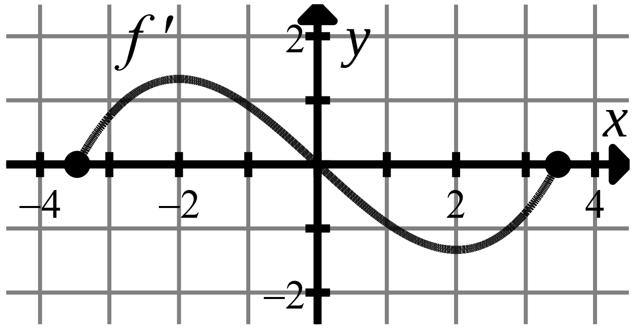
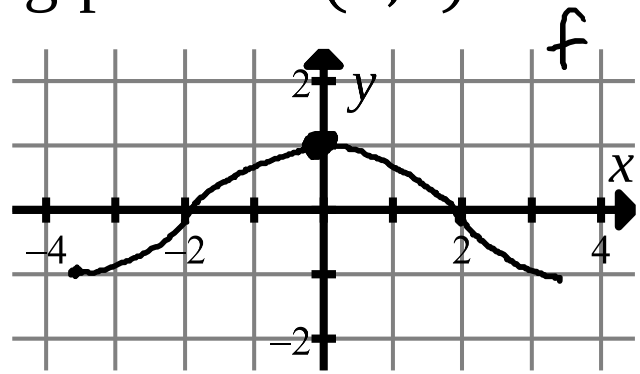
New seats today, you may sit where you wish.

- Blue part is out of 60
- Green part is out of 140
- Grade is out of 200

Antiderivatives

<u>Ex.</u> Use number lines for f' and f'' to sketch a graph of f with a starting point of $(0,1)$.





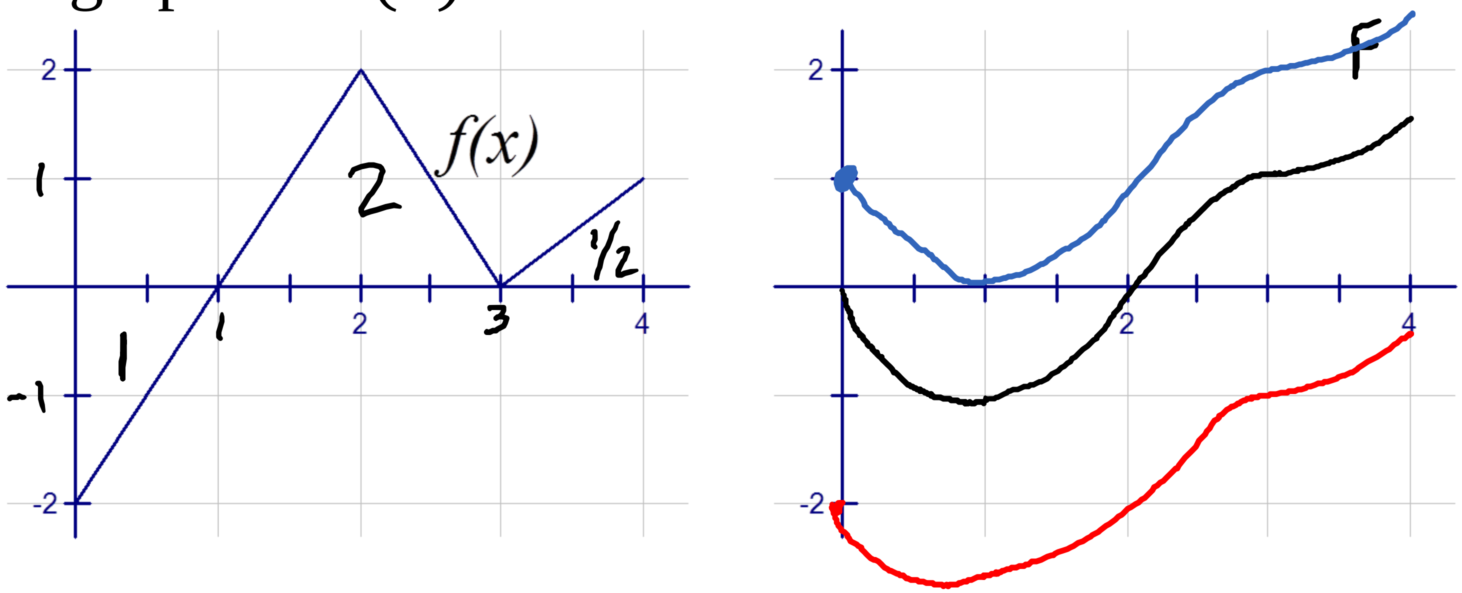
f': $+$ on $(-3.5, 0)$, $-$ on $(0, 3.5)$

-3.5   0   3.5

f'': $+$ on $(-3.5, -2)$, $-$ on $(-2, 2)$, $+$ on $(2, 3.5)$

-3.5   -2   2   3.5

Ex. Let $F(x)$ be such that $F'(x) = f(x)$. Sketch a graph of $F(x)$.



$F(x)$ is called an antiderivative of $f(x)$.

→Notice that antiderivatives are not unique.

