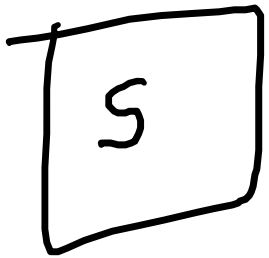
Counting Principles

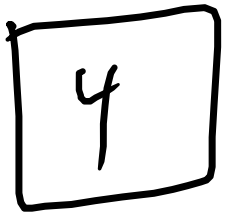
Ex. Eight pieces of paper are numbered 1 to 8 and placed in a box. One piece of paper is drawn from the box, its number is written down, and the piece of paper is replaced in the box. A second piece of paper is drawn from the box, and its number is written down. How many different ways can a sum of 12 be obtained?

4	8
5	7
6	6
7	5
8	4



Ex. Eight pieces of paper are numbered 1 to 8 and placed in a box. One piece of paper is drawn from the box, its number is written down, and the piece of paper is NOT replaced in the box. A second piece of paper is drawn from the box, and its number is written down. How many different ways can a sum of 12 be obtained?

4 8
5 7
7 5
8 4



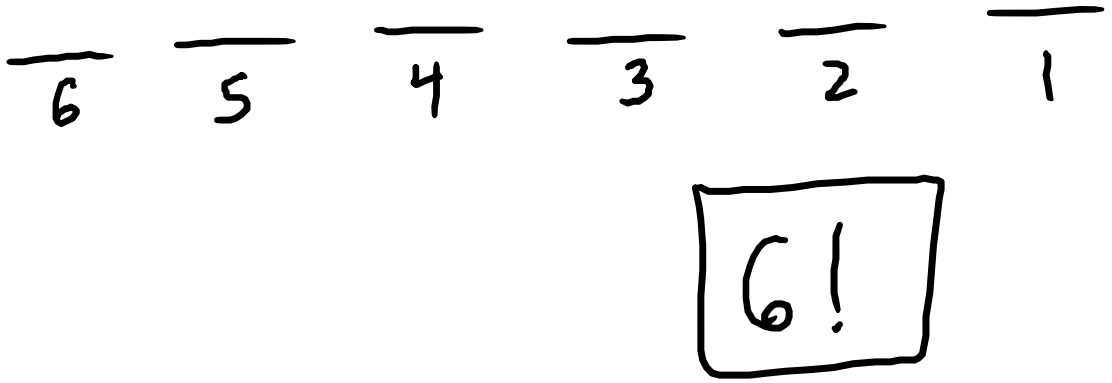
Ex. How many different phone numbers are there in one area code? [Keep in mind that numbers can't start with 0 or 1.]

$8 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10$

$8{,}000{,}000$

A <u>permutation</u> is the rearrangement of elements.

<u>Ex.</u> How many permutation are there of the letters A, B, C, D, E, and F?



The number of permutations of n elements is $n!$

Ex. How many distinguishable ways can the letters in BANANA be written?

$$\frac{6!}{2!\,3!}$$

N repeats 2 times

A repeats 3 times

Ex. Eight horses are running in a race. How many different ways are there for the horses to take 1^{st}, 2^{nd}, and 3^{rd} place?

8	7	6

$$8 \cdot 7 \cdot 6 = \frac{8!}{5!}$$

The number of permutations of n elements taken r at a time is

$$_nP_r = \frac{n!}{(n-r)!}$$

In the last example,

$$_8P_3 = \frac{8!}{5!} = \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{5\cdot 4\cdot 3\cdot 2\cdot 1} = 8\cdot 7\cdot 6$$

A combination is a list of elements where order is not important.

{A, B, C} and {B, C, A}

are different **permutations** (where order matters) but the same **combination** (where order doesn't matter)

The number of combinations of n elements taken r at a time is ${}_nC_r = \frac{n!}{r!(n-r)!}$

Combinations

Order doesn't matter

Permutations

Order matters

Who won a prize?

Who came in 1^{st}, 2^{nd}, 3^{rd}?

