First part is out of 40 pts.
Second part is out of 65 pts.
Total points possible is 105 pts.
→ Grade is out of 100 pts.

$500 \cdot (\text{desired \%}) = \text{total points needed}$

Total points needed

- 3 undropped exams

- Percent in quiz category

Points needed on final (out of 100)

<u>Chapter 1</u>

- Graphing Equations
- *x*- and *y*-intercepts

$y = 0$ $\quad$ $x = 0$

- Solving Linear Equations

<u>Ex.</u> Solve $\frac{x}{3} + \frac{3x}{4} = 2$

$$\overset{4}{\cancel{12}} \cdot \frac{x}{\cancel{3}} + \overset{3}{\cancel{12}} \cdot \frac{3x}{\cancel{4}} = 12 \cdot 2$$

$$4x + 9x = 24$$

$$13x = 24$$

$$x = \frac{24}{13}$$

- Word problems with one variable equations

Ex. You have a job for which your annual salary will be $32,300. This includes a year-end bonus of $500. You will be paid twice a month. What is your pay (before taxes) for each paycheck?

x = amount of each paycheck

$24x + 500 = 32300$

$24x = 31800$

$x = 1325$

- Solving quadratic equations → factoring → quad. formula
- Using complex numbers

<u>Ex.</u> Simplify $\frac{2+3i}{4-2i} \cdot \frac{(4+2i)}{(4+2i)}$

$i^2 = -1$

$$= \frac{8+4i+12i+6i^2}{16+8i-8i-4i^2}$$

$i^2 = -1$

$$= \frac{2+16i}{20}$$

$$= \frac{2}{20} + \frac{16}{20}i$$

$$= \frac{1}{10} + \frac{4}{5}i$$

- Solving other types of equations

Ex. Solve $\sqrt{2x+7} - x = 2$

$+x \quad +x$

$\sqrt{2x+7}^2 = (x+2)^2$

$2x+7 = x^2+4x+4$

$-2x \quad -7 \qquad -2x \quad -7$

$0 = x^2+2x-3$

$0 = (x+3)(x-1) = 0$

$x+3=0 \qquad x-1=0$

$\cancel{x=-3} \qquad x=1$

$x=-3$

$\sqrt{2(-3)+7} - (-3) = 2$

$\sqrt{-6+7} + 3 = 2$

$\sqrt{1} + 3 = 2$ ✗

$x=1$

$\sqrt{2(1)+7} - 1 = 2$

$\sqrt{2+7} - 1 = 2$

$\sqrt{9} - 1 = 2$ ✓

check

div. by x

log. in orig. equ.

$\sqrt{}$ in orig. equ.

$(x+2)^2 = (x+2)(x+2)$

$= x^2+2x+2x+4$

$= x^2+4x+4$

- Inequalities of one variable

Ex. Solve and graph $|x + 3| \geq 7$

$\boxed{(-\infty, -10] \cup [4, \infty)}$

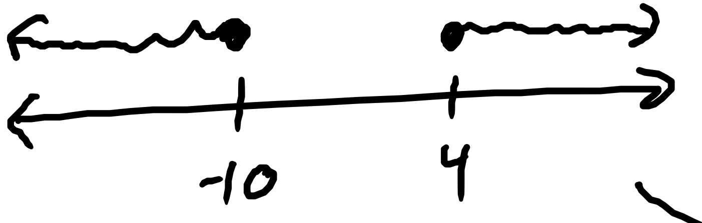
$|x+3| = 7$

$x+3 = 7$

$x = 4$

$x+3 = -7$

$x = -10$



$x = -12$

$|-12+3| \geq 7$

$|-9| \geq 7$

true

$x = 0$

$|0+3| \geq 7$

$|3| \geq 7$

false

$x = 6$

$|6+3| \geq 7$

$|9| \geq 7$

true

Ex. Solve $\dfrac{2x-7}{x-5} \le 3$

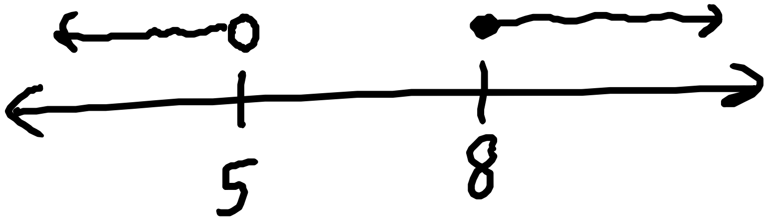
$$\frac{2x-7}{x-5} = \frac{3}{1}$$

$$3(x-5) = 2x-7$$
$$3x-15 = 2x-7$$
$$x-15 = -7$$
$$x = 8$$

undefined: $x-5=0$

$x=5$

$$\boxed{(-\infty, 5) \cup [8, \infty)}$$



$x=0$: $\dfrac{-7}{-5} \le 3$ true

$x=6$: $\dfrac{2(6)-7}{6-1} \le 3$ false

$x=10$: $\dfrac{2(10)-7}{10-5} \le 3$ true

Chapter 2

- Graph and equation of a line

Ex. Find equation of the line that passes through (2,-1) and is perpendicular to the line $2x - 3y = 5$

$y - y_1 = m(x - x_1)$

$\cancel{2x}$ $-2x$ $\qquad -2x$

$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{5}{-3}$

$y = \frac{2}{3}x - \frac{5}{3}$

my slope $= \frac{-3}{2}$

my point $= (2,-1)$

$$\boxed{y - (-1) = \frac{-3}{2}(x - 2)}$$

- Functions
  - x's can't repeat (vertical line test of the graph)
  - function notation $f(x)$
  - domain and range

all x's that can be plugged in

all y's we can get

- Zeroes of a function $\rightarrow$ What x's make $f(x)=0$?

Ex. Find the zeroes of the function $h(t)=\dfrac{2t-3}{t+5}=\dfrac{0}{1}$

$2t-3=0$

$2t=3$

$t=\frac{3}{2}$

- Average rate of change of a function

Ex. Find the average rate of change of $f(x) = x^3 - 3x$ from $x_1 = -2$ to $x_2 = 0$.

