1. (a) $\mathbf{A} + \mathbf{B} = \begin{pmatrix} 4-2 & 5+6 \\ -6+8 & 9-10 \end{pmatrix} = \begin{pmatrix} 2 & 11 \\ 2 & -1 \end{pmatrix}$

   (b) $\mathbf{B} - \mathbf{A} = \begin{pmatrix} -2-4 & 6-5 \\ 8+6 & -10-9 \end{pmatrix} = \begin{pmatrix} -6 & 1 \\ 14 & -19 \end{pmatrix}$

   (c) $2\mathbf{A} + 3\mathbf{B} = \begin{pmatrix} 8 & 10 \\ -12 & 18 \end{pmatrix} + \begin{pmatrix} -6 & 18 \\ 24 & -30 \end{pmatrix} = \begin{pmatrix} 2 & 28 \\ 12 & -12 \end{pmatrix}$

3. (a) $\mathbf{AB} = \begin{pmatrix} -2-9 & 12-6 \\ 5+12 & -30+8 \end{pmatrix} = \begin{pmatrix} -11 & 6 \\ 17 & -22 \end{pmatrix}$

   (b) $\mathbf{BA} = \begin{pmatrix} -2-30 & 3+24 \\ 6-10 & -9+8 \end{pmatrix} = \begin{pmatrix} -32 & 27 \\ -4 & -1 \end{pmatrix}$

   (c) $\mathbf{A}^2 = \begin{pmatrix} 4+15 & -6-12 \\ -10-20 & 15+16 \end{pmatrix} = \begin{pmatrix} 19 & -18 \\ -30 & 31 \end{pmatrix}$

   (d) $\mathbf{B}^2 = \begin{pmatrix} 1+18 & -6+12 \\ -3+6 & 18+4 \end{pmatrix} = \begin{pmatrix} 19 & 6 \\ 3 & 22 \end{pmatrix}$

5. (a) $\mathbf{BC} = \begin{pmatrix} 9 & 24 \\ 3 & 8 \end{pmatrix}$

   (b) $\mathbf{A}(\mathbf{BC}) = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} 9 & 24 \\ 3 & 8 \end{pmatrix} = \begin{pmatrix} 3 & 8 \\ -6 & -16 \end{pmatrix}$

   (c) $\mathbf{C}(\mathbf{BA}) = \begin{pmatrix} 0 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

   (d) $\mathbf{A}(\mathbf{B} + \mathbf{C}) = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} 6 & 5 \\ 5 & 5 \end{pmatrix} = \begin{pmatrix} -4 & -5 \\ 8 & 10 \end{pmatrix}$

7. (a) $\mathbf{A}^T\mathbf{A} = (4 \quad 8 \quad -10)\begin{pmatrix} 4 \\ 8 \\ -10 \end{pmatrix} = (180)$

(b) $\mathbf{B}^T\mathbf{B} = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}(2 \quad 4 \quad 5) = \begin{pmatrix} 4 & 8 & 10 \\ 8 & 16 & 20 \\ 10 & 20 & 25 \end{pmatrix}$

(c) $\mathbf{A} + \mathbf{B}^T = \begin{pmatrix} 4 \\ 8 \\ -10 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ 12 \\ -5 \end{pmatrix}$

9. (a) $(\mathbf{AB})^T = \begin{pmatrix} 7 & 10 \\ 38 & 75 \end{pmatrix}^T = \begin{pmatrix} 7 & 38 \\ 10 & 75 \end{pmatrix}$

(b) $\mathbf{B}^T\mathbf{A}^T = \begin{pmatrix} 5 & -2 \\ 10 & -5 \end{pmatrix}\begin{pmatrix} 3 & 8 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 38 \\ 10 & 75 \end{pmatrix}$

11. $\begin{pmatrix} -4 \\ 8 \end{pmatrix} - \begin{pmatrix} 4 \\ 16 \end{pmatrix} + \begin{pmatrix} -6 \\ 9 \end{pmatrix} = \begin{pmatrix} -14 \\ 1 \end{pmatrix}$

13. $\begin{pmatrix} -19 \\ 18 \end{pmatrix} - \begin{pmatrix} 19 \\ 20 \end{pmatrix} = \begin{pmatrix} -38 \\ -2 \end{pmatrix}$

