- Blue part is out of 29
- Green part is out of 71

→Total of 100 points possible

L'Hopital's Rule

Ex. $\lim_{x \to 1} \frac{\ln x}{x-1} = \frac{0}{0}$

We can use a table of values, but there's an easier way.

Thm. L'Hopital's Rule

Consider $\lim_{x \to a} \frac{f(x)}{g(x)}$. If $f(x) \to 0$ and $g(x) \to 0$ as $x \to a$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

→Do not write "$= \frac{0}{0}$"

→You must evaluate top and bottom limits individually

→Also works when the limit is of the form $\frac{\infty}{\infty}$

Ex. $\lim_{x\to 1} \frac{\ln x}{x-1} \overset{L}{=} \lim_{x\to 1} \frac{1/x}{1} = 1$

$\lim_{x\to 1} \ln x = 0$

$\lim_{x\to 1} (x-1) = 0$

- This is not the quotient rule!
- Always plug in the number first to be sure that LHR applies.

Ex. $\lim_{x\to 2} \frac{x^4-16}{x-2} \overset{L}{=} \lim_{x\to 2} \frac{4x^3}{1} = \boxed{32}$

$\lim_{x\to 2} (x^4-16) = 0$

$\lim_{x\to 2} (x-2) = 0$

Ex. $\lim_{x\to\infty} \frac{x^2}{e^x} \overset{L}{=} \lim_{x\to\infty} \frac{2x}{e^x} \overset{L}{=} \lim_{x\to\infty} \frac{2}{e^x} = \boxed{0}$

$\lim_{x\to\infty} x^2 = \infty$

$\lim_{x\to\infty} e^x = \infty$

$\lim_{x\to\infty} 2x = \infty$

$\lim_{x\to\infty} e^x = \infty$

Ex. $\lim_{x\to 5} \frac{2x-3}{x+4} = \frac{7}{9}$

Ex. $\lim_{x\to 0} \frac{e^x}{x^2} = \frac{1}{+0} = \infty$

$\lim_{x\to 0} \frac{e^x}{x^3} = \frac{1}{\pm 0} = DNE$

L'Hôpital (I have Calculus in the Heart)

Once upon a time I had trouble with math,
But now they all think that I am smart,
There's nothing I can't do,
I have Calculus in the heart.
Once upon a time I was crying all night,
But now I do my math in the dark,
Nothing I can say,
I have Calculus in the heart.

L'Hôpital:
Of the top and of the bottom,
Every now and then I get a little bit of trouble when I'm taking a limit.
We're just go-ing to compare them,
L'Hôpital:
And you know that we're making this strange,
Cause we take the limit again so for the numerator and the denominator.
If we find out that we cannot define
L'Hôpital:
Oh then, we will have to go repeat one more time.
They almost always work, but if you're confused in some kind of indeterminate form.
An Oscillating Function may be leaving its mark!
L'Hôpital:
Every now and then I get a little bit terrified but then I think of all your advice.
And when something doesn't work!
L'Hôpital:
But most of the time, well it does,
For most of the time our song works.
Guillaume François Antoine Marquis de
L'Hôpital:
Guillaume François Antoine Marquis de L'Hôp!

Pract. $\lim_{x \to 0} \frac{\sin x}{x}$

Pract. $\lim_{x \to \infty} \frac{\ln x}{x^2}$

Pract. $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$

Pract. $\lim_{x \to 0} \frac{x^3}{\cos x}$