## The Binomial Theorem

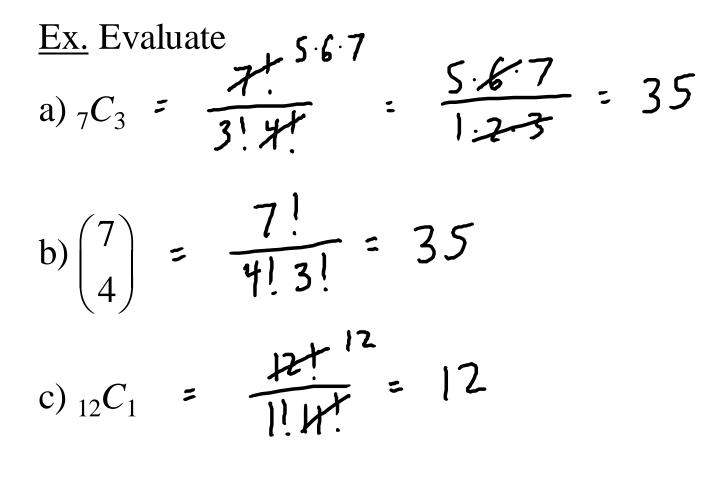
We are going to discuss how to expand something like  $(x + y)^8$  without a ton of work

First, we need to define the "choose function"

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

We will also write  ${}_{n}C_{r}$  for this function

<u>Ex.</u> Evaluate  $7.8^{4}$ a)  $_{8}C_{2} = \frac{8!}{2!6!} = 28$  $b)\binom{10}{3} = \frac{10!}{3! 7!} \frac{8 \cdot 9 \cdot 10}{3! 7!} = \frac{43}{8 \cdot 9 \cdot 10} = \frac{43}{8 \cdot 9 \cdot 10} = \frac{120}{1 \cdot 7 \cdot 3} = 120$ 1.7.3 c)  $_7C_0 = \frac{7!}{0!7!} = 1$ d)  $\binom{8}{8} = \frac{8!}{8!0!} = 1$ 



d) 
$$\begin{pmatrix} 12\\11 \end{pmatrix}$$
 = 12

#### When we expand $(x + y)^n$

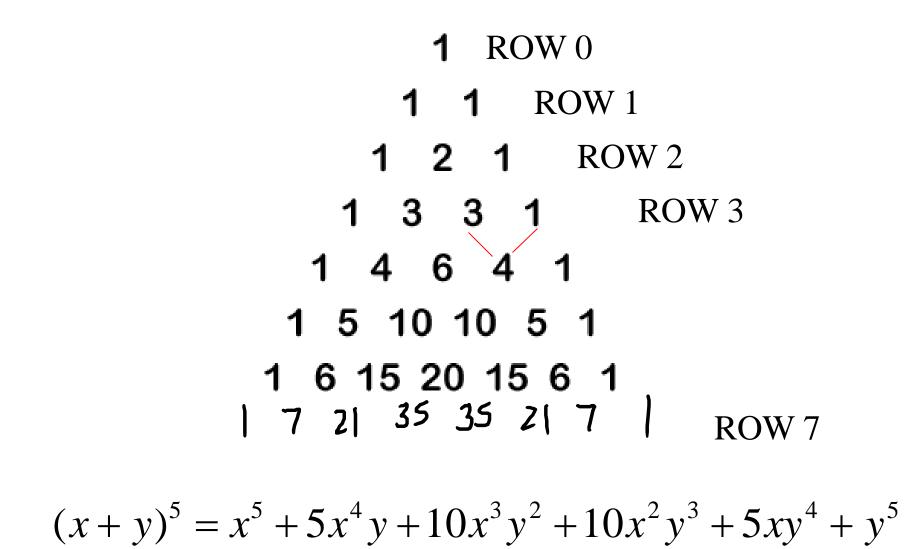
- There will be n + 1 terms
- As the powers of *x* decrease, the powers of *y* increase
- The powers of x and y will add up to n in each term
- The coefficient of the term with  $x^r$  is  ${}_nC_r$

### Consider the expansion

$$(x+y)^{5} = x^{5} + 5x^{4}y + 10x^{3}y^{2} + 10x^{2}y^{3} + 5xy^{4} + y^{5}$$

$${}_{\underline{5}}C_{\underline{3}} = 10$$

#### An easy way to find these coefficients is Pascal's Triangle



<u>Ex.</u> Expand the binomial  $(\underline{x} + \underline{1})^3$ 

 $|\cdot_{X}^{3} + 3\cdot_{X}^{2}\cdot|' + 3\cdot_{X}^{\prime}\cdot|^{2} + |\cdot|^{3}$  $x^{3} + 3x^{2} + 3x + 1$ 

<u>Ex.</u> Expand the binomial  $(2x-3)^4$ 

$$\frac{1(2x)^{4} + 4(2x)^{3}(-3)^{2} + 6(2x)^{2}(-3)^{2} + 4(2x)^{4}(-3)^{3} + 1(-3)^{4}}{2^{4}x^{4} + 4(2)^{3}x^{3}(-3) + 6(2)^{2}x^{2}(-3)^{2} + 4(2)x(-3)^{3} + (-3)^{4}}{16x^{4} - 96x^{3} + 216x^{2} - 216x + 8|}$$

<u>Ex.</u> Expand the binomial  $(x^2 + 4)^3$ 

$$\frac{1(x^{2})^{3} + 3(x^{2})^{2}(4) + 3(x^{2})(4)^{2} + 1 \cdot 4^{3}}{x^{6} + 12x^{4} + 48x^{2} + 64}$$

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# <u>Ex.</u> Find the coefficient of the term $x^3$ in the expansion of $(x + 2)^8$

 $\binom{3}{8} \binom{3}{2} \times \binom{3}{2} \binom{2}{5} \frac{56}{5} \times \binom{3}{2} \binom{2}{5}$  $1792 x^{3}$ 

Ex. Find the coefficient of the term  $x^6y^5$  in the expansion of  $(3x - 2y)^{11}$