1. From $dy = \sin 5x\,dx$ we obtain $y = -\frac{1}{5}\cos 5x + c$.

3. From $dy = -e^{-3x}\,dx$ we obtain $y = \frac{1}{3}e^{-3x} + c$.

5. From $\dfrac{1}{y}\,dy = \dfrac{4}{x}\,dx$ we obtain $\ln|y| = 4\ln|x| + c$ or $y = c_1x^4$.

7. From $e^{-2y}dy = e^{3x}dx$ we obtain $3e^{-2y} + 2e^{3x} = c$.

9. From $\left(y + 2 + \dfrac{1}{y}\right) dy = x^2 \ln x\,dx$ we obtain $\dfrac{y^2}{2} + 2y + \ln|y| = \dfrac{x^3}{3}\ln|x| - \dfrac{1}{9}x^3 + c$.

11. From $\dfrac{1}{\csc y}\,dy = -\dfrac{1}{\sec^2 x}\,dx$ or $\sin y\,dy = -\cos^2 x\,dx = -\frac{1}{2}(1 + \cos 2x)\,dx$ we obtain $-\cos y = -\frac{1}{2}x - \frac{1}{4}\sin 2x + c$ or $4\cos y = 2x + \sin 2x + c_1$.

13. From $\dfrac{e^y}{(e^y + 1)^2}\,dy = \dfrac{-e^x}{(e^x + 1)^3}\,dx$ we obtain $-(e^y + 1)^{-1} = \frac{1}{2}(e^x + 1)^{-2} + c$.

15. From $\dfrac{1}{S}\,dS = k\,dr$ we obtain $S = ce^{kr}$.

17. From $\dfrac{1}{P - P^2}dP = \left(\dfrac{1}{P} + \dfrac{1}{1 - P}\right) dP = dt$ we obtain $\ln|P| - \ln|1 - P| = t + c$ so that $\ln\left|\dfrac{P}{1 - P}\right| = t + c$ or $\dfrac{P}{1 - P} = c_1e^t$. Solving for P we have $P = \dfrac{c_1e^t}{1 + c_1e^t}$.

19. From $\dfrac{y - 2}{y + 3}\,dy = \dfrac{x - 1}{x + 4}\,dx$ or $\left(1 - \dfrac{5}{y + 3}\right) dy = \left(1 - \dfrac{5}{x + 4}\right) dx$ we obtain $y - 5\ln|y + 3| = x - 5\ln|x + 4| + c$ or $\left(\dfrac{x + 4}{y + 3}\right)^5 = c_1e^{x-y}$.

21. From $x\,dx = \dfrac{1}{\sqrt{1 - y^2}}\,dy$ we obtain $\frac{1}{2}x^2 = \sin^{-1} y + c$ or $y = \sin\left(\dfrac{x^2}{2} + c_1\right)$.

23. From $\dfrac{1}{x^2 + 1}\,dx = 4\,dt$ we obtain $\tan^{-1} x = 4t + c$. Using $x(\pi/4) = 1$ we find $c = -3\pi/4$. The solution of the initial-value problem is $\tan^{-1} x = 4t - \dfrac{3\pi}{4}$ or $x = \tan\left(4t - \dfrac{3\pi}{4}\right)$.

25. From $\dfrac{1}{y}\,dy = \dfrac{1-x}{x^2}\,dx = \left(\dfrac{1}{x^2} - \dfrac{1}{x}\right)dx$ we obtain $\ln|y| = -\dfrac{1}{x} - \ln|x| = c$ or $xy = c_1 e^{-1/x}$. Using $y(-1) = -1$ we find $c_1 = e^{-1}$. The solution of the initial-value problem is $xy = e^{-1-1/x}$ or $y = e^{-(1+1/x)}/x$.

27. Separating variables and integrating we obtain

$$\frac{dx}{\sqrt{1-x^2}} - \frac{dy}{\sqrt{1-y^2}} = 0 \quad \text{and} \quad \sin^{-1} x - \sin^{-1} y = c.$$

Setting $x = 0$ and $y = \sqrt{3}/2$ we obtain $c = -\pi/3$. Thus, an implicit solution of the initial-value problem is $\sin^{-1} x - \sin^{-1} y = -\pi/3$. Solving for y and using an addition formula from trigonometry, we get

$$y = \sin\left(\sin^{-1} x + \frac{\pi}{3}\right) = x\cos\frac{\pi}{3} + \sqrt{1-x^2}\,\sin\frac{\pi}{3} = \frac{x}{2} + \frac{\sqrt{3}\sqrt{1-x^2}}{2}.$$

