The Definite Integral

Def. The definite integral gives the exact area under the curve.

$$\int_a^b f(x)dx = \begin{pmatrix} \text{signed area under } f(x) \\ \text{from } x = a \text{ to } x = b \end{pmatrix}$$

- We've been approximating this with Riemann sums.
- The calculator will evaluate the definite integral for us.

Ex. Set up an integral to find the area under $y = \ln x$ on the interval $[1,5]$, then evaluate it.

$$\int_1^5 \ln x \, dx = 4.047$$

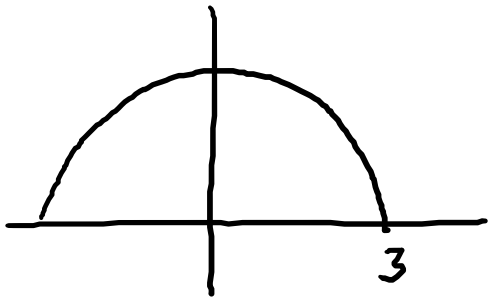
Pract. Set up an integral to find the area under $y = \sin x$ on the interval $[0, \pi]$, then evaluate it.

$$\int_0^\pi \sin x \, dx = 2$$

→ If the problem asks for a Riemann sum, you need to do that.

Some integrals can be done using geometry.

Ex. Find the area between the x-axis and $y = \sqrt{9 - x^2}$

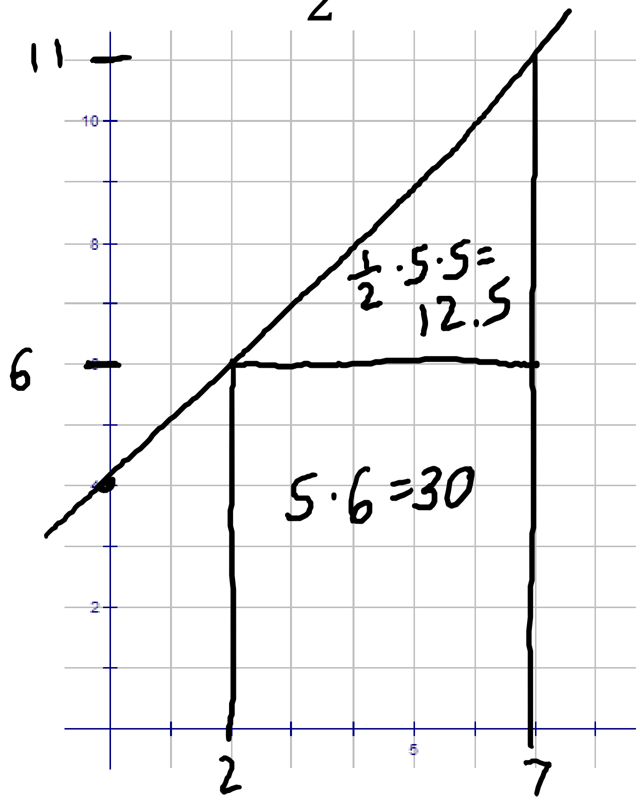


$$y^2 = 9 - x^2$$

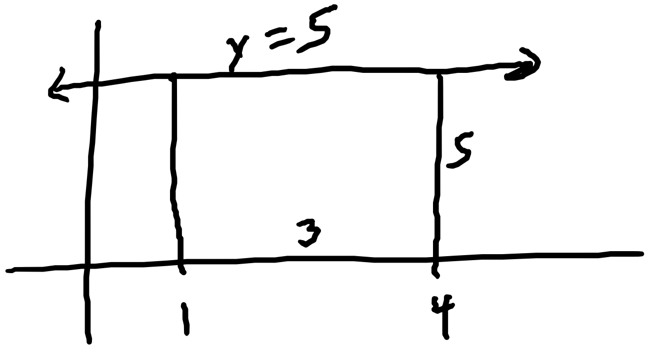
$$x^2 + y^2 = 9$$

$$\int_{-3}^{3} \sqrt{9 - x^2}\, dx = \frac{1}{2}\pi(3)^2 = \frac{9\pi}{2}$$

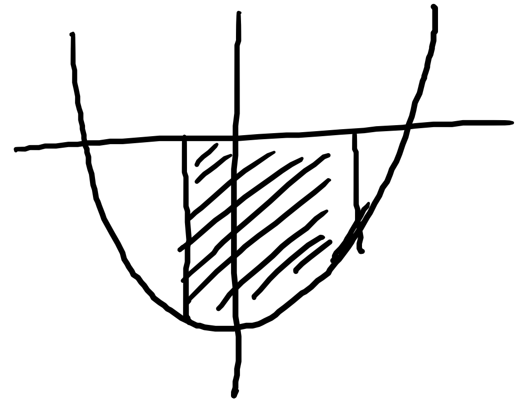
$$\underline{\text{Ex.}} \quad \int_2^7 (x+4)\,dx = 12.5 + 30 = 42.5$$