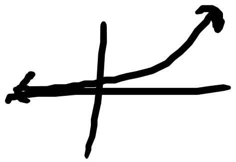
$f(-2) = (-2)^3 - 3(-2) = -8 + 6 = -2$ $\quad (-2, -2)$

$f(0) = 0^3 - 3(0) = 0$ $\quad (0, 0)$

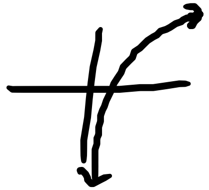
ave. rate $= \frac{-2-0}{-2-0} = 1$

- Using parent functions
  - Know the basic power, exponential, reciprocal, logarithmic functions

$y = 2^x$



$y = \ln x$



Vertical and Horizontal Shifts

Let c be a positive real number. **Vertical and horizontal shifts** in the graph of $y = f(x)$ are represented as follows.

1. Vertical shift c units *upward:* $h(x) = f(x) + c$
2. Vertical shift c units *downward:* $h(x) = f(x) - c$

outside ⇒ vertical

3. Horizontal shift c units to the *right:* $h(x) = f(x - c)$
4. Horizontal shift c units to the *left:* $h(x) = f(x + c)$

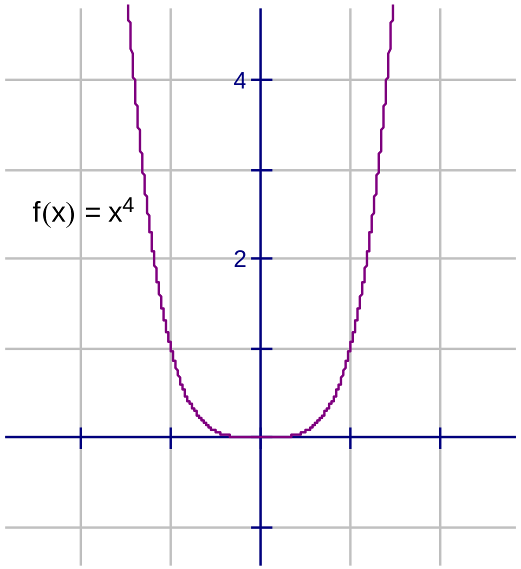
inside ⇒ horiz.

Reflections in the Coordinate Axes

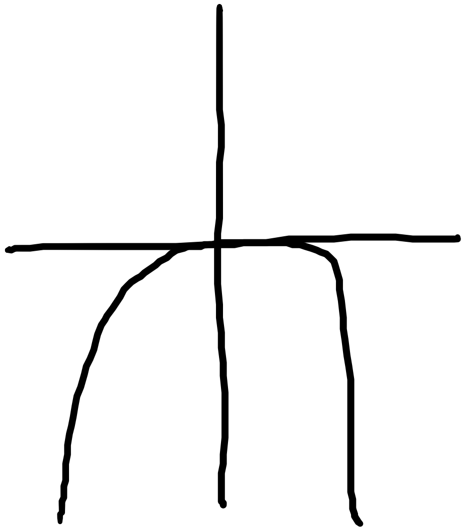
Reflections in the coordinate axes of the graph of $y = f(x)$ are represented as follows.

1. Reflection in the x-axis: $h(x) = -f(x)$ → vert. flip
2. Reflection in the y-axis: $h(x) = f(-x)$ → horiz. flip

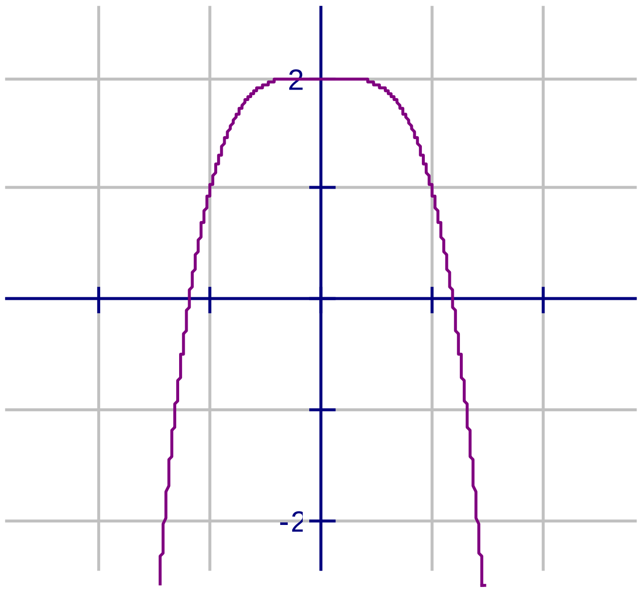
$\underline{\text{Ex.}}$ Given the graph of $y = x^4$ below, identify the equation of the second graph.



$y = x^4$



$y = -x^4$



$y = -x^4 + 2$

- Composite functions

$$(f \circ g)(x) \quad \text{means} \quad f(g(x))$$

<u>Ex.</u> Given $f(x) = x + 2$ and $g(x) = 4 - x^2$

find $(f \circ g)(x)$

$= f(g(x))$

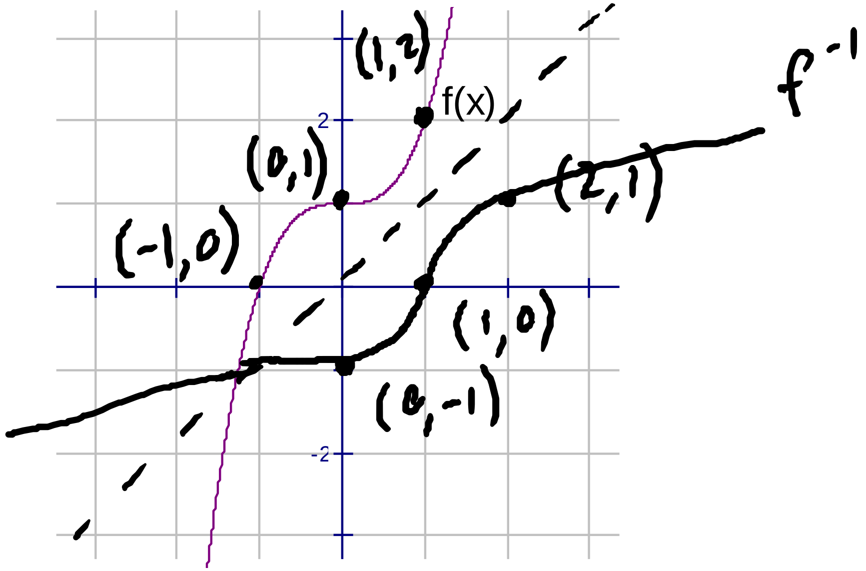
$= f(4 - x^2)$

$= (4 - x^2) + 2$

$= 6 - x^2$

- Inverse functions
  - To find the equation for f^{-1}, switch the roles of x and y and then solve for y.
  - To find the graph of f^{-1}, reflect the graph of f over the line $y = x$.