29. Separating variables, integrating from 4 to x, and using t as a dummy variable of integration gives

$$\begin{aligned}\int_4^x \frac{1}{y}\frac{dy}{dt}\,dt &= \int_4^x e^{-t^2}\,dt\\ \ln y(t)\Big|_4^x &= \int_4^x e^{-t^2}\,dt\\ \ln y(x) - \ln y(4) &= \int_4^x e^{-t^2}\,dt\end{aligned}$$

Using the initial condition we have

$$\ln y(x) = \ln y(4) + \int_4^x e^{-t^2}\,dt = \ln 1 + \int_4^x e^{-t^2}\,dt = \int_4^x e^{-t^2}\,dt.$$

Thus,

$$y(x) = e^{\int_4^x e^{-t^2}\,dt}.$$

31. **(a)** The equilibrium solutions $y(x)=2$ and $y(x)=-2$ satisfy the initial conditions $y(0)=2$ and $y(0)=-2$, respectively. Setting $x=\frac{1}{4}$ and $y=1$ in $y=2(1+ce^{4x})/(1-ce^{4x})$ we obtain

$$1=2\frac{1+ce}{1-ce}, \quad 1-ce=2+2ce, \quad -1=3ce, \quad \text{and} \quad c=-\frac{1}{3e}.$$

The solution of the corresponding initial-value problem is

$$y=2\frac{1-\frac{1}{3}e^{4x-1}}{1+\frac{1}{3}e^{4x-1}}=2\frac{3-e^{4x-1}}{3+e^{4x-1}}.$$

(b) Separating variables and integrating yields

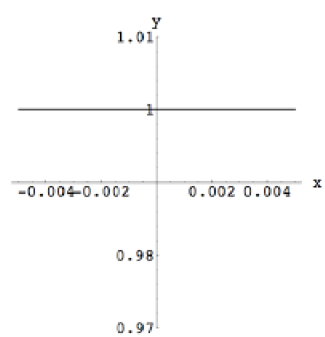
$$\frac{1}{4}\ln|y-2|-\frac{1}{4}\ln|y+2|+\ln c_1=x$$
$$\ln|y-2|-\ln|y+2|+\ln c=4x$$
$$\ln\left|\frac{c(y-2)}{y+2}\right|=4x$$
$$c\frac{y-2}{y+2}=e^{4x}.$$

Solving for y we get $y=2(c+e^{4x})/(c-e^{4x})$. The initial condition $y(0)=-2$ implies $2(c+1)/(c-1)=-2$ which yields $c=0$ and $y(x)=-2$. The initial condition $y(0)=2$ does not correspond to a value of c, and it must simply be recognized that $y(x)=2$ is a solution of the initial-value problem. Setting $x=\frac{1}{4}$ and $y=1$ in $y=2(c+e^{4x})/(c-e^{4x})$ leads to $c=-3e$. Thus, a solution of the initial-value problem is

$$y=2\frac{-3e+e^{4x}}{-3e-e^{4x}}=2\frac{3-e^{4x-1}}{3+e^{4x-1}}.$$

33. Singular solutions of $dy/dx=x\sqrt{1-y^2}$ are $y=-1$ and $y=1$. A singular solution of $(e^x+e^{-x})dy/dx=y^2$ is $y=0$.

35. The singular solution $y=1$ satisfies the initial-value problem.

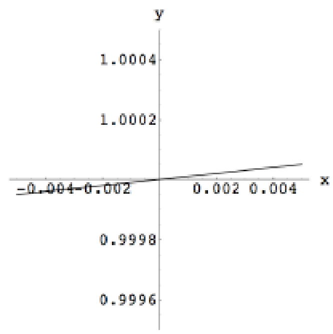


37. Separating variables we obtain $\dfrac{dy}{(y-1)^2+0.01} = dx$. Then

$$10\tan^{-1}10(y-1) = x + c \quad \text{and} \quad y = 1 + \frac{1}{10}\tan\frac{x+c}{10}.$$

Setting $x = 0$ and $y = 1$ we obtain $c = 0$. The solution is

$$y = 1 + \frac{1}{10}\tan\frac{x}{10}.$$



39. Separating variables, we have

$$\frac{dy}{y-y^3} = \frac{dy}{y(1-y)(1+y)} = \left(\frac{1}{y} + \frac{1/2}{1-y} - \frac{1/2}{1+y}\right) dy = dx.$$

Integrating, we get

$$\ln|y| - \frac{1}{2}\ln|1-y| - \frac{1}{2}\ln|1+y| = x + c.$$

When $y > 1$, this becomes

$$\ln y - \frac{1}{2}\ln(y-1) - \frac{1}{2}\ln(y+1) = \ln\frac{y}{\sqrt{y^2-1}} = x + c.$$

Letting $x = 0$ and $y = 2$ we find $c = \ln(2/\sqrt{3})$. Solving for y we get $y_1(x) = 2e^x/\sqrt{4e^{2x}-3}$, where $x > \ln(\sqrt{3}/2)$.

