longrightarrow \quad 4x-7y=2$

$$\begin{array}{r} -4x+7y=-4\\ 4x-7y=2\\ \hline 0=-2 \end{array} \longrightarrow \text{false}$$

$\boxed{\text{No Solution}}$

Ex. Solve the system
$$\begin{cases} x + y - 3z = -1 & (1) \\ y - z = 0 & (2) \\ -x + 2y = 1 & (3) \end{cases}$$

(1) + (3) $\quad 3y - 3z = 0 \rightarrow 3y - 3z = 0$

(2) $\quad y - z = 0 \xrightarrow{\times -3} -3y + 3z = 0$

$0 = 0 \rightarrow$ true

Many Solutions

$\boxed{\text{Let } y = k} \rightarrow$ (2) $k - z = 0$ $\quad \boxed{z = k}$

(1) $x + k - 3k = -1$

$x - 2k = -1$

$\boxed{x = 2k - 1}$

$\boxed{(2k-1, k, k)}$

This system has many solutions, and we can describe what the solutions look like

These systems have been square (same number of equations and variables)

- If there are less equations than variables, then there will not be a unique solution

- We can still describe the solutions, as we did in the last example

Ex. Solve the system $\begin{cases} x - 2y + z = 2 \\ 2x - y - z = 1 \end{cases}$

$3x - 3y = 3$

Let $\boxed{x = k}$ $\longrightarrow$ $3k - 3y = 3$

$-3y = -3k + 3$

$\boxed{y = k - 1}$

$k - 2(k-1) + z = 2$

$k - 2k + 2 + z = 2$

$-k + 2 + z = 2$

$\boxed{z = k}$

$\boxed{(k, k-1, k)}$

<u>Ex.</u> Find a quadratic equation $y = ax^2 + bx + c$ that passes through the points (-1,3), (1,1), and (2,6).

$(-1,3)$: $3 = a(-1)^2 + b(-1) + c \longrightarrow$ ① $a - b + c = 3 \longrightarrow 2a - 2b + 2c = 6$

$(1,1)$: $1 = a(1)^2 + b(1) + c \longrightarrow$ ② $a + b + c = 1$

$(2,6)$: $6 = a(2)^2 + b(2) + c \longrightarrow$ ③ $4a + 2b + c = 6 \longrightarrow 4a + 2b + c = 6$

① + ② : $2a + 2c = 4 \xrightarrow{\times -3} -6a - 6c = -12$

2① + ③ : $6a + 3c = 12 \longrightarrow 6a + 3c = 12$

$-3c = 0$

$\boxed{c = 0}$

$6a + 3(0) = 12$

$6a = 12$

$\boxed{a = 2}$

② $2 + b + 0 = 1$

$\boxed{b = -1}$

$\boxed{y = 2x^2 - x}$

Ex. A total of \$12,000 is invested in three funds that pay money market (5%), municipal (6%), and mutual (12%). The amount in mutual funds is \$4000 more than the amount in municipal. If the total interest is \$1120, how much was invested in each fund?

z

y

x: amt. in money market

y: amt. in municipal

z: amt. in mutual

$.05x$ = int. from money market

$.06y$ = int. from municipal

$.12z$ = int. from mutual

$$x + y + z = 12000 \longrightarrow x + y + z = 12000$$

$$.05x + .06y + .12z = 1120 \xrightarrow{\times 100} 5x + 6y + 12z = 112000$$

$$z = 4000 + y \longrightarrow -y + z = 4000$$

① $x + y + z = 12000 \xrightarrow{\times -5} -5x - 5y - 5z = -60000$

② $5x + 6y + 12z = 112000 \longrightarrow 5x + 6y + 12z = 112000$

③ $-y + z = 4000$

$-5①+②:$ $y + 7z = 52000$

③ $-y + z = 4000$

$8z = 56000$

$\boxed{z = \$7000}$

$\longrightarrow -y + 7000 = 4000$

$\boxed{y = \$3000}$

① $x + 3000 + 7000 = 12000$

$x + 10000 = 12000$

$\boxed{x = \$2000}$