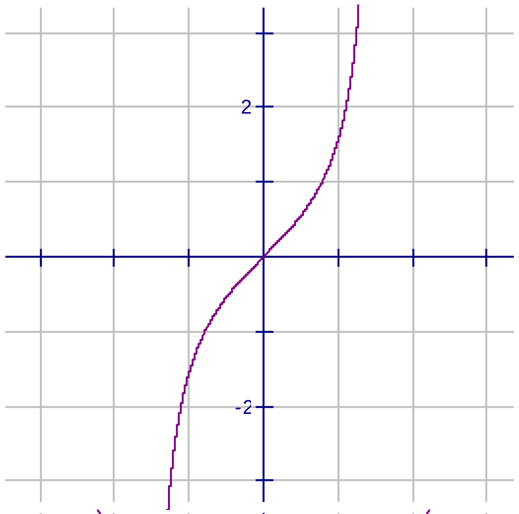
<u>Ex.</u> Given the graph of $f(x)$ below, sketch f^{-1}.



- A function is invertible if no y’s repeat (graph passes the horizontal line test)

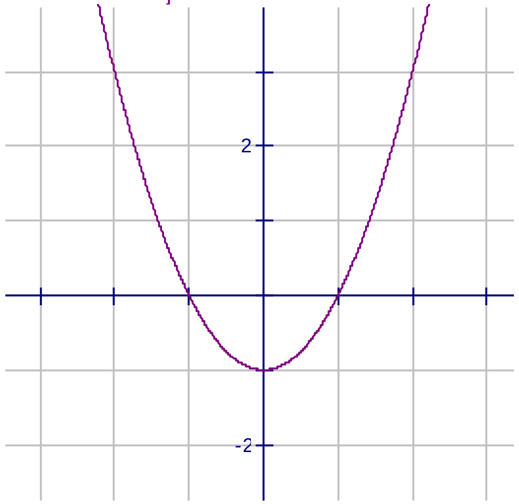
Ex. Are these functions invertible?

a)



yes

b)



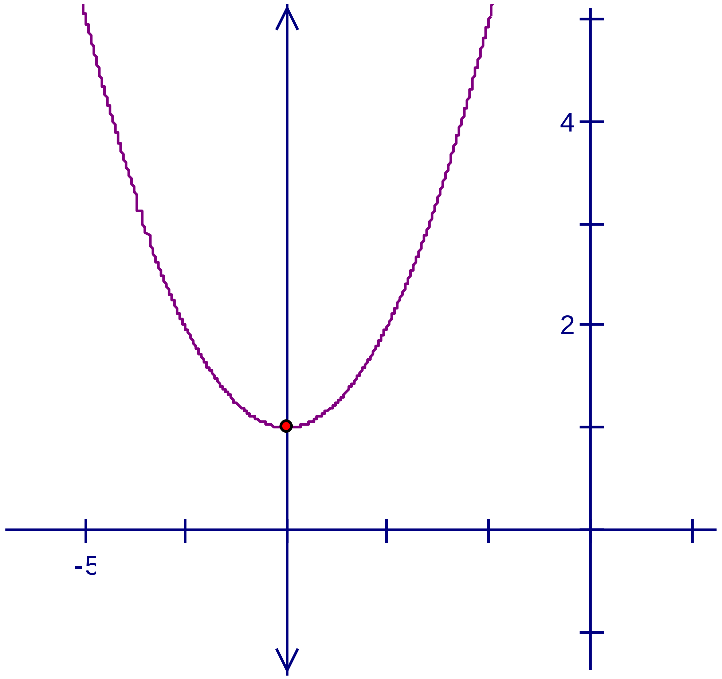
no

Even if the function is invertible, you still may not be able to find an equation.

Chapter 3

- Quadratic functions

$$f(x) = ax^2 + bx + c$$



Parabolas are symmetric with respect to a vertical line, called the axis of symmetry

This line has the equation

$$x = -\frac{b}{2a}$$

The turning point of a parabola is called the vertex.

Notice that the *x*-coordinate of the vertex is also $x = -\frac{b}{2a}$

If the lead coefficient, a, is positive, the parabola opens upward.

The <u>standard form</u> of a quadratic function is

$$f(x) = a(x-h)^2 + k$$

The vertex of the parabola is the point (h,k).

If $a > 0$, the parabola opens upward.

If $a < 0$, the parabola opens downward.

Ex. Write the equation of the parabola whose vertex is (1,2) and that contains the point (0,0).

h k

$$y = a(x-h)^2 + k$$

$$y = a(x-1)^2 + 2$$

$$0 = a(0-1)^2 + 2$$

$$0 = a(-1)^2 + 2$$

$$0 = a + 2$$

$$a = -2$$

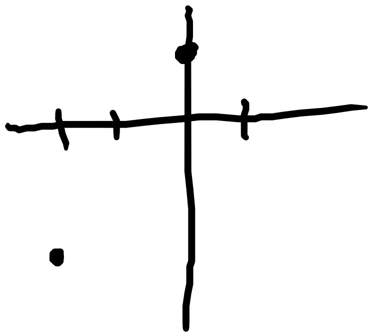
$$\boxed{y = -2(x-1)^2 + 2}$$

- End behavior of a polynomial
  - What happens off to the left (as $x \to -\infty$) and to the right (as $x \to \infty$)
  - Intermediate Value Theorem

Ex. Use the Intermediate Value Theorem to show that $f(x) = x^3 - x^2 + 1$ has a zero on the interval [-2,0].

$f(-2) = (-2)^3 - (-2)^2 + 1 = -8 - 4 + 1 = -11 < 0$

$f(0) = 0^3 - 0^2 + 1 = 1 > 0$



- Polynomial division (or synthetic division)

<u>Ex.</u> Divide $x^4 - 10x^2 - 2x + 4$ by $x + 3$.

$$\boxed{x^3 - 3x^2 - x + 1 + \frac{1}{x+3}}$$

$$x+3\,\overline{)\,x^4 + 0x^3 - 10x^2 - 2x + 4}$$

$$\underline{-x^4 + {}^-3x^3}$$

$$-3x^3 - 10x^2$$

$$\underline{+3x^3 + 9x^2}$$

$$-x^2 - 2x$$

$$\underline{+x^2 + 3x}$$

$$x + 4$$

$$\underline{-x + {}^-3}$$

$$1$$

factor $x+3$ $\rightarrow$ root $x = -3$

$$\begin{array}{r|rrrr|r} -3 & 1 & 0 & -10 & -2 & 4 \\ & & -3 & 9 & 3 & -3 \\ \hline & 1 & -3 & -1 & 1 & \boxed{1} \\ & x^3 & x^2 & x & \text{const} & \end{array}$$

$$\boxed{x^3 - 3x^2 - x + 1 + \frac{1}{x+3}}$$

Ex. Find the rational zeroes of $f(x) = x^4 - x^3 + x^2 - 3x - 6$, then factor.