When $0 < y < 1$ we have

$$\ln y - \frac{1}{2}\ln(1-y) - \frac{1}{2}\ln(1+y) = \ln\frac{y}{\sqrt{1-y^2}} = x + c.$$

Letting $x = 0$ and $y = \frac{1}{2}$ we find $c = \ln(1/\sqrt{3})$. Solving for y we get $y_2(x) = e^x/\sqrt{e^{2x}+3}$, where $-\infty < x < \infty$.

When $-1 < y < 0$ we have

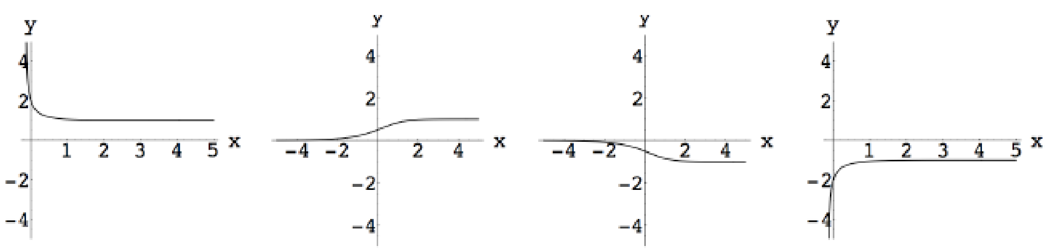
$$\ln(-y) - \frac{1}{2}\ln(1-y) - \frac{1}{2}\ln(1+y) = \ln\frac{-y}{\sqrt{1-y^2}} = x + c.$$

Letting $x = 0$ and $y = -\frac{1}{2}$ we find $c = \ln(1/\sqrt{3})$. Solving for y we get $y_3(x) = -e^x/\sqrt{e^{2x}+3}$, where $-\infty < x < \infty$.

When $y < -1$ we have

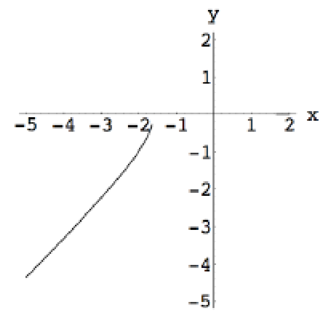
$$\ln(-y) - \frac{1}{2}\ln(1-y) - \frac{1}{2}\ln(-1-y) = \ln\frac{-y}{\sqrt{y^2-1}} = x + c.$$

Letting $x = 0$ and $y = -2$ we find $c = \ln(2/\sqrt{3})$. Solving for y we get $y_4(x) = -2e^x/\sqrt{4e^{2x}-3}$, where $x > \ln(\sqrt{3}/2)$.



41. **(a)** Separating variables we have $2y\,dy = (2x+1)dx$. Integrating gives $y^2 = x^2 + x + c$. When $y(-2) = -1$ we find $c = -1$, so $y^2 = x^2 + x - 1$ and $y = -\sqrt{x^2+x-1}$. The negative square root is chosen because of the initial condition.

(b) From the figure, the largest interval of definition appears to be approximately $(-\infty, -1.65)$.



(c) Solving $x^2 + x - 1 = 0$ we get $x = -\frac{1}{2} \pm \frac{1}{2}\sqrt{5}$, so the largest interval of definition is $(-\infty,\ -\frac{1}{2} - \frac{1}{2}\sqrt{5})$. The right-hand endpoint of the interval is excluded because $y = -\sqrt{x^2+x-1}$ is not differentiable at this point.

47. We are looking for a function $y(x)$ such that

$$y^2 + \left(\frac{dy}{dx}\right)^2 = 1.$$

Using the positive square root gives

$$\frac{dy}{dx} = \sqrt{1-y^2} \implies \frac{dy}{\sqrt{1-y^2}} = dx \implies \sin^{-1} y = x + c.$$

Thus a solution is $y = \sin(x+c)$. If we use the negative square root we obtain

$$y = \sin(c-x) = -\sin(x-c) = -\sin(x+c_1).$$

Note that when $c = c_1 = 0$ and when $c = c_1 = \pi/2$ we obtain the well known particular solutions $y = \sin x$, $y = -\sin x$, $y = \cos x$, and $y = -\cos x$. Note also that $y = 1$ and $y = -1$ are singular solutions.