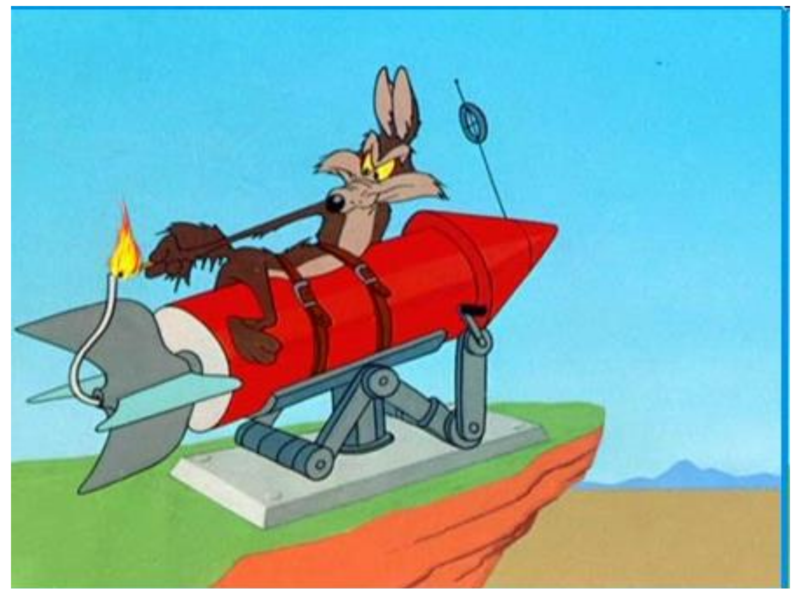
Warm up Problem

Wile E. Coyote's rocket has a position given by $s(t) = 3t^2 - 2$. Find the average velocity of the rocket on the interval $[2,5]$. What is the rocket's instantaneous velocity at $t = 4$?



The Derivative

- Remember, velocity is the rate of change when dealing with the position function.
  - When we generalize, the rate of change of a function is called the <u>derivative</u>.

Def. The derivative of a function $f(x)$ at $x = a$, denoted $f'(a)$, is the rate of change of the function at the point. It is defined as:

$$\boxed{f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

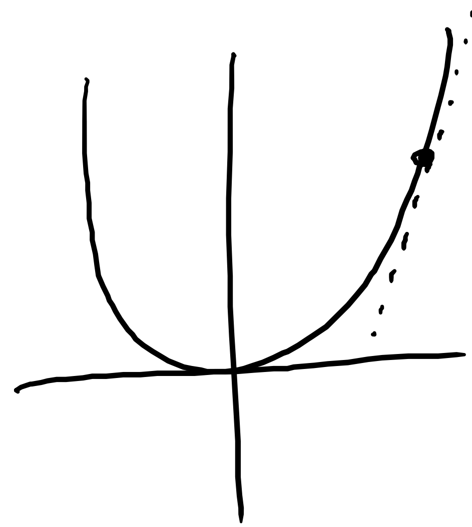
If the limit exists, we say that $f(x)$ is differentiable at $x = a$. The graph will be "smooth" at this point.

Ex. If $f(x) = x^2$, find $f'(3)$.

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} \lim_{h \to 0} \frac{(3+h)^2 - 3^2}{h} = \lim_{h \to 0} \frac{9 + 6h + h^2 - 9}{h}$$

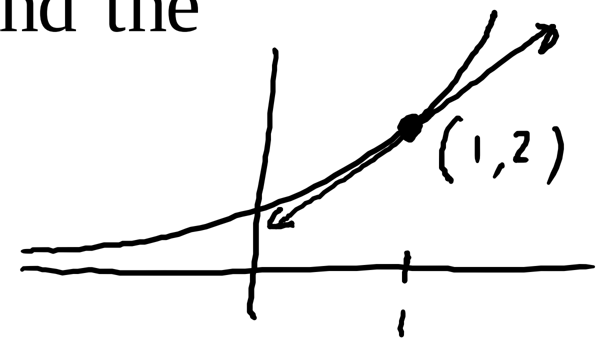
$$= \lim_{h \to 0} \frac{6h + h^2}{h} = \lim_{h \to 0} \frac{h(6+h)}{h} = \lim_{h \to 0} (6+h) = 6$$

$$f'(3) = 6$$



<u>Thm.</u> $f'(a)$ is the slope of the tangent line of $f(x)$ at $x = a$.

Ex. Approximate the slope of the tangent line to $f(x) = 2^x$ at $x = 1$, then find the equation of the line.



$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{2^{1+h} - 2^1}{h}$$

$$\approx 1.386$$

$$f(1) = 2^1 = 2$$

$$\boxed{y - 2 = 1.386(x - 1)}$$

Pract. Find the slope of the tangent line to $f(x) = x^2 + x$ at $x = 3$, then find the equation of the line.

$$f'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h\to 0} \frac{[(3+h)^2 + (3+h)] - 12}{h} = \lim_{h\to 0} \frac{9+6h+h^2+3+h-12}{h}$$

$$= \lim_{h\to 0} \frac{7h+h^2}{h} = \lim_{h\to 0} \frac{h(7+h)}{h} = \lim_{h\to 0} (7+h) = 7 = m$$

$$f(3) = 3^2 + 3 = 12$$

$$(3, 12)$$

$$\boxed{y - 12 = 7(x-3)}$$

<u>Ex.</u> The table below gives selected values of $f(x)$. Use these values to approximate $f'(7)$.

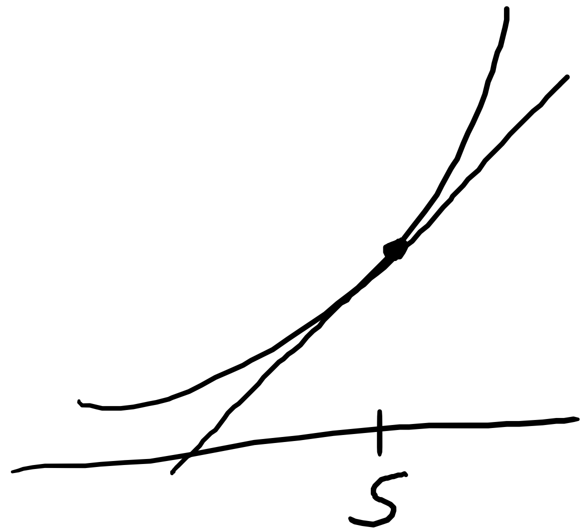
x	0	2	4	6	8	10	12
$f(x)$	0	.25	.48	.68	.84	.95	1

$$\frac{f(8)-f(6)}{8-6} = \frac{.84-.68}{2} = .08$$

Ex. The equation of the line tangent to $f(x)$ at $x = 5$ is $y = 6x - 3$. Find $f(5)$ and $f'(5)$.

$f(5) = 6(5) - 3 = 27$

$f'(5) = 6$



Alternate Form

$$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$