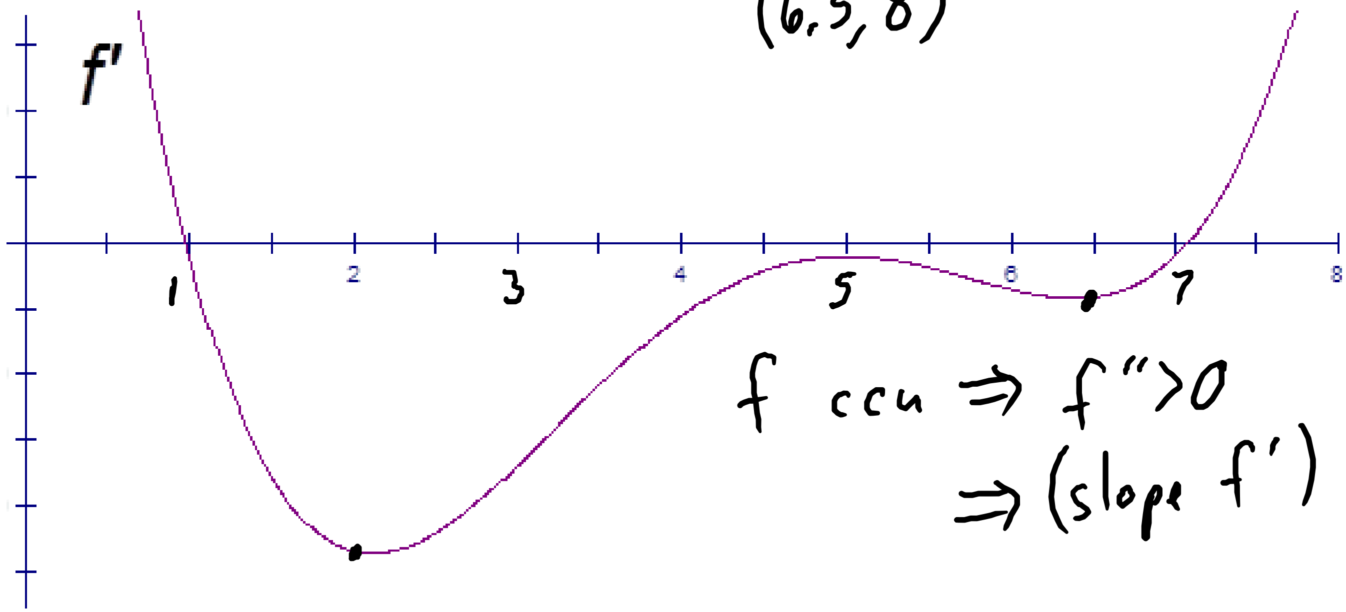
Graph of a Function

Ex. Given the graph of f', answer the following:

a) Where is f decreasing? $(1, 7)$ $f' < 0$

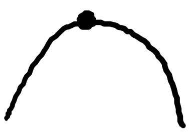
b) Where is f concave up? $(2, 5)$ $(6.5, 8)$ f' inc.



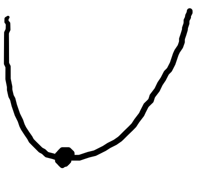
f ccu $\Rightarrow f'' > 0$

$\Rightarrow (\text{slope } f') > 0$

Def. A function $f(x)$ has a local maximum (relative max) at $x = p$ if $f(x) < f(p)$ for all points near p.



Def. A function $f(x)$ has a local minimum (relative min) at $x = p$ if $f(x) > f(p)$ for all points near p.



Def. We say that p is a critical point of $f(x)$ if $f'(p) = 0$ or is undefined.

Thm. All local max./min. points of a function are critical points.

→ The converse is not true.

First Derivative Test

Assume p is an ~~inflection~~ critical point of $f(x)$:

If $f'(x)$ is positive before p and negative after p, then p is a local maximum.

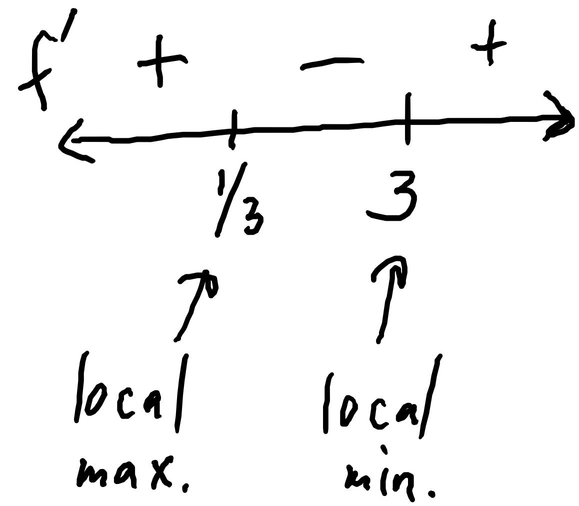
If $f'(x)$ is negative before p and positive after p, then p is a local minimum.

Ex. Find and classify all critical points of
$f(x) = x^3 - 5x^2 + 3x - 1.$

$$f'(x) = 3x^2 - 10x + 3 = 0$$

$$(3x-1)(x-3) = 0$$

$$\boxed{x = \tfrac{1}{3} \;\Big|\; x = 3}$$



If $f'' > 0$, then f is concave up.
If $f'' < 0$, then f is concave down.

