Warm up Problems

Assume for a continuous and differentiable function, $f(-1) = -2$ and $f(3) = 6$. Determine if the following statement is guaranteed by one of our theorems, then state the theorem that was used:

1) $f(c) = 1$ for some c on the interval $[-1,3]$. yes, IVT
2) $f'(c) = 0$ for some c on the interval $[-1,3]$. no
3) $-2 \leq f(c) \leq 6$ for all c on the interval $[-1,3]$. no
4) $f'(c) = 2$ for some c on the interval $[-1,3]$. yes, MVT

Review of Chapter 4

Ex. $\lim_{x\to 0}\frac{\cos x - 1}{x} \overset{L}{=} \lim_{x\to 0}\frac{-\sin x}{1} = 0$

$\lim_{x\to 0}(\cos x - 1) = 0$

$\lim_{x\to 0}(x) = 0$

Ex. $\lim_{x\to\infty}\frac{e^x}{x^2} \overset{L}{=} \lim_{x\to\infty}\frac{e^x}{2x} \overset{L}{=} \lim_{x\to\infty}\frac{e^x}{2} = \infty$

$\lim_{x\to\infty} e^x = \infty$

$\lim_{x\to\infty} x^2 = \infty$

$\lim_{x\to\infty} e^x = \infty$

$\lim_{x\to\infty} 2x = \infty$

Ex. $\lim_{x\to 0}\frac{e^x}{x^2} = \frac{1}{+0} = \infty$

Strategy for Related Rates

"PGWEDA"

P) Draw a picture → Change = variable; Fixed = n[…]ber

G) Ident[…] given i[…]uding rates

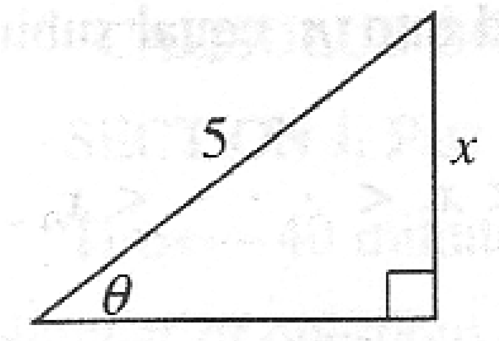
"Changes at a rate of … "

Involves time

E) […]es → Use the picture

D) Tak[…]e der[…]ative with respect to time

A) Plug in values to get your answer



Ex. In the triangle shown, θ increases at a constant rate of 3 radians per minute. At what rate is x increasing when x equals 3 units?

$\frac{d\theta}{dt} = 3$ $\qquad \frac{dx}{dt} = ?$

$x = 3$

$\sin\theta = \frac{3}{5}$

$\theta = .644$

$\sin\theta = \frac{x}{5}$

$\cos\theta \frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt}$

$\cos(.644)(3) = \frac{1}{5}\frac{dx}{dt}$

$\frac{dx}{dt} = 12$

Ex. Let $f(x) = \frac{x-4}{x^2}$ for $x \neq 0$. Find and classify all critical points. Find all inflection points. Find the global max/min values on [1,100].

$$f(x) = \frac{x}{x^2} - \frac{4}{x^2} = x^{-1} - 4x^{-2}$$

$$f'(x) = -x^{-2} + 8x^{-3} = \frac{-1}{x^2} + \frac{8}{x^3} = \frac{-x+8}{x^3} = 0$$

$$-x + 8 = 0$$

$$\boxed{x = 8}$$

f': − (0) + (8) −

8 ← local max.

$$f''(x) = 2x^{-3} - 24x^{-4} = \frac{2}{x^3} - \frac{24}{x^4} = \frac{2x-24}{x^4} = 0$$

$$2x - 24 = 0$$

$$x = 12$$

f'': − (0) − (12) +

12 ← inf. pt.

$$f(1) = \frac{-3}{1} = -3$$

$$f(8) = \frac{4}{8^2} = .0625$$

$$f(100) = \frac{96}{100^2} = .0096$$

global max. value $= .0625$

global min. value $= -3$

Strategy for Optimization

1) Draw a picture, if appropriate

2) Writ... give... ncluding an

Find the largest, smallest, closest

Maximize, minimize

4) ...

5) Take the derivative

6) Set equal to zero and solve

Calculators Allowed

1. B
2. B
3. B
4. A
5. B
6. C
7. 375,000 $ft.^2$
8. .321 rad/sec.
9. -5.890 cm^3/hr.
10. -1.014 m/s

No Calculators

1. D
2. C
3. E
4. A
5. a. (1,3) b. -3

6. a. $x = -2$ because f' goes from pos. to neg.

   b. $x = 4$ because f' goes from pos. to neg. to pos.

   c. $-1 < x < 1$ and $3 < x < 5$ because f ○ is incr.

7. a. horiz. tan at $x = \sqrt{2}$ and $x = -\sqrt{2}$, both local min. because f' goes from neg. to pos.

   b. concave up for all $x \neq 0$ because $f'' > 0$

   c. below, because graph is concave up

8. f twice-diff. → f continuous → IVT applies

$f(2) < 0 < f(4)$ → $f(c) = 0$ on interval

9. a. $x = 1$ and $x = 3$ because slope of f' changes signs

b. $x = 4$ because f decreases a lot from $x = 0$ to $x = 4$, and only increases a little after $x = 4$.

c. $y - 30 = 16(x - 5)$