Pract. $\int_1^4 5dx = 15$



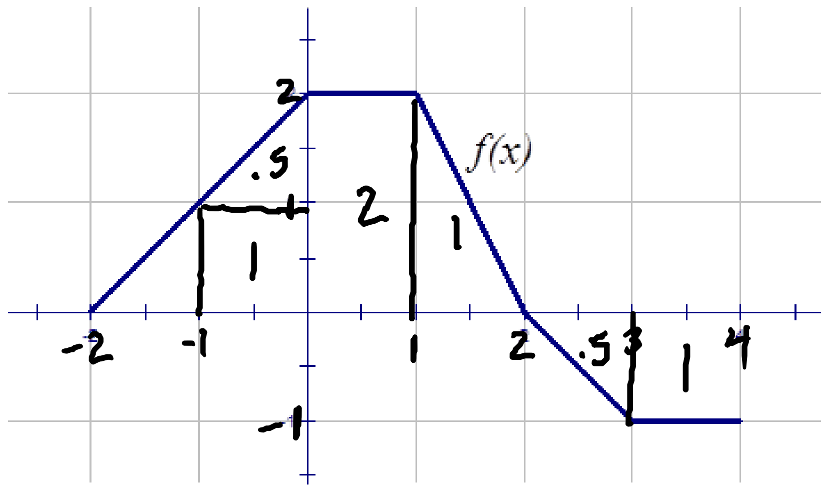
Ex. $\displaystyle\int_{-1}^{2} (x^2 - 4)\,dx = -9$



Regions under the x-axis count negatively.

Pract. The graph of $f(x)$ is shown below.

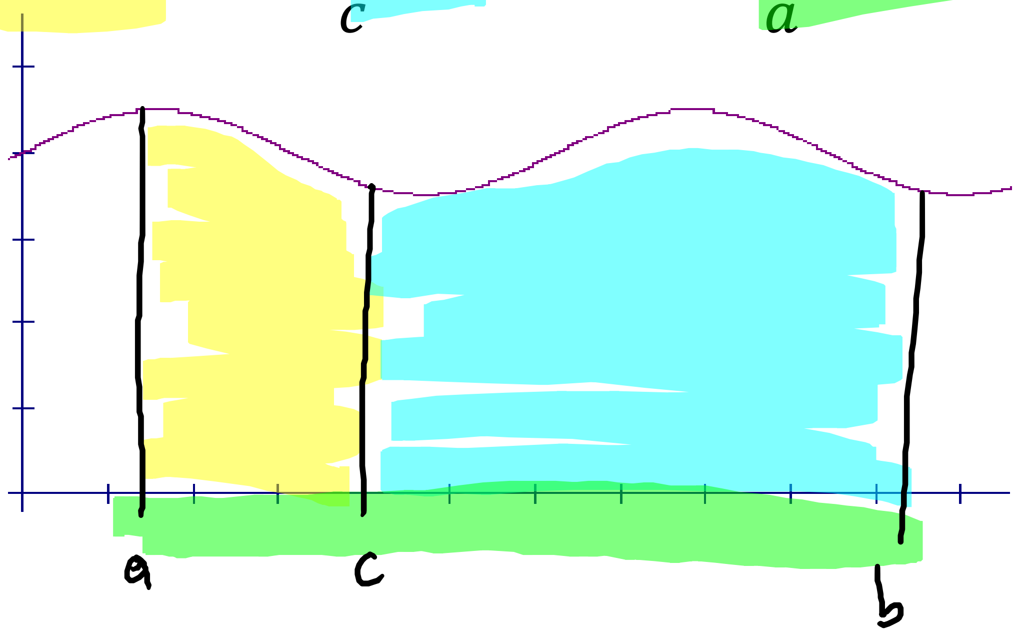
Find $\int_{-1}^{4} f(x)dx = 4.5 - 1.5$



Properties of Integrals

$$i)\ \int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$ii)\ \int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$$



$$iii) \int_a^b cf(x)dx = c\int_a^b f(x)dx$$

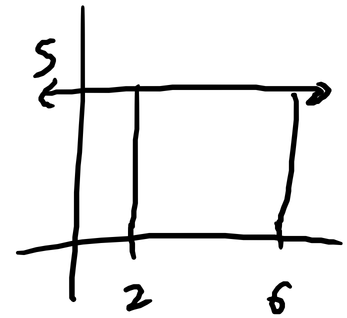
$$iv) \int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