Secretariat, Seabiscuit, Maximus
Seabiscuit, Maximus, Secretariat

Secretariat, Seabiscuit, Maximus
Seabiscuit, Maximus, Secretariat

$$_8C_3 = \frac{8!}{3!5!}$$

$$_8P_3 = \frac{8!}{5!}$$

Ex. A standard poker hand consists of five cards dealt from a deck of 52. How many different poker hands are possible?

$$_{52}C_5 = \frac{52!}{5!\,47!} = 2,598,960$$

Ex. On a 15-member baseball team, a batting order of 9 players needs to be created. How many different batting orders are possible?

$$_{15}P_9 = 1,816,214,400$$

Probability

A sample space is a list of possible outcomes

Ex. Find the sample space if

a) one coin is tossed {H, T}

b) two coins are tossed {HH, HT, TT, TH}

The probability of an event is

$$P(\text{event}) = \frac{\text{number favorable}}{\text{number possible}}$$

Probabilities are always between
0 (impossible) and 1 (certain)

Ex. Two coins are tossed. What is the probability of both landing on heads?

$$\frac{\#\ \text{favorable}}{\#\ \text{possible}} = \frac{1}{4}$$

Ex. A card is drawn from a deck, what is the probability that it is an ace?

$$\frac{\#\ \text{favorable}}{\#\ \text{possible}} = \frac{4}{52} = \frac{1}{13}$$

1 6
2 5
3 4
4 3
5 2
6 1

Ex. Two dice are tossed. What is the probability that the sum of the tosses is 7?

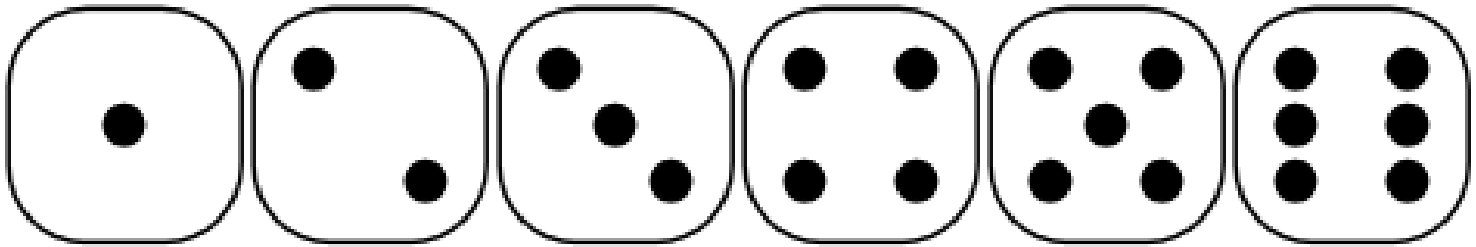
$$\frac{\#\ \text{favorable}}{\#\ \text{possible}} = \frac{6}{36} = \boxed{\frac{1}{6}}$$

$\frac{}{6}$ $\frac{}{6}$

52 Cards

Face Cards

♣	2	3	4	5	6	7	8	9	10	J	Q	K	A
♠	2	3	4	5	6	7	8	9	10	J	Q	K	A
♦	2	3	4	5	6	7	8	9	10	J	Q	K	A
♥	2	3	4	5	6	7	8	9	10	J	Q	K	A



Ex. What is the probability of winning a lottery where a player chooses six numbers (in any order) between 1 and 41?

$$\frac{\#\ \text{favorable}}{\#\ \text{possible}} = \frac{1}{{}_{41}C_{6}} = \frac{1}{4,496,388}$$

$$= .00000022$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Ex. When drawing a card from a standard deck, what is the probability that the card is a heart or a face card?

$$\frac{\#\text{ favorable}}{\#\text{ possible}} = \frac{13 + 12 - 3}{52} = \frac{22}{52} = \frac{11}{26}$$

♣	2	3	4	5	6	7	8	9	10	J	Q	K	A
♠	2	3	4	5	6	7	8	9	10	J	Q	K	A
♦	2	3	4	5	6	7	8	9	10	J	Q	K	A
♥	2	3	4	5	6	7	8	9	10	J	Q	K	A

Two events are independent if their occurrences don't affect each other

If A and B are independent, then

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Ex. A random number generator selects three integers from 1 to 20. What is the probability that all three numbers are less than or equal to 5?

$$\frac{\#\text{ favorable}}{\#\text{ possible}} = \underset{1^{st}\ \#}{\frac{5}{20}} \cdot \underset{2^{nd}\ \#}{\frac{5}{20}} \cdot \underset{3^{rd}\ \#}{\frac{5}{20}} = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$$