$\to 1$

$\to 1, 2, 3, 6$

$$\boxed{\begin{array}{l} \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{6}{1} \\ -\frac{1}{1}, -\frac{2}{1}, -\frac{3}{1}, -\frac{6}{1} \end{array}}$$

$f(1) = 1 - 1 + 1 - 3 - 6 = \cancel{-8}$

$f(-1) = 1 + 1 + 1 + 3 - 6 = 0 \leftarrow x = -1$ is zero

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 1 & -3 & -6 \\ & & -1 & 2 & -3 & 6 \\ \hline 2 & 1 & -2 & 3 & -6 & \boxed{0} \\ & & 2 & 0 & 6 & \\ \hline & 1 & 0 & 3 & \boxed{0} & \end{array}$$

$f(2) = 16 - 8 + 4 - 6 - 6 = 0$

$x = 2$ is a zero

$f(x) = (x+1)(x^3 - 2x^2 + 3x - 6)$

$$\boxed{f(x) = (x+1)(x-2)(x^2+3)}$$

Ex. Find a fourth-degree polynomial that has 1, -1, and $3i$ as zeroes.

$-3i$

$$f(x) = (x-1)(x+1)(x-3i)(x+3i)$$

$$f(x) = (x^2-1)(x^2+9)$$

$$f(x) = x^4 + 8x^2 - 9$$

y is <u>directly proportional</u> (varies directly) to x if

$y = kx$ for some constant k

y is <u>directly proportional to the n^{th} power</u> to x if

$y = kx^n$ for some constant k

y is <u>inversely proportional</u> (varies inversely) to x if

$y = \dfrac{k}{x}$ for some constant k

z is <u>jointly proportional</u> (varies jointly) to x and y

if $z = kxy$ for some constant k

Ex. The state income tax is directly proportional to gross income. If the tax is \$46.05 for an income of \$1500, write a mathematical model for income tax.

Chapter 4

- Rational Functions
  - Vertical asymptotes (when we divide by 0, maybe)
  - Horizontal asymptotes (value as graph goes left or right) – look at lead terms on top and bottom
  - If degree on top is one more than degree on the bottom, look for a slant asymptote (divide)

Ex. Graph $f(x)=\dfrac{x}{x^2-x-2} = \dfrac{x}{(x-2)(x+1)}$

1) Plug in 0: $f(0)=\dfrac{0}{0^2-0-2}=0$ ← y-int.

2) Factor

3) Set top $=0$: $x=0$ ← x-int.

4) Set bottom $=0$: $(x-2)(x+1)=0$ ← vert. asymp.

$x=2 \quad x=-1$

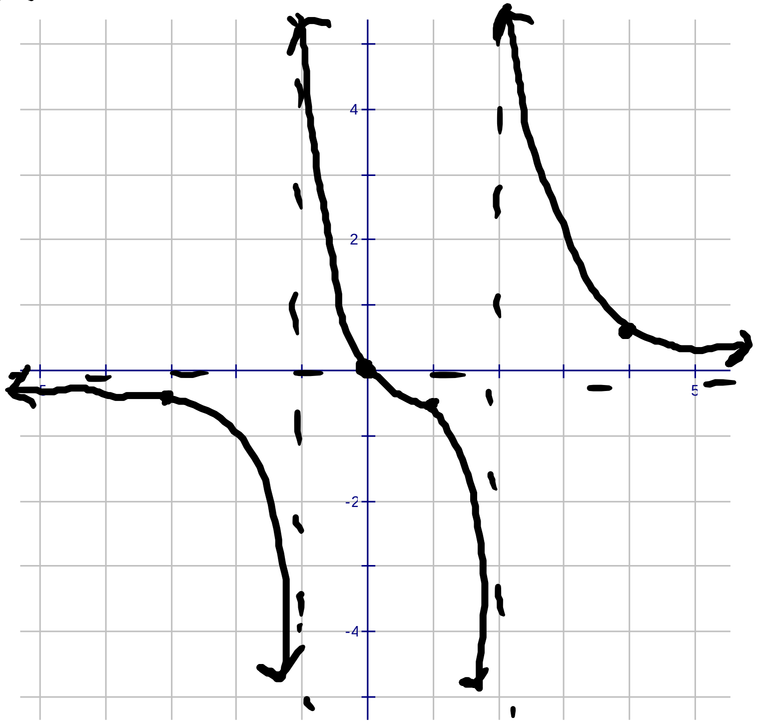
5) As $x\to\infty$, $f \to \dfrac{x}{x^2}=\dfrac{1}{x}\to 0$ $\quad y=0$ ← horiz asymp.

6) Plug in more if needed

$f(4)=\dfrac{4}{10}=\dfrac{2}{5}$

$f(1)=\dfrac{1}{-2}$

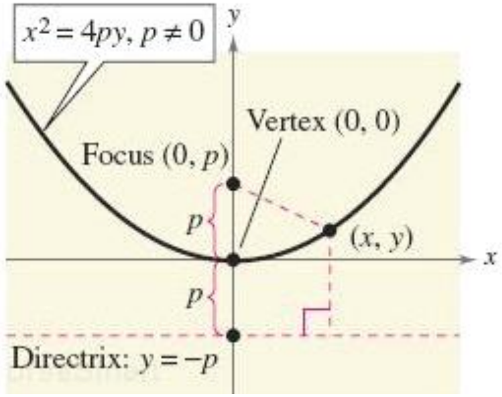
$f(-3)=\dfrac{-3}{10}$



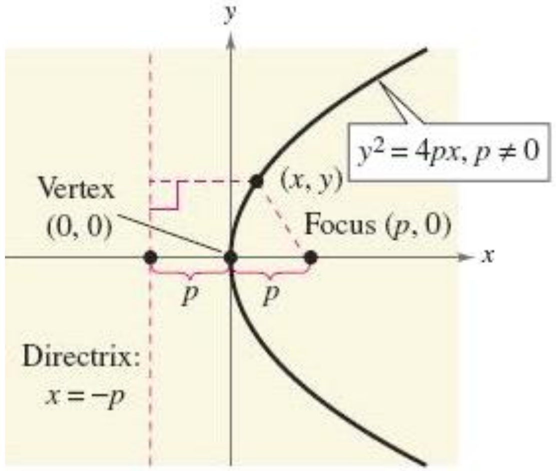
- Conics (circles, parabolas, ellipses, hyperbolas)

