

Determinant of a Matrix

The determinant of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by

$$\det A = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

We can only find the determinant of a square matrix

Ex. Find the determinant

a. $\begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix}$

b. $\begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix}$

To find the determinant of a larger matrix, we will use minors and cofactors

The minor of entry a_{ij} , written M_{ij} , is the determinant of the matrix found by removing row i and column j of matrix A

The cofactor of entry a_{ij} , written C_{ij} , is

$$C_{ij} = (-1)^{i+j} M_{ij}$$

Ex. Find some minors and cofactors of $A = \begin{bmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 4 & 0 & 1 \end{bmatrix}$

The determinant of a square matrix is the sum of the entries of any row (or column) multiplied by the corresponding cofactors

Ex. Find the determinant of $A = \begin{bmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 4 & 0 & 1 \end{bmatrix}$

Ex. Find the determinant of $A = \begin{bmatrix} -2 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{bmatrix}$

Ex. Find the determinant of

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 4 \\ -1 & 1 & 3 & 2 & 1 \\ 0 & 2 & 0 & 3 & -2 \\ 3 & 4 & 0 & 2 & 3 \\ 1 & -2 & 0 & 2 & 1 \end{bmatrix}$$

Ex. Find the determinant of $A = \begin{bmatrix} 5 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$

The determinant of a triangular matrix is the product of the entries on the main diagonal.