<u>Geometric Sequences and Series</u>

A sequence is <u>geometric</u> if we *multiply* the same amount each time to get a new term.

This amount, *r*, is called the <u>common ratio</u>

$1, 4, 16, 64, 256, \ldots$

$\times 4 \quad \times 4 \quad \times 4 \quad \times 4$

Ex. Find the first 4 terms of the geometric sequence.

a) $a_n = 2^n$ $\quad$ 2, 4, 8, 16, ... $\quad r = 2$

$a_1 = 2^1 = 2 \quad a_2 = 2^2 = 4 \quad a_3 = 2^3 = 8 \quad a_4 = 2^4 = 16$

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b) $a_n = 4(3^n)$ $\quad$ 12, 36, 108, 324, ... $\quad r = 3$

$a_1 = 4(3^1) = 12 \quad a_2 = 4(3^2) = 36 \quad a_3 = 4(3^3) = 108 \quad a_4 = 4(3^4) = 324$

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c) $a_n = (-\frac{1}{3})^n$ $\quad -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \frac{1}{81}, \ldots \quad r = -\frac{1}{3}$

$a_1 = (-\frac{1}{3})^1 = -\frac{1}{3} \quad a_2 = (-\frac{1}{3})^2 = \frac{1}{9} \quad a_3 = (-\frac{1}{3})^3 = -\frac{1}{27} \quad a_4 = (-\frac{1}{3})^4 = \frac{1}{81}$

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not geom. ~~d)~~ $a_n = n^2$ $\quad$ 1, 4, 9, 16, ... $\quad r$: ??

$a_1 = 1^2 = 1 \quad a_2 = 2^2 = 4 \quad a_3 = 3^2 = 9 \quad a_4 = 4^2 = 16$

To find the n^{th} term of a geometric sequence, we use the formula

$$a_n = a_1 r^{n-1}$$

where a_1 is the first term and r is the common ratio

Ex. Find the n^{th} term of the geometric sequence: 3, 6, 12, 24, 48,...

×2 ×2 ×2 ×2

$r = 2$

$a_n = 3(2)^{n-1}$

Ex. Write the 15th term of the geometric sequence whose 1st term is 20 and whose common ratio is 1.05.

$$a_n = 20(1.05)^{n-1}$$

$$a_{15} = 20(1.05)^{15-1} = 39.6$$

Ex. Write the 12th term of the geometric sequence 6, 18, 54,...

$$a_n = 6(3)^{n-1}$$

$$a_{12} = 6(3)^{12-1} = 1062882$$

Ex. The 4th term of a geometric sequence is 125 and the 10th term is $\frac{125}{64}$. Find the 14th term.

To find the sum of finite geometric sequence with n terms, we use the formula

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

Ex. Find the sum $\sum_{i=2}^{12} 4(1.03)^{i-1}$

$r = 1.03$

first term $= 4(1.03)^{2-1}$

≈ 4.12

$n = 11$

$$\text{Sum} = 4.12\left(\frac{1-1.03^{11}}{1-1.03}\right) = \boxed{52.8}$$

It is possible to take the sum of an infinite geometric sequence and get a finite answer.

Consider $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$

We say that this geometric series converges

A geometric series will converge if $|r| < 1$, and the sum is given by the formula

$$S = \frac{a_1}{1-r}$$

Ex. Find the sum

a) $\sum_{n=1}^{\infty} 4(.06)^{n-1} = \frac{4}{1-.06}$

$= 4.3$

first term $= 4(.06)^{1-1} = 4$

$r = .06$

b) $0.3 + 0.03 + 0.003 + \ldots = \frac{.3}{1-.1} = \frac{.3}{.9} = \frac{3}{9} = \frac{1}{3}$

$\times .1$ $\times .1$

$r = .1$

Arithmetic

$$a_n = a_1 + d(n-1)$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Geometric

$$a_n = a_1 r^{n-1}$$

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

$$S = \frac{a_1}{1-r} \text{ if } |r| < 1$$