

In Problems 33-46 the form of the answer will vary according to the choice of eigenvector. For example, in Problem 33, if  $\mathbf{K}_1$  is chosen to be  $\begin{pmatrix} 1 \\ 2-i \end{pmatrix}$  the solution has the form

$$\mathbf{X} = c_1 \begin{pmatrix} \cos t \\ 2 \cos t + \sin t \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} \sin t \\ 2 \sin t - \cos t \end{pmatrix} e^{4t}.$$

33. We have  $\det(\mathbf{A} - \lambda\mathbf{I}) = \lambda^2 - 8\lambda + 17 = 0$ . For  $\lambda_1 = 4 + i$  we obtain

$$\mathbf{K}_1 = \begin{pmatrix} 2+i \\ 5 \end{pmatrix}$$

so that

$$\mathbf{X}_1 = \begin{pmatrix} 2+i \\ 5 \end{pmatrix} e^{(4+i)t} = \begin{pmatrix} 2 \cos t - \sin t \\ 5 \cos t \end{pmatrix} e^{4t} + i \begin{pmatrix} \cos t + 2 \sin t \\ 5 \sin t \end{pmatrix} e^{4t}.$$

Then

$$\mathbf{X} = c_1 \begin{pmatrix} 2 \cos t - \sin t \\ 5 \cos t \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} \cos t + 2 \sin t \\ 5 \sin t \end{pmatrix} e^{4t}.$$

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35. We have  $\det(\mathbf{A} - \lambda\mathbf{I}) = \lambda^2 - 8\lambda + 17 = 0$ . For  $\lambda_1 = 4 + i$  we obtain

$$\mathbf{K}_1 = \begin{pmatrix} -1-i \\ 2 \end{pmatrix}$$

so that

$$\mathbf{X}_1 = \begin{pmatrix} -1-i \\ 2 \end{pmatrix} e^{(4+i)t} = \begin{pmatrix} \sin t - \cos t \\ 2 \cos t \end{pmatrix} e^{4t} + i \begin{pmatrix} -\sin t - \cos t \\ 2 \sin t \end{pmatrix} e^{4t}.$$

Then

$$\mathbf{X} = c_1 \begin{pmatrix} \sin t - \cos t \\ 2 \cos t \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} -\sin t - \cos t \\ 2 \sin t \end{pmatrix} e^{4t}.$$

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37. We have  $\det(\mathbf{A} - \lambda\mathbf{I}) = \lambda^2 + 9 = 0$ . For  $\lambda_1 = 3i$  we obtain

$$\mathbf{K}_1 = \begin{pmatrix} 4 + 3i \\ 5 \end{pmatrix}$$

so that

$$\mathbf{X}_1 = \begin{pmatrix} 4 + 3i \\ 5 \end{pmatrix} e^{3it} = \begin{pmatrix} 4 \cos 3t - 3 \sin 3t \\ 5 \cos 3t \end{pmatrix} + i \begin{pmatrix} 4 \sin 3t + 3 \cos 3t \\ 5 \sin 3t \end{pmatrix}.$$

Then

$$\mathbf{X} = c_1 \begin{pmatrix} 4 \cos 3t - 3 \sin 3t \\ 5 \cos 3t \end{pmatrix} + c_2 \begin{pmatrix} 4 \sin 3t + 3 \cos 3t \\ 5 \sin 3t \end{pmatrix}.$$

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39. We have  $\det(\mathbf{A} - \lambda\mathbf{I}) = -\lambda(\lambda^2 + 1) = 0$ . For  $\lambda_1 = 0$  we obtain

$$\mathbf{K}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

For  $\lambda_2 = i$  we obtain

$$\mathbf{K}_2 = \begin{pmatrix} -i \\ i \\ 1 \end{pmatrix}$$

so that

$$\mathbf{X}_2 = \begin{pmatrix} -i \\ i \\ 1 \end{pmatrix} e^{it} = \begin{pmatrix} \sin t \\ -\sin t \\ \cos t \end{pmatrix} + i \begin{pmatrix} -\cos t \\ \cos t \\ \sin t \end{pmatrix}.$$

Then

$$\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} \sin t \\ -\sin t \\ \cos t \end{pmatrix} + c_3 \begin{pmatrix} -\cos t \\ \cos t \\ \sin t \end{pmatrix}.$$

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$$\mathbf{X} = c_1 \begin{pmatrix} \cos t \\ 2 \cos t + \sin t \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} \sin t \\ 2 \sin t - \cos t \end{pmatrix} e^{4t}.$$