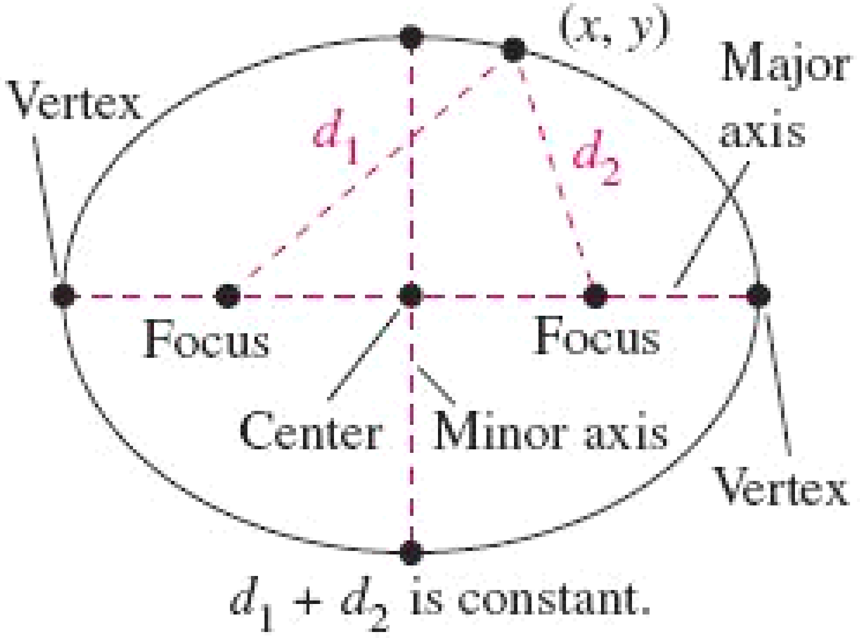
A parabola with vertex (0,0) and directrix $y = -p$ has the equation $x^2 = 4py$

variable that's not squared is direction parabola opens



A parabola with vertex (0,0) and directrix $x = -p$ has the equation $y^2 = 4px$

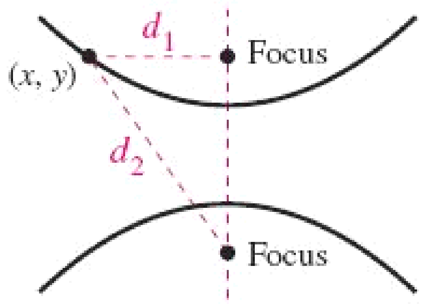




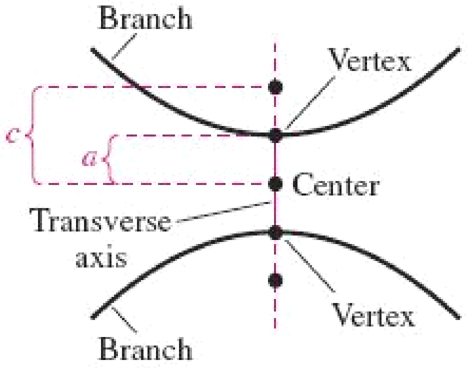
An ellipse centered at (0,0) with horizontal axis length 2*a* and vertical axis length 2*b* has equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

The vertices and foci lie on the major (longer) axis. The foci lie *c* units from the center, where $c^2 = a^2 - b^2$.



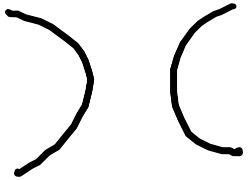
$d_2 - d_1$ is a positive constant.



positive term
is direction
hyperbola opens

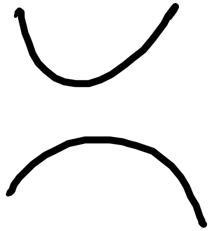
A hyperbola centered at (0,0) with a horizontal transverse axis has equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



A hyperbola centered at (0,0) with a vertical transverse axis has equation

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$



The foci lie c units from the center, where $c^2 = a^2 + b^2$.

Ex. Sketch a graph of the hyperbola $\longrightarrow c^2 = a^2 + b^2$

$\frac{4x^2}{16} - \frac{y^2}{16} = \frac{16}{16}$, and identify the foci.

