1. From $4m^2 + m = 0$ we obtain $m = 0$ and $m = -1/4$ so that $y = c_1 + c_2e^{-x/4}$.

3. From $m^2 - m - 6 = 0$ we obtain $m = 3$ and $m = -2$ so that $y = c_1e^{3x} + c_2e^{-2x}$.

5. From $m^2 + 8m + 16 = 0$ we obtain $m = -4$ and $m = -4$ so that $y = c_1e^{-4x} + c_2xe^{-4x}$.

7. From $12m^2 - 5m - 2 = 0$ we obtain $m = -1/4$ and $m = 2/3$ so that $y = c_1e^{-x/4} + c_2e^{2x/3}$.

9. From $m^2 + 9 = 0$ we obtain $m = 3i$ and $m = -3i$ so that $y = c_1 \cos 3x + c_2 \sin 3x$.

11. From $m^2 - 4m + 5 = 0$ we obtain $m = 2 \pm i$ so that $y = e^{2x}(c_1 \cos x + c_2 \sin x)$.

13. From $3m^2 + 2m + 1 = 0$ we obtain $m = -1/3 \pm \sqrt{2}\,i/3$ so that

$$y = e^{-x/3}[c_1 \cos(\sqrt{2}x/3) + c_2 \sin(\sqrt{2}x/3)].$$

15. From $m^3 - 4m^2 - 5m = 0$ we obtain $m = 0$, $m = 5$, and $m = -1$ so that

$$y = c_1 + c_2e^{5x} + c_3e^{-x}.$$

17. From $m^3 - 5m^2 + 3m + 9 = 0$ we obtain $m = -1$, $m = 3$, and $m = 3$ so that

$$y = c_1e^{-x} + c_2e^{3x} + c_3xe^{3x}.$$

19. From $m^3 + m^2 - 2 = 0$ we obtain $m = 1$ and $m = -1 \pm i$ so that

$$u = c_1e^t + e^{-t}(c_2 \cos t + c_3 \sin t).$$

21. From $m^3 + 3m^2 + 3m + 1 = 0$ we obtain $m = -1$, $m = -1$, and $m = -1$ so that

$$y = c_1e^{-x} + c_2xe^{-x} + c_3x^2e^{-x}.$$

23. From $m^4 + m^3 + m^2 = 0$ we obtain $m = 0$, $m = 0$, and $m = -1/2 \pm \sqrt{3}\,i/2$ so that

$$y = c_1 + c_2x + e^{-x/2}[c_3 \cos(\sqrt{3}x/2) + c_4 \sin(\sqrt{3}x/2)].$$

25. From $16m^4 + 24m^2 + 9 = 0$ we obtain $m = \pm\sqrt{3}\,i/2$ and $m = \pm\sqrt{3}\,i/2$ so that

$$y = c_1 \cos(\sqrt{3}x/2) + c_2 \sin(\sqrt{3}x/2) + c_3x \cos(\sqrt{3}x/2) + c_4x \sin(\sqrt{3}x/2).$$

27. From $m^5 + 5m^4 - 2m^3 - 10m^2 + m + 5 = 0$ we obtain $m = -1$, $m = -1$, $m = 1$, and $m = 1$, and $m = -5$ so that

$$u = c_1e^{-r} + c_2re^{-r} + c_3e^{r} + c_4re^{r} + c_5e^{-5r}.$$

29. From $m^2 + 16 = 0$ we obtain $m = \pm 4i$ so that $y = c_1 \cos 4x + c_2 \sin 4x$. If $y(0) = 2$ and $y'(0) = -2$ then $c_1 = 2$, $c_2 = -1/2$, and $y = 2\cos 4x - \frac{1}{2}\sin 4x$.

31. From $m^2 - 4m - 5 = 0$ we obtain $m = -1$ and $m = 5$, so that $y = c_1e^{-t} + c_2e^{5t}$. If $y(1) = 0$ and $y'(1) = 2$, then $c_1e^{-1} + c_2e^5 = 0$, $-c_1e^{-1} + 5c_2e^5 = 2$, so $c_1 = -e/3$, $c_2 = e^{-5}/3$, and $y = -\frac{1}{3}e^{1-t} + \frac{1}{3}e^{5t-5}$.

33. From $m^2+m+2 = 0$ we obtain $m = -1/2 \pm \sqrt{7}\,i/2$ so that $y = e^{-x/2}[c_1\cos(\sqrt{7}\,x/2) + c_2\sin(\sqrt{7}\,x/2)]$. If $y(0) = 0$ and $y'(0) = 0$ then $c_1 = 0$ and $c_2 = 0$ so that $y = 0$.