15. Since $\det \mathbf{A} = 0$, $\mathbf{A}$ is singular.

17. Since $\det \mathbf{A} = 4$, $\mathbf{A}$ is nonsingular.

$$\mathbf{A}^{-1} = \frac{1}{4}\begin{pmatrix} -5 & -8 \\ 3 & 4 \end{pmatrix}$$

19. Since $\det \mathbf{A} = 2$, $\mathbf{A}$ is nonsingular. The cofactors are

$$\begin{array}{lll} A_{11} = 0 & A_{12} = 2 & A_{13} = -4 \\ A_{21} = -1 & A_{22} = 2 & A_{23} = -3 \\ A_{31} = 1 & A_{32} = -2 & A_{33} = 5. \end{array}$$

Then

$$\mathbf{A}^{-1} = \frac{1}{2}\begin{pmatrix} 0 & 2 & -4 \\ -1 & 2 & -3 \\ 1 & -2 & 5 \end{pmatrix}^T = \frac{1}{2}\begin{pmatrix} 0 & -1 & 1 \\ 2 & 2 & -2 \\ -4 & -3 & 5 \end{pmatrix}.$$

**21.** Since $\det \mathbf{A} = -9$, $\mathbf{A}$ is nonsingular. The cofactors are

$$
\begin{array}{lll}
A_{11} = -2 & A_{12} = -13 & A_{13} = 8 \\
A_{21} = -2 & A_{22} = 5 & A_{23} = -1 \\
A_{31} = -1 & A_{32} = 7 & A_{33} = -5.
\end{array}
$$

Then

$$
\mathbf{A}^{-1} = -\frac{1}{9}\begin{pmatrix} -2 & -13 & 8 \\ -2 & 5 & -1 \\ -1 & 7 & -5 \end{pmatrix}^{T} = -\frac{1}{9}\begin{pmatrix} -2 & -2 & -1 \\ -13 & 5 & 7 \\ 8 & -1 & -5 \end{pmatrix}.
$$

**23.** Since $\det \mathbf{A}(t) = 2e^{3t} \neq 0$, $\mathbf{A}$ is nonsingular.

$$
\mathbf{A}^{-1} = \frac{1}{2}e^{-3t}\begin{pmatrix} 3e^{4t} & -e^{4t} \\ -4e^{-t} & 2e^{-t} \end{pmatrix}
$$

**25.** $\dfrac{d\mathbf{X}}{dt} = \begin{pmatrix} -5e^{-t} \\ -2e^{-t} \\ 7e^{-t} \end{pmatrix}$

**27.** $\mathbf{X} = \begin{pmatrix} 2e^{2t} + 8e^{-3t} \\ -2e^{2t} + 4e^{-3t} \end{pmatrix}$ so that $\dfrac{d\mathbf{X}}{dt} = \begin{pmatrix} 4e^{2t} - 24e^{-3t} \\ -4e^{2t} - 12e^{-3t} \end{pmatrix}.$

**29. (a)** $\dfrac{d\mathbf{A}}{dt} = \begin{pmatrix} 4e^{4t} & -\pi\sin \pi t \\ 2 & 6t \end{pmatrix}$

**(b)** $\displaystyle\int_0^2 \mathbf{A}(t)\,dt = \begin{pmatrix} \frac{1}{4}e^{4t} & \frac{1}{\pi}\sin \pi t \\ t^2 & t^3 - t \end{pmatrix}\Big|_{t=0}^{t=2} = \begin{pmatrix} \frac{1}{4}e^8 - \frac{1}{4} & 0 \\ 4 & 6 \end{pmatrix}$

**(c)** $\displaystyle\int_0^t \mathbf{A}(s)\,ds = \begin{pmatrix} \frac{1}{4}e^{4s} & \frac{1}{\pi}\sin \pi s \\ s^2 & s^3 - s \end{pmatrix}\Big|_{s=0}^{s=t} = \begin{pmatrix} \frac{1}{4}e^{4t} - \frac{1}{4} & \frac{1}{\pi}\sin \pi t \\ t^2 & t^3 - t \end{pmatrix}$

**47.** We solve
$$\det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} -1-\lambda & 2 \\ -7 & 8-\lambda \end{vmatrix} = (\lambda - 6)(\lambda - 1) = 0.$$

For $\lambda_1 = 6$ we have
$$\begin{pmatrix} -7 & 2 & | & 0 \\ -7 & 2 & | & 0 \end{pmatrix} \Longrightarrow \begin{pmatrix} 1 & -2/7 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix}$$

so that $k_1 = \frac{2}{7}k_2$. If $k_2 = 7$ then
$$\mathbf{K}_1 = \begin{pmatrix} 2 \\ 7 \end{pmatrix}.$$