41. We have  $\det(\mathbf{A} - \lambda\mathbf{I}) = (1 - \lambda)(\lambda^2 - 2\lambda + 2) = 0$ . For  $\lambda_1 = 1$  we obtain

$$\mathbf{K}_1 = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}.$$

For  $\lambda_2 = 1 + i$  we obtain

$$\mathbf{K}_2 = \begin{pmatrix} 1 \\ i \\ i \end{pmatrix}$$

so that

$$\mathbf{X}_2 = \begin{pmatrix} 1 \\ i \\ i \end{pmatrix} e^{(1+i)t} = \begin{pmatrix} \cos t \\ -\sin t \\ -\sin t \end{pmatrix} e^t + i \begin{pmatrix} \sin t \\ \cos t \\ \cos t \end{pmatrix} e^t.$$

Then

$$\mathbf{X} = c_1 \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} \cos t \\ -\sin t \\ -\sin t \end{pmatrix} e^t + c_3 \begin{pmatrix} \sin t \\ \cos t \\ \cos t \end{pmatrix} e^t.$$

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$$\mathbf{X} = c_1 \begin{pmatrix} \cos t \\ 2 \cos t + \sin t \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} \sin t \\ 2 \sin t - \cos t \end{pmatrix} e^{4t}.$$

43. We have  $\det(\mathbf{A} - \lambda\mathbf{I}) = (2 - \lambda)(\lambda^2 + 4\lambda + 13) = 0$ . For  $\lambda_1 = 2$  we obtain

$$\mathbf{K}_1 = \begin{pmatrix} 28 \\ -5 \\ 25 \end{pmatrix}.$$

For  $\lambda_2 = -2 + 3i$  we obtain

$$\mathbf{K}_2 = \begin{pmatrix} 4 + 3i \\ -5 \\ 0 \end{pmatrix}$$

so that

$$\mathbf{X}_2 = \begin{pmatrix} 4 + 3i \\ -5 \\ 0 \end{pmatrix} e^{(-2+3i)t} = \begin{pmatrix} 4 \cos 3t - 3 \sin 3t \\ -5 \cos 3t \\ 0 \end{pmatrix} e^{-2t} + i \begin{pmatrix} 4 \sin 3t + 3 \cos 3t \\ -5 \sin 3t \\ 0 \end{pmatrix} e^{-2t}.$$

Then

$$\mathbf{X} = c_1 \begin{pmatrix} 28 \\ -5 \\ 25 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 4 \cos 3t - 3 \sin 3t \\ -5 \cos 3t \\ 0 \end{pmatrix} e^{-2t} + c_3 \begin{pmatrix} 4 \sin 3t + 3 \cos 3t \\ -5 \sin 3t \\ 0 \end{pmatrix} e^{-2t}.$$

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$$\mathbf{X} = c_1 \begin{pmatrix} \cos t \\ 2 \cos t + \sin t \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} \sin t \\ 2 \sin t - \cos t \end{pmatrix} e^{4t}.$$

45. We have  $\det(\mathbf{A} - \lambda\mathbf{I}) = (1 - \lambda)(\lambda^2 + 25) = 0$ . For  $\lambda_1 = 1$  we obtain

$$\mathbf{K}_1 = \begin{pmatrix} 25 \\ -7 \\ 6 \end{pmatrix}.$$

For  $\lambda_2 = 5i$  we obtain

$$\mathbf{K}_2 = \begin{pmatrix} 1 + 5i \\ 1 \\ 1 \end{pmatrix}$$

so that

$$\mathbf{X}_2 = \begin{pmatrix} 1 + 5i \\ 1 \\ 1 \end{pmatrix} e^{5it} = \begin{pmatrix} \cos 5t - 5 \sin 5t \\ \cos 5t \\ \cos 5t \end{pmatrix} + i \begin{pmatrix} \sin 5t + 5 \cos 5t \\ \sin 5t \\ \sin 5t \end{pmatrix}.$$

Then

$$\mathbf{X} = c_1 \begin{pmatrix} 25 \\ -7 \\ 6 \end{pmatrix} e^t + c_2 \begin{pmatrix} \cos 5t - 5 \sin 5t \\ \cos 5t \\ \cos 5t \end{pmatrix} + c_3 \begin{pmatrix} \sin 5t + 5 \cos 5t \\ \sin 5t \\ \sin 5t \end{pmatrix}.$$

If

$$\mathbf{X}(0) = \begin{pmatrix} 4 \\ 6 \\ -7 \end{pmatrix}$$

then  $c_1 = c_2 = -1$  and  $c_3 = 6$ .