**1.** $\lim_{n\to\infty}\left|\frac{2^{n+1}x^{n+1}/(n+1)}{2^n x^n/n}\right| = \lim_{n\to\infty}\frac{2n}{n+1}|x| = 2|x|$

The series is absolutely convergent for $2|x|<1$ or $|x|<\frac{1}{2}$. The radius of convergence is $R=\frac{1}{2}$. At $x=-\frac{1}{2}$, the series $\sum_{n=1}^{\infty}(-1)^n/n$ converges by the alternating series test. At $x=\frac{1}{2}$, the series $\sum_{n=1}^{\infty}1/n$ is the harmonic series which diverges. Thus, the given series converges on $[-\frac{1}{2},\frac{1}{2})$.

**3.** By the ratio test,

$$\lim_{k\to\infty}\left|\frac{(x-5)^{k+1}/10^{k+1}}{(x-5)^k/10^k}\right| = \lim_{k\to\infty}\frac{1}{10}|x-5| = \frac{1}{10}|x-5|.$$

The series is absolutely convergent for $\frac{1}{10}|x-5|<1$, $|x-5|<10$, or on $(-5,15)$. The radius of convergence is $R=10$. At $x=-5$, the series $\sum_{k=1}^{\infty}(-1)^k(-10)^k/10^k=\sum_{k=1}^{\infty}1$ diverges by the $n$th term test. At $x=15$, the series $\sum_{k=1}^{\infty}(-1)^k 10^k/10^k=\sum_{k=1}^{\infty}(-1)^k$ diverges by the $n$th term test. Thus, the series converges on $(-5,15)$.

**5.** $\sin x\cos x = \left(x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{x^7}{5040}+\cdots\right)\left(1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\cdots\right) = x-\frac{2x^3}{3}+\frac{2x^5}{15}-\frac{4x^7}{315}+\cdots$

**7.** $\frac{1}{\cos x} = \frac{1}{1-\frac{x^2}{2}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots} = 1+\frac{x^2}{2}+\frac{5x^4}{4!}+\frac{61x^6}{6!}+\cdots$

Since $\cos(\pi/2)=\cos(-\pi/2)=0$, the series converges on $(-\pi/2,\pi/2)$.

**9.** Let $k=n+2$ so that $n=k-2$ and

$$\sum_{n=1}^{\infty} nc_n x^{n+2} = \sum_{k=3}^{\infty}(k-2)c_{k-2}x^k.$$

**11.** 
$$\sum_{n=1}^{\infty}2nc_n x^{n-1}+\sum_{n=0}^{\infty}6c_n x^{n+1} = 2\cdot 1\cdot c_1x^0+\underbrace{\sum_{n=2}^{\infty}2nc_n x^{n-1}}_{k=n-1}+\underbrace{\sum_{n=0}^{\infty}6c_n x^{n+1}}_{k=n+1}$$

$$= 2c_1+\sum_{k=1}^{\infty}2(k+1)c_{k+1}x^k+\sum_{k=1}^{\infty}6c_{k-1}x^k$$

$$= 2c_1+\sum_{k=1}^{\infty}[2(k+1)c_{k+1}+6c_{k-1}]x^k$$

**13.** $y' = \sum_{n=1}^{\infty} (-1)^{n+1} x^{n-1}, \qquad y'' = \sum_{n=2}^{\infty} (-1)^{n+1} (n-1) x^{n-2}$

$$\begin{aligned}
(x+1)y'' + y' &= (x+1) \sum_{n=2}^{\infty} (-1)^{n+1} (n-1) x^{n-2} + \sum_{n=1}^{\infty} (-1)^{n+1} x^{n-1} \\
&= \sum_{n=2}^{\infty} (-1)^{n+1} (n-1) x^{n-1} + \sum_{n=2}^{\infty} (-1)^{n+1} (n-1) x^{n-2} + \sum_{n=1}^{\infty} (-1)^{n+1} x^{n-1} \\
&= -x^0 + x^0 + \underbrace{\sum_{n=2}^{\infty} (-1)^{n+1} (n-1) x^{n-1}}_{k=n-1} + \underbrace{\sum_{n=3}^{\infty} (-1)^{n+1} (n-1) x^{n-2}}_{k=n-2} + \underbrace{\sum_{n=2}^{\infty} (-1)^{n+1} x^{n-1}}_{k=n-1} \\
&= \sum_{k=1}^{\infty} (-1)^{k+2} k x^k + \sum_{k=1}^{\infty} (-1)^{k+3} (k+1) x^k + \sum_{k=1}^{\infty} (-1)^{k+2} x^k \\
&= \sum_{k=1}^{\infty} \left[ (-1)^{k+2} k - (-1)^{k+2} k - (-1)^{k+2} + (-1)^{k+2} \right] x^k = 0
\end{aligned}$$