$$\frac{x^2}{4} - \frac{y^2}{16} = 1$$

a^2 b^2

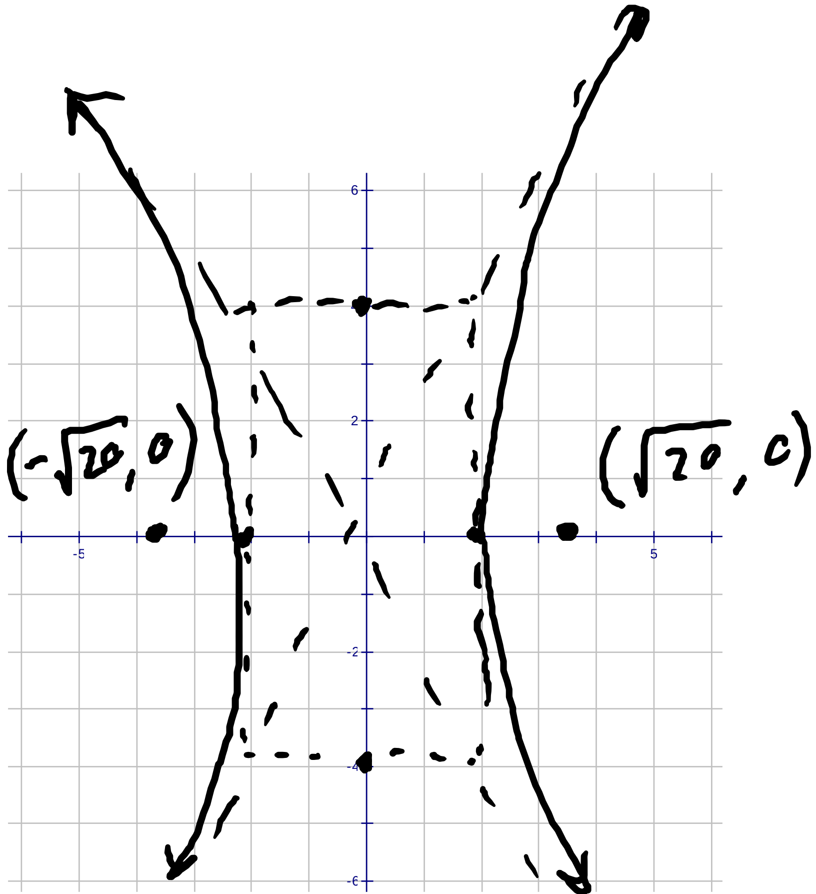
$a = 2$

$b = 4$

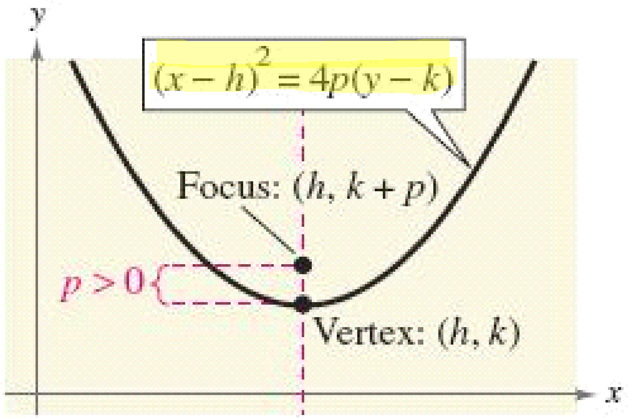
$c^2 = 2^2 + 4^2$

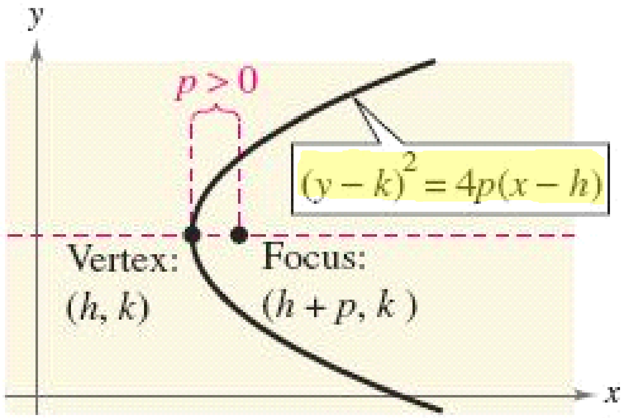
$c^2 = 4 + 16$

$c = \sqrt{20}$

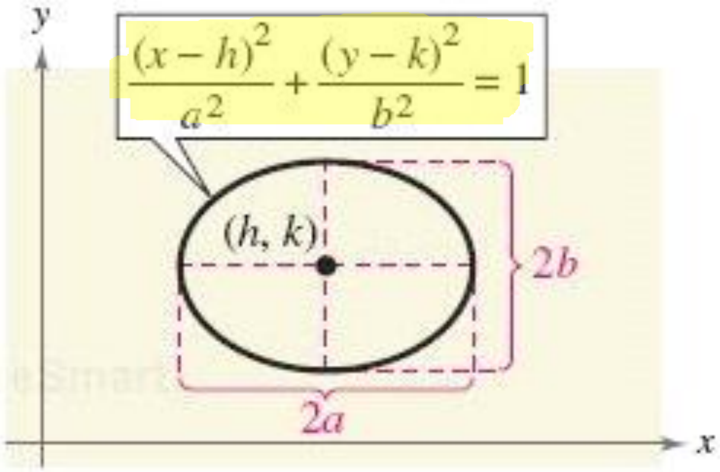


Parabola: Vertex = (h, k)

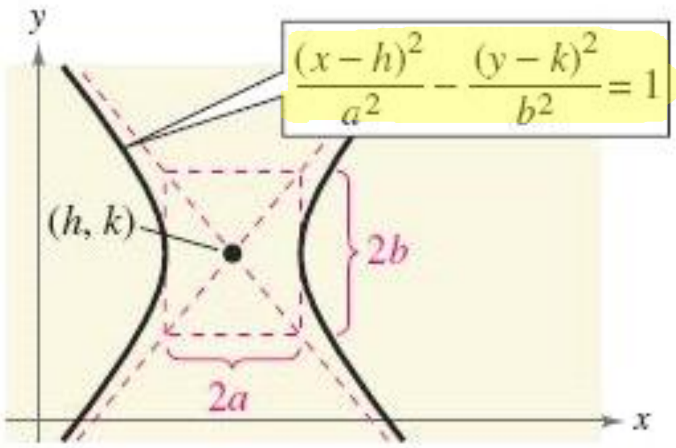




Ellipse: Center = (h, k)



Hyperbola: Center = (h, k)



$\underline{\text{Ex.}}$ Sketch $x^2 + 4y^2 + 6x - 8y + 9 = 0$ ellipse

$-9 \quad -9$

$x^2 + 6x + 4y^2 - 8y = -9$

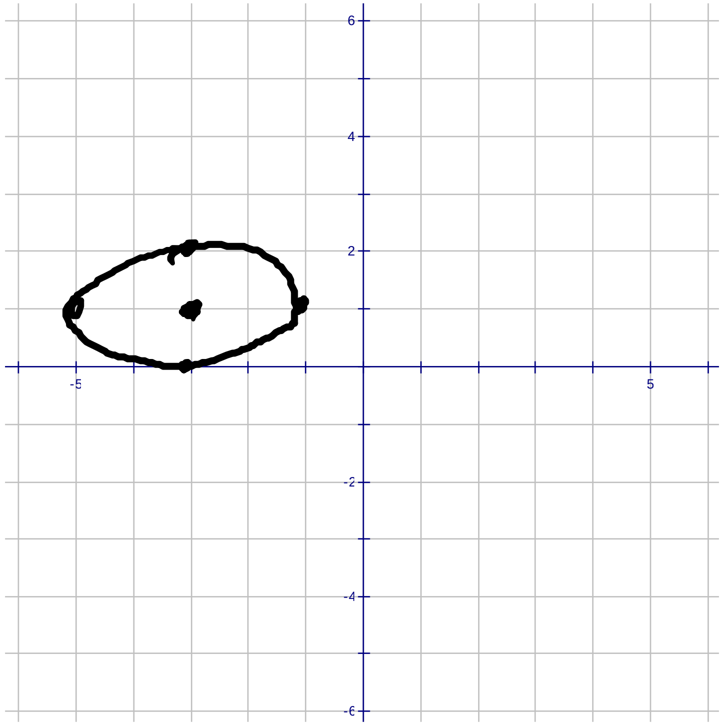
$(x^2 + 6x + \underline{9}) + 4(y^2 - 2y + \underline{1}) = -9 + \underline{9} + \underline{4}$