Computing Antiderivatives

Ex. Find an antiderivative of $3x^2$.

x^3

Ex. Find an antiderivative of x^5.

$\frac{1}{6}x^6$

The last answer was $\frac{1}{6}x^6$

→It could have been $\frac{1}{6}x^6 + 9$ or $\frac{1}{6}x^6 - 58$

To describe all possible answers, we write

$$\frac{1}{6}x^6 + c$$

→ This is called the <u>general antiderivative</u>.

Pract. Find the general antiderivative of $x^2 - 4$.

$\frac{1}{3}x^3 - 4x + C$

Def. The indefinite integral of $f(x)$, written $\int f(x)dx$, is the general antiderivative of $f(x)$.

Ex. $\int x^5 dx = \frac{1}{6}x^6 + c$

"find the integral" requires "+ c"
"find an antiderivative" doesn't need "+ c"

$\int f(x)dx$	vs.	$\int_a^b f(x)dx$
Indefinite integral		Definite integral
Has no endpoints		Has endpoints
Is a function		Is a number
General antiderivative		Area under the curve

Integral Rules

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

$$\int cf(x)dx = c\int f(x)dx$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c \quad \text{for } n \neq -1$$

Ex. $\int (4x^2 - x^3)\,dx$

$\frac{4}{3}x^3 - \frac{1}{4}x^4 + C$

$2x^{-2}$

Ex. $\int \left(5x^3 - \frac{2}{x^2} + 10\right) dx$

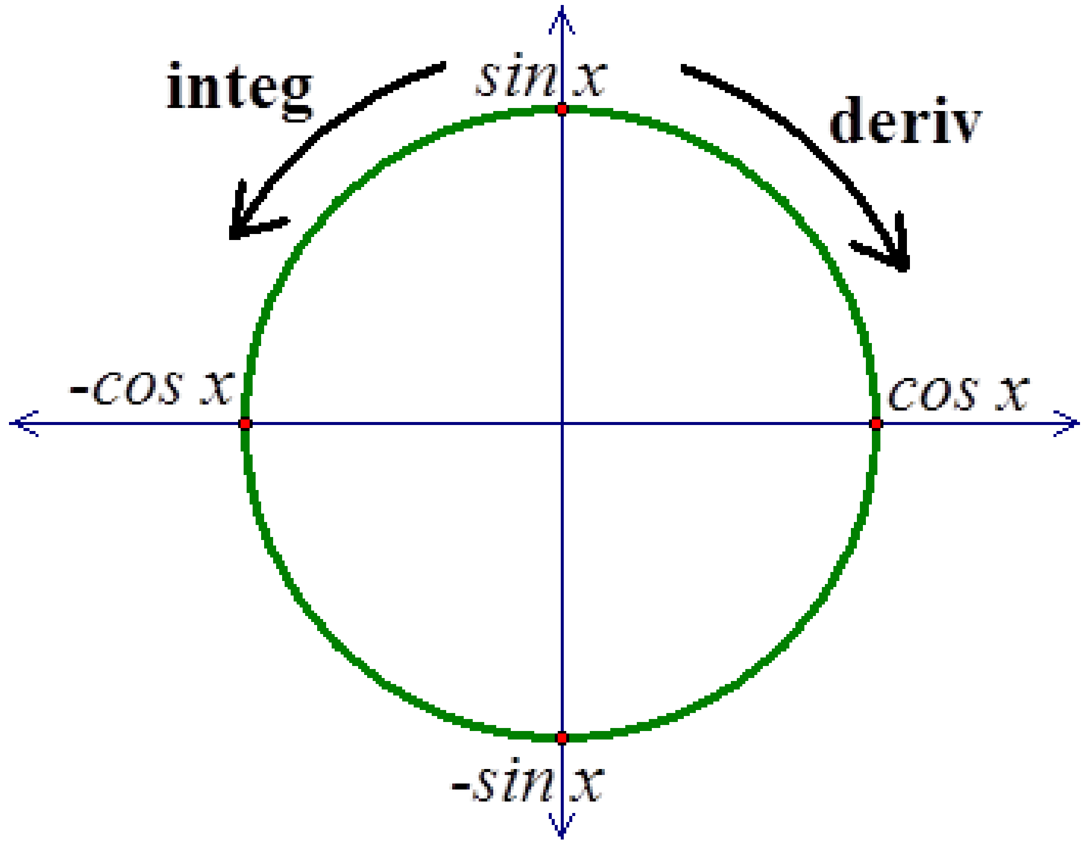
$= \frac{5}{4}x^4 + 2x^{-1} + 10x + C$

Pract. Find the following in groups:

$$\int \sin x \, dx = -\cos x + C \qquad \int \cos x \, dx = \sin x + C$$

$$\int e^x \, dx = e^x + C \qquad \int \sec^2 x \, dx = \tan x + C$$

$$\int \frac{1}{x} \, dx = \ln|x| + C \qquad \int 1 \, dx = x + C$$


integ
sin x
deriv
-cos x
cos x
-sin x

Ex. If $f'(x) = \frac{1}{x}$ and $f(1) = 3$, find $f(x)$.

$f(x) = \ln|x| + C$ ← general solution

$f(1) = \ln|1| + C = 3$

$C = 3$

$f(x) = \ln|x| + 3$ ← particular solution