Arithmetic Sequences and Series

A sequence is arithmetic if we add the same amount each time to get a new term.

This amount, *d*, is called the common difference

$3, 7, 11, 15, 19, \ldots \longrightarrow d = 4$

Ex. Find the first 4 terms of the arithmetic sequence.

a) $a_n = 4n + 3$

$a_1 = 4(1) + 3 = 7$

$a_2 = 4(2) + 3 = 11$

$a_3 = 4(3) + 3 = 15$

$a_4 = 4(4) + 3 = 19$

$d = 4$

$7, 11, 15, 19, \ldots$

b) $a_n = 7 - 5n$

$a_1 = 7 - 5(1) = 2$

$a_2 = 7 - 5(2) = -3$

$a_3 = 7 - 5(3) = -8$

$a_4 = 7 - 5(4) = -13$

$d = -5$

$2, -3, -8, -13, \ldots$

c) $a_n = \frac{1}{4}(n + 3)$

$a_1 = \frac{1}{4}(1 + 3) = \frac{4}{4}$

$a_2 = \frac{1}{4}(2 + 3) = \frac{5}{4}$

$a_3 = \frac{1}{4}(3 + 3) = \frac{6}{4}$

$a_4 = \frac{1}{4}(4 + 3) = \frac{7}{4}$

$d = \frac{1}{4}$

$\frac{4}{4}, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}, \ldots$

To find the n^{th} term of an arithmetic sequence, we use the formula

$$a_n = a_1 + d(n - 1)$$

where a_1 is the first term and d is the common difference

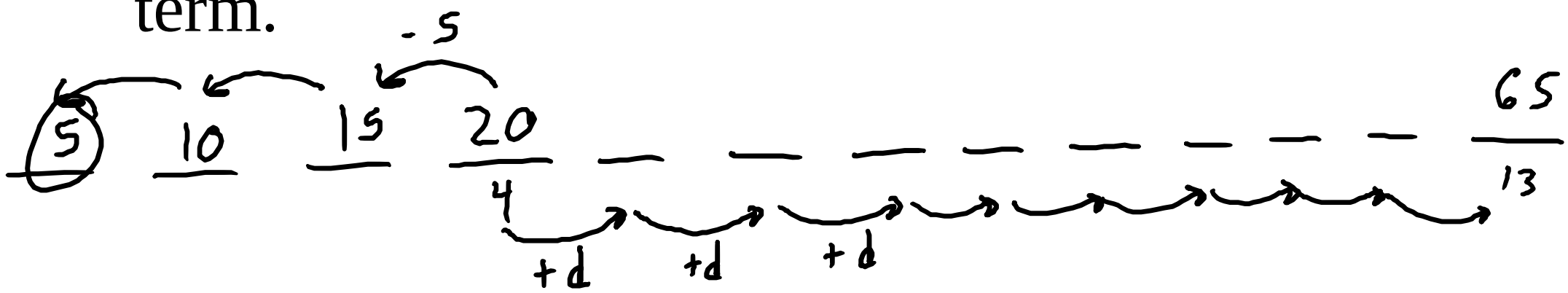
Ex. Find the n^{th} term of the arithmetic sequence: 2, 5, 8, 11, 14,...

+3 +3 +3

$d = 3$

$a_n = 2 + 3(n - 1)$

Ex. The fourth term of an arithmetic sequence is 20 and the 13th term is 65. Find the n^{th} term.



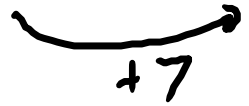
$a_n = 5 + 5(n-1)$

$20 + 9d = 65$

$9d = 45$

$d = 5$

Ex. Find a_9 of the arithmetic sequence that starts with 2 and 9.



$d = 7$

$a_n = 2 + 7(n-1)$

$a_9 = 2 + 7(9-1) = \boxed{58}$

To find the sum of a finite arithmetic sequence with n terms, we use the formula

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Ex. Find the sum of the first 10 odd numbers.

1, 3, 5, 7, 9, 11, 13, 15, 17, 19

$$S = \frac{10}{2}(1 + 19) = 5(20) = 100$$

Ex. Find the 150^{th} partial sum of the arithmetic sequence: 5, 16, 27, 38, 49,...

$a_n = 5 + 11(n-1)$ $\quad +11 \quad d = 11$

term $\rightarrow$ $a_{150} = 5 + 11(150-1) = 1644$

sum $\rightarrow$ $S = \frac{150}{2}(5 + 1644) = 123675$

Ex. Find the sum $\sum_{n=51}^{100} 7n = 7(51) + 7(52) + 7(53) + \ldots + 7(100)$

$+7$

$d = 7$

$$S = \frac{50}{2}(7 \cdot 51 + 7 \cdot 100) = 26425$$

Ex. In a golf tournament, 16 golfers win cash prizes. First place gets $1000, second place gets $950, third place gets $900, and so on. What is the total amount of prize money?

$$a_n = 1000 - 50(n-1)$$

term → $a_{16} = 1000 - 50(16-1) = 250$

sum → $S = \frac{16}{2}(1000 + 250) = 10{,}000$