For $\lambda_2 = 1$ we have
$$\begin{pmatrix} -2 & 2 & | & 0 \\ -7 & 7 & | & 0 \end{pmatrix} \Longrightarrow \begin{pmatrix} 1 & -1 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix}$$

so that $k_1 = k_2$. If $k_2 = 1$ then
$$\mathbf{K}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

**49.** We solve
$$\det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} -8-\lambda & -1 \\ 16 & -\lambda \end{vmatrix} = (\lambda + 4)^2 = 0.$$

For $\lambda_1 = \lambda_2 = -4$ we have
$$\begin{pmatrix} -4 & -1 & | & 0 \\ 16 & 4 & | & 0 \end{pmatrix} \Longrightarrow \begin{pmatrix} 1 & 1/4 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix}$$

so that $k_1 = -\frac{1}{4}k_2$. If $k_2 = 4$ then
$$\mathbf{K}_1 = \begin{pmatrix} -1 \\ 4 \end{pmatrix}.$$

**51.** We solve

$$\det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} 5-\lambda & -1 & 0 \\ 0 & -5-\lambda & 9 \\ 5 & -1 & -\lambda \end{vmatrix} = \lambda(4-\lambda)(\lambda+4) = 0.$$

If $\lambda_1 = 0$ then

$$\begin{pmatrix} 5 & -1 & 0 & | & 0 \\ 0 & -5 & 9 & | & 0 \\ 5 & -1 & 0 & | & 0 \end{pmatrix} \implies \begin{pmatrix} 1 & 0 & -9/25 & | & 0 \\ 0 & 1 & -9/5 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

so that $k_1 = \frac{9}{25}k_3$ and $k_2 = \frac{9}{5}k_3$. If $k_3 = 25$ then

$$\mathbf{K}_1 = \begin{pmatrix} 9 \\ 45 \\ 25 \end{pmatrix}.$$

If $\lambda_2 = 4$ then

$$\begin{pmatrix} 1 & -1 & 0 & | & 0 \\ 0 & -9 & 9 & | & 0 \\ 5 & -1 & -4 & | & 0 \end{pmatrix} \implies \begin{pmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

so that $k_1 = k_3$ and $k_2 = k_3$. If $k_3 = 1$ then

$$\mathbf{K}_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

If $\lambda_3 = -4$ then

$$\begin{pmatrix} 9 & -1 & 0 & | & 0 \\ 0 & -1 & 9 & | & 0 \\ 5 & -1 & 4 & | & 0 \end{pmatrix} \implies \begin{pmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 1 & -9 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

so that $k_1 = k_3$ and $k_2 = 9k_3$. If $k_3 = 1$ then

$$\mathbf{K}_3 = \begin{pmatrix} 1 \\ 9 \\ 1 \end{pmatrix}.$$

**53.** We solve

$$\det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} -\lambda & 4 & 0 \\ -1 & -4-\lambda & 0 \\ 0 & 0 & -2-\lambda \end{vmatrix} = -(\lambda+2)^3 = 0.$$

For $\lambda_1 = \lambda_2 = \lambda_3 = -2$ we have

$$\begin{pmatrix} 2 & 4 & 0 & | & 0 \\ -1 & -2 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \implies \begin{pmatrix} 1 & 2 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

so that $k_1 = -2k_2$. If $k_2 = 1$ and $k_3 = 1$ then

$$\mathbf{K}_1 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \mathbf{K}_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

**55.** We solve

$$\det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} -1-\lambda & 2 \\ -5 & 1-\lambda \end{vmatrix} = \lambda^2 + 9 = (\lambda - 3i)(\lambda + 3i) = 0.$$

For $\lambda_1 = 3i$ we have

$$\begin{pmatrix} -1-3i & 2 & | & 0 \\ -5 & 1-3i & | & 0 \end{pmatrix} \implies \begin{pmatrix} 1 & -(1/5)+(3/5)i & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix}$$

so that $k_1 = \left(\frac{1}{5} - \frac{3}{5}i\right)k_2$. If $k_2 = 5$ then

$$\mathbf{K}_1 = \begin{pmatrix} 1-3i \\ 5 \end{pmatrix}.$$

For $\lambda_2 = -3i$ we have

$$\begin{pmatrix} -1+3i & 2 & | & 0 \\ -5 & 1+3i & | & 0 \end{pmatrix} \implies \begin{pmatrix} 1 & -\frac{1}{5} - \frac{3}{5}i & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix}$$

so that $k_1 = \left(\frac{1}{5} + \frac{3}{5}i\right)k_2$. If $k_2 = 5$ then

$$\mathbf{K}_2 = \begin{pmatrix} 1+3i \\ 5 \end{pmatrix}.$$