35. From $m^3+12m^2+36m = 0$ we obtain $m = 0$, $m = -6$, and $m = -6$ so that $y = c_1 + c_2e^{-6x} + c_3xe^{-6x}$. If $y(0) = 0$, $y'(0) = 1$, and $y''(0) = -7$ then

$$c_1 + c_2 = 0, \quad -6c_2 + c_3 = 1, \quad 36c_2 - 12c_3 = -7,$$

so $c_1 = 5/36$, $c_2 = -5/36$, $c_3 = 1/6$, and $y = \frac{5}{36} - \frac{5}{36}e^{-6x} + \frac{1}{6}xe^{-6x}$.

37. From $m^2 - 10m + 25 = 0$ we obtain $m = 5$ and $m = 5$ so that $y = c_1e^{5x} + c_2xe^{5x}$. If $y(0) = 1$ and $y(1) = 0$ then $c_1 = 1$, $c_1e^5 + c_2e^5 = 0$, so $c_1 = 1$, $c_2 = -1$, and $y = e^{5x} - xe^{5x}$.

39. From $m^2 + 1 = 0$ we obtain $m = \pm i$ so that $y = c_1\cos x + c_2\sin x$ and $y' = -c_1\sin x + c_2\cos x$. From $y'(0) = c_1(0) + c_2(1) = c_2 = 0$ and $y'(\pi/2) = -c_1(1) = 0$ we find $c_1 = c_2 = 0$. A solution of the boundary-value problem is $y = 0$.

41. The auxiliary equation is $m^2 - 3 = 0$ which has roots $-\sqrt{3}$ and $\sqrt{3}$. By (10) the general solution is $y = c_1e^{\sqrt{3}x} + c_2e^{-\sqrt{3}x}$. By (11) the general solution is $y = c_1\cosh\sqrt{3}x + c_2\sinh\sqrt{3}x$. For $y = c_1e^{\sqrt{3}x} + c_2e^{-\sqrt{3}x}$ the initial conditions imply $c_1 + c_2 = 1$, $\sqrt{3}c_1 - \sqrt{3}c_2 = 5$. Solving for c_1 and c_2 we find $c_1 = \frac{1}{2}(1 + 5\sqrt{3})$ and $c_2 = \frac{1}{2}(1 - 5\sqrt{3})$ so $y = \frac{1}{2}(1 + 5\sqrt{3})e^{\sqrt{3}x} + \frac{1}{2}(1 - 5\sqrt{3})e^{-\sqrt{3}x}$. For $y = c_1\cosh\sqrt{3}x + c_2\sinh\sqrt{3}x$ the initial conditions imply $c_1 = 1$, $\sqrt{3}c_2 = 5$. Solving for c_1 and c_2 we find $c_1 = 1$ and $c_2 = \frac{5}{3}\sqrt{3}$ so $y = \cosh\sqrt{3}x + \frac{5}{3}\sqrt{3}\sinh\sqrt{3}x$.

43. The auxiliary equation should have two positive roots, so that the solution has the form $y = c_1e^{k_1x} + c_2e^{k_2x}$. Thus, the differential equation is (f).

45. The auxiliary equation should have a pair of complex roots $\alpha \pm \beta i$ where $\alpha < 0$, so that the solution has the form $e^{\alpha x}(c_1\cos\beta x + c_2\sin\beta x)$. Thus, the differential equation is (e).

47. The differential equation should have the form $y'' + k^2y = 0$ where $k = 1$ so that the period of the solution is 2π. Thus, the differential equation is (d).

49. Since $(m-4)(m+5)^2 = m^3+6m^2-15m-100$ the differential equation is $y'''+6y''-15y'-100y = 0$. The differential equation is not unique since any constant multiple of the left-hand side of the differential equation would lead to the auxiliary roots.

51. From the solution $y_1 = e^{-4x}\cos x$ we conclude that $m_1 = -4 + i$ and $m_2 = -4 - i$ are roots of the auxiliary equation. Hence another solution must be $y_2 = e^{-4x}\sin x$. Now dividing the polynomial $m^3 + 6m^2 + m - 34$ by $[m - (-4 + i)][m - (-4 - i)] = m^2 + 8m + 17$ gives $m - 2$. Therefore $m_3 = 2$ is the third root of the auxiliary equation, and the general solution of the differential equation is

$$y = c_1 e^{-4x}\cos x + c_2 e^{-4x}\sin x + c_3 e^{2x}.$$