Diagonalization

Ex. If $D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$, find D^3.

If a matrix is diagonal, it's easy to find a power of the matrix

Ex. Consider the matrices $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$, $P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$, and $D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$. It can be shown that $A = PDP^{-1}$, find A^3.

Def. A matrix A is diagonalizable if it is similar to a diagonal matrix D.

$$A = PDP^{-1}$$

Thm. The Diagonalization Theorem

An $n \times n$ matrix A is diagonalizable if and only if its eigenvectors form a basis for $\mathbb{R}^n$.

- This is called an eigenvector basis of $\mathbb{R}^n$.
- The columns of P are the eigenvectors of A and the diagonal entries of D are the corresponding eigenvalues (remember, similar matrices have the same eigenvalues).

To "diagonalize" a matrix, we need to find D and P.

Ex. Diagonalize, if possible, $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$

We could check by using $A = PDP^{-1}$ → $AP = PD$

Ex. Determine if $A = \begin{bmatrix} 9 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 5 \end{bmatrix}$ is diagonalizable.

If there are n distinct eigenvalues, then there are n independent eigenvectors → diagonalizable.

Ex. Diagonalize, if possible, $A = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$

Ex. Diagonalize, if possible, $A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 1 & 4 & -3 & 0 \\ -1 & -2 & 0 & -3 \end{bmatrix}$