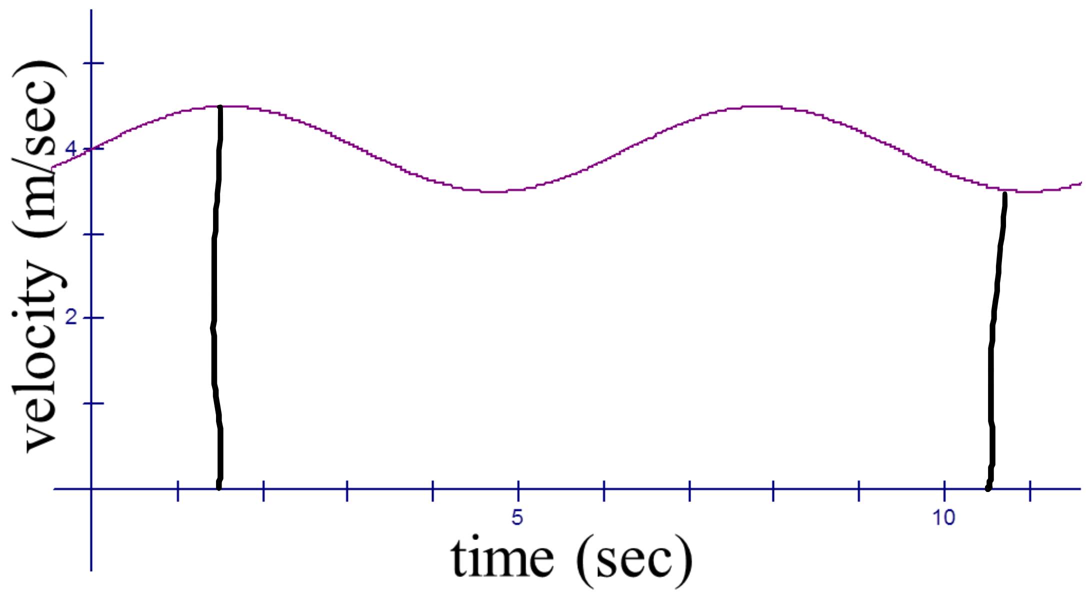
Ex. If $\int_2^6 f(x)dx = 19$, find $\int_2^6 [f(x)+5]dx$

$$\int_2^6 [f(x)+5]dx = \int_2^6 f(x)dx + \int_2^6 5\,dx$$

$$= 19 + 4 \cdot 5$$

$$= 39$$



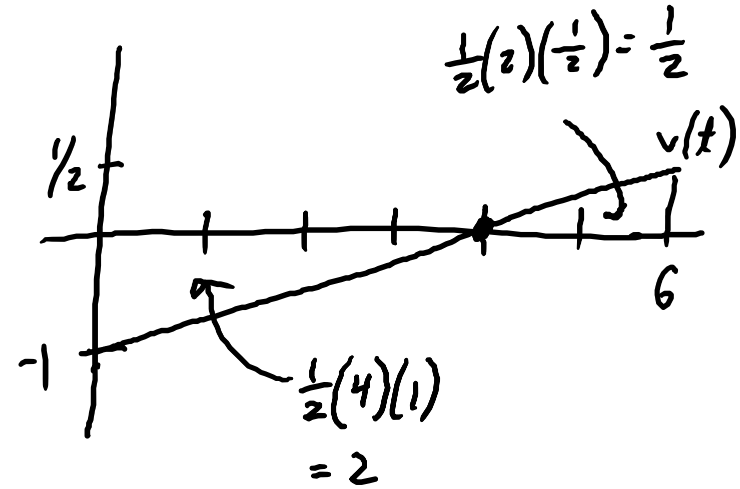

velocity (m/sec)
4
2
5
10
time (sec)

$$\text{If } f(t) = \frac{\text{meters}}{\text{seconds}}, \text{then } \int_a^b f(t)dt = \text{meters}$$

$$\boxed{\begin{array}{l} \int \text{velocity} = \text{displacement} \\ \\ \int \text{speed} = \text{dist. travelled} \end{array}}$$

Ex. Let $v(t) = \frac{1}{4}t - 1$, find the displacement and distance travelled from $t = 0$ to $t = 6$.

$$\text{disp.} = \int_0^6 v(t)\,dt = -2 + \frac{1}{2} = -1.5$$



$$\text{dist. trav.} = \int_0^6 |v(t)|\,dt = 2 + \frac{1}{2} = 2.5$$

Ex. Let $v(t) = \cos(t^3)