$$\frac{(x+3)^2}{4} + \frac{4(y-1)^2}{4} = \frac{4}{4}$$

$$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{1} = 1$$

$a = 2$ $b = 1$

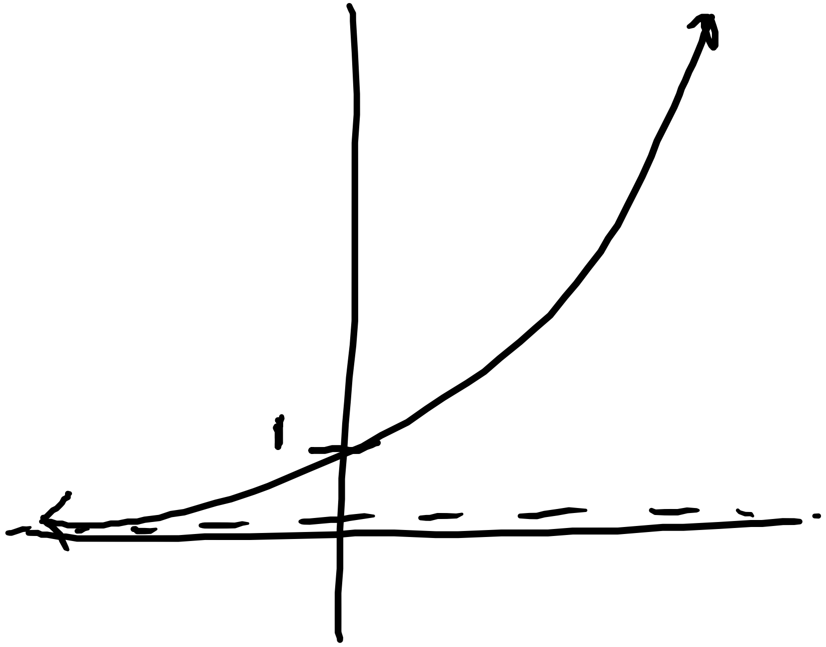
center: $(-3, 1)$



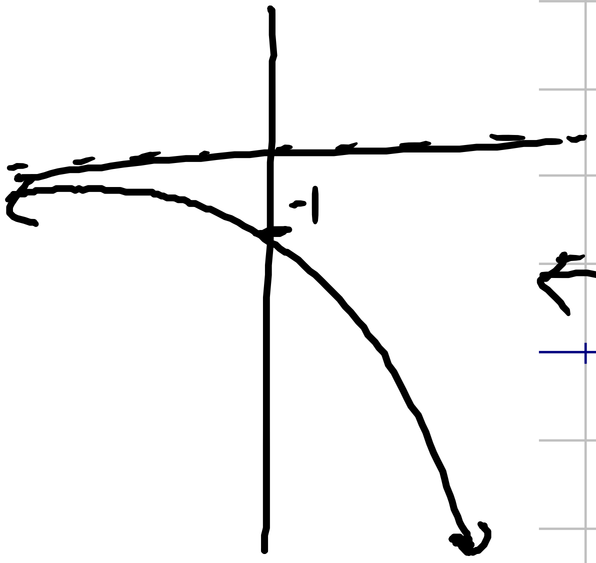
Chapter 5

- Exponential Functions

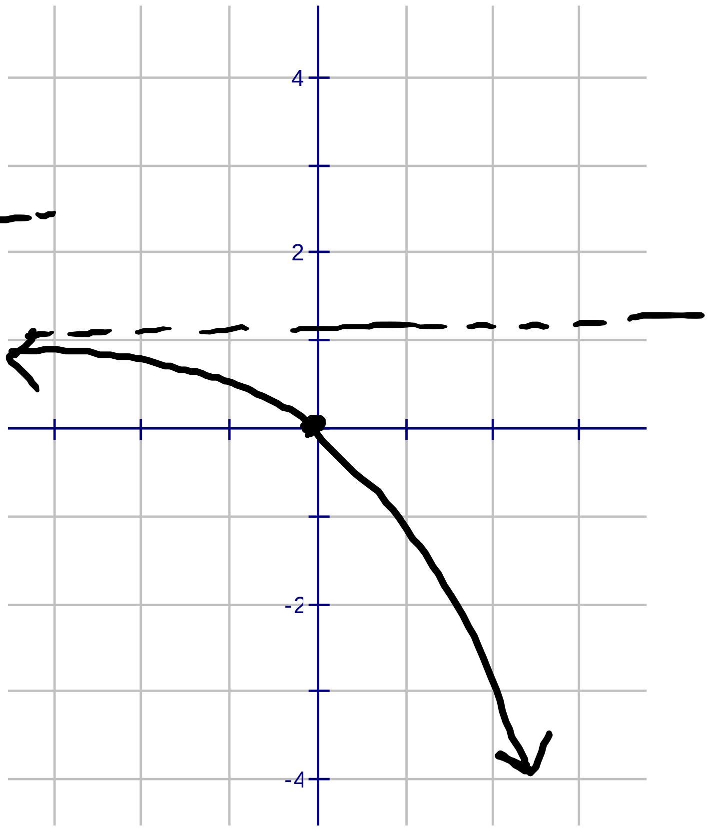
Ex. Sketch the graph of $f(x) = 1 - 3^x$. $= -3^x + 1$



$y = 3^x$



$y = -3^x$



Ex. Solve for x.

$$9^x = 3^{x+1}$$

$$(3^2)^x = 3^{x+1}$$

$$3^{2x} = 3^{x+1}$$

$$2x = x + 1$$

$$x = 1$$

When interest is compounded n times per year, we used the formula

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$A = \text{amount in bank}$

$P = \text{principal invested}$

$r = \text{interest rate}$

$t = \text{time (in years)}$

If interest is compounded continually, we use the formula

$$A = Pe^{rt}$$

- Logarithmic functions

$$y = \cancel{\log}_{a} x \quad \longleftrightarrow \quad x = a^y$$

$$a^y = x$$

<u>Ex.</u> Evaluate by hand.

a) $\log_2 32 = x \rightarrow 32 = 2^x \rightarrow x = 5$

b) $\log_3 1 = 0$

c) $\log_9 3 = x \rightarrow 3 = 9^x \rightarrow x = \frac{1}{2}$

The logarithmic function with base 10 is called the <u>common logarithm</u> is can be written $f(x) = \log x$

The function $f(x) = \log_e x$ is called the <u>natural logarithm function</u>, and it is often written $f(x) = \ln x$

Properties of Logarithms

$a^0 = 1 \quad \leftrightarrow \quad \log_a 1 = 0$

$a^1 = a \quad \leftrightarrow \quad \log_a a = 1$

Since exponents and logarithms are inverse, $a^{\log_a x} = x$ and $\log_a a^x = x$

Since logarithms are one-to-one, we know:
If $\log_a x = \log_a y$, then $x = y$.