<u>Concave up</u> means that the graph lies above its tangent line and below its secant line

Def. We say that p is an inflection point of $f(x)$ if the concavity of f changes at p.

Thm. If p is an inflection point of $f(x)$, then $f''(p) = 0$ or is undefined.

→ The converse is not true.

<u>Ex.</u> Find all inflection points of

$$f(x) = \frac{1}{4}x^4 - x^3 + \frac{3}{2}x^2 - 3$$

$$f'(x) = x^3 - 3x^2 + 3x$$

$$f''(x) = 3x^2 - 6x + 3$$

$$= 3(x^2 - 2x + 1)$$

$$= 3(x-1)^2 = 0$$

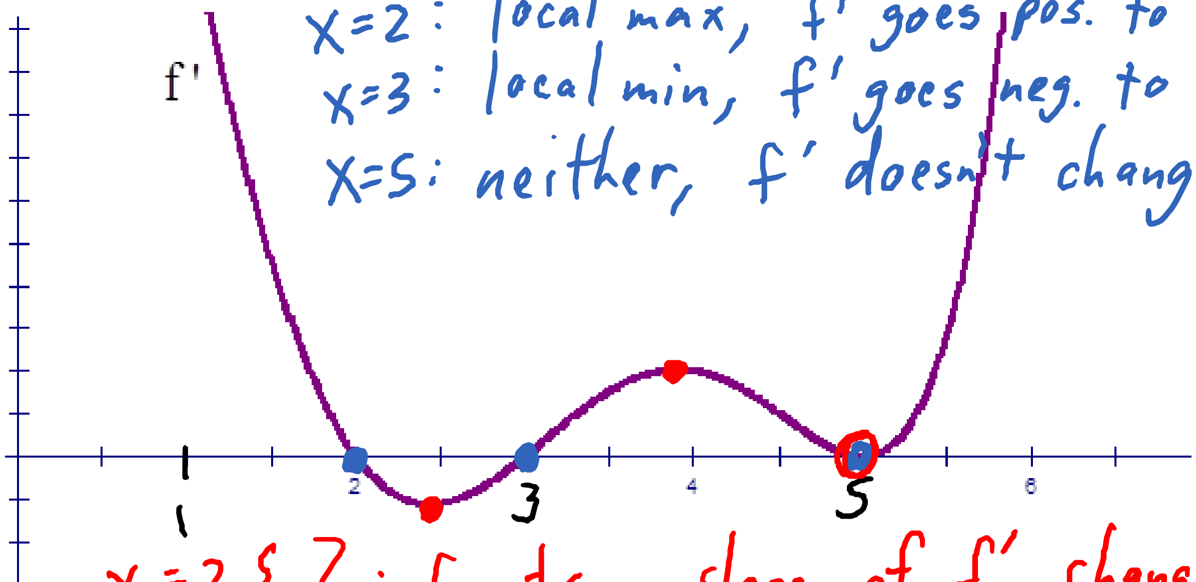
$$x = 1$$

f'': $\quad + \quad \underset{1}{|} \quad +$

no inf. pts.

Ex. Identify the x-coordinate of all points where $f(x)$ has a local max., local min., and inflection point. Justify your answer.

x=2: local max, f' goes pos. to neg.
x=3: local min, f' goes neg. to pos.
x=5: neither, f' doesn't change signs



x = 2.5
x = 3
x = 5

} inf pts., slope of f' changes signs

<u>Be careful!</u>

The terms velocity, acceleration, and speed should ONLY be used in motion problems.

x	-3	-2	-1	0	1	2	3
$f(x)$	-	0	+	+	+	+	-
$f'(x)$	+	+	0	+	+	0	-

0 -

<u>Ex.</u> The table contains selected values of the differentiable function $f(x)$ and its derivative. Which of the following is true?

A. f has a local minimum at $x = 2$.

B. f has a local maximum at $x = 2$.

C. f does not have a local extremum at $x = 2$.

D. It is not possible to determine the local extrema of f from the information in the table.

Ex. If $f(x) = ax^2 + bx$, find values of a and b that would result in a local max at $(1,5)$.

$f'(x) = 2ax + b$

$f'(1) = 0$

$f'(1) = 2a(1) + b = 0$

$2a + b = 0$

$f(1) = 5$

$f(1) = a(1)^2 + b(1) = 5$

$a + b = 5$

$$\begin{array}{r} 2a + b = 0 \\ -\quad a + b = 5 \\ \hline \boxed{a = -5} \end{array}$$

$-5 + b = 5$

$\boxed{b = 10}$

A Critical Point

The first derivative will show you
increase or decrease.
It’s positive or negative
y’know respectively.

If the d’rivative is zero or it’s undefined,
Then that point is critical,
and always on your mind!

You know a saddle is a critical point,
that also is an inflection point.
You know, you know a saddle
is an inflection point,
that also is a critical point.

The second d’rivative will tell you
concave up or down.
Concave up, positive smile
and down negative frown.

The second d’rivative
is zero in between the sections,
Of concave up or down
and they are called
points of inflection.

You know a saddle is a critical point,
that also is an inflection point.
You know, you know a saddle
is an inflection point.
that also is a critical point.