Ex. Evaluate $\log_4 25 = \dfrac{\ln 25}{\ln 4} = 2.3$

Properties of Logarithms

$$\log(AB) = \log A + \log B$$ → mult. inside ⇒ add logs

$$\log\left(\frac{A}{B}\right) = \log A - \log B$$ → div. inside ⇒ subtr. logs.

$$\log(A^n) = n \log A$$ → exp. inside ⇒ coeff. outside

These apply to all logarithms, not just the common log

<u>Ex.</u> Find the exact value without a calculator

a) $\log_5 \sqrt[3]{5} = \log_5(5^{1/3}) = \frac{1}{3}$

- Exponential and Logarithmic Equations

Ex. $\ln x - \cancel{\ln 3} = 0$ $\rightarrow$ $\ln x = \ln 3$

$+\cancel{\ln 3}$ $+\ln 3$

$\boxed{x = 3}$

Ex. $\cancel{\log} x = -1$ $\Rightarrow$ $\boxed{x = 10^{-1}}$

10

Ex. $e^{x^2} = e^{-3x+4}$ $\Rightarrow$ $x^2 = -3x + 4$

$\vdots$

$\underline{\text{Ex.}}$ $2(3^{2x-5}) - \cancel{4} = 11$

$+4 \qquad +4$

$\frac{\cancel{2}(3^{2x-5})}{\cancel{2}} = \frac{15}{2}$

$\ln(3^{2x-5}) = \ln\left(\frac{15}{2}\right)$

$(2x-5)\ln 3 = \ln \frac{15}{2}$

$2x-5 = \frac{\ln\left(\frac{15}{2}\right)}{\ln 3}$

$2x = \frac{\ln\left(\frac{15}{2}\right)}{\ln 3} + 5$

$x = \frac{\frac{\ln\left(\frac{15}{2}\right)}{\ln 3} + 5}{2}$

$\underline{\text{Ex.}}$ $5e^{2x} = e^{x-3}$

Ex. $\log_3(5x - 1) = \log_3(x + 7)$

$$5x - 1 = x + 7$$
$$4x - 1 = 7$$
$$4x = 8$$
$$x = 2$$

Ex. $\log_6(3x + 14) - \log_6 5 = \log_6 2x$

$$\log_6\left(\frac{3x+14}{5}\right) = \log_6 2x$$

$$\frac{3x+14}{5} = 2x$$

$$\vdots$$

- Exponential Models

Ex. The number N of bacteria in a culture is modeled by $N = 450e^{kt}$, where t is time in hours. If $N = 600$ when $t = 3$, ~~estimate the time required for the population to double.~~ find pop. at $t=5$.

$$600 = 450e^{k(3)}$$

$$\ln\left(\frac{600}{450}\right) = \ln\left(e^{3k}\right)$$

$$\ln\left(\frac{600}{450}\right) = 3k$$

$$k = \frac{\ln\left(\frac{600}{450}\right)}{3} = .096$$

$$N = 450e^{.096t}$$

$$N = 450e^{.096(5)}$$

$$= 726.85$$

Chapter 6

- Solving systems of equations
  - Substitution
  - Elimination
  - Augmented matrix
- No solutions (all variables drop out and you’re left with a false statement)
- Many solutions (all variables drop out and you’re left with a true statement)

<u>Ex.</u> Solve the system $\begin{cases} x+y=4 \\ x-y=2 \end{cases}$

$2x = 6$

$x = 3$

$x=3$

$3+y=4$

$y=1$

$\boxed{(3,1)}$

Ex. Solve the system $\begin{cases} -x + y = -4 \\ x^2 + y = -2 \end{cases}$

$-x + y = -4$
$+x \qquad\qquad +x$
$\longrightarrow y = x - 4$

$x^2 + y = -2$
$-x^2 \qquad\qquad -x^2$
$\longrightarrow y = -x^2 - 2$

$x - 4 = -x^2 - 2$
$+x^2 \quad +2 \quad +x^2 + 2$

$x^2 + x - 2 = 0$
$(x+2)(x-1) = 0$

$x = -2$

$y = -2 - 4$
$= -6$
$(-2, -6)$

$x = 1$

$y = 1 - 4$
$= -3$
$(1, -3)$

Ex. Solve the system $\begin{cases} 5x+3y=9 \\ 2x-4y=14 \end{cases}$

$$5x+3y=9 \xrightarrow{\times 4} 20x+12y=36$$

$$2x-4y=14 \xrightarrow{\times 3} 6x-12y=42$$

$$26x=78$$

$$x=3$$

$$2(3)-4y=14$$

$$6-4y=14$$

$$-4y=8$$

$$y=-2$$

$$\boxed{(3,-2)}$$

Ex. Solve the system $\begin{cases} x-3y+z=1 \\ 2x-y-2z=2 \\ x+2y-3z=-1 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 2 & -1 & -2 & 2 \\ 1 & 2 & -3 & -1 \end{array}\right]$

$\vdots$

Ex. Find a quadratic equation $y = ax^2 + bx + c$ that passes through the points (-1,3), (1,1), and (2,6).

$(-1,3): \quad 3 = a(-1)^2 + b(-1) + c \longrightarrow 3 = a - b + c$

$(1,1): \quad 1 = a(1)^2 + b(1) + c \longrightarrow 1 = a + b + c$

$(2,6): \quad 6 = a(2)^2 + b(2) + c \longrightarrow 6 = 4a + 2b + c$

Chapter 7

- Using matrices to solve systems of equations
- Adding, subtracting, multiplying matrices
- Finding inverse matrix (shortcut for 2 x 2)
- Determinant of a matrix

Chapter 8

- Sequences and series
- Arithmetic sequences/series
- Geometric sequences/series
- Binomial Theorem (including ${}_{n}C_{r}$)
- Counting Principal (${}_{n}P_{r}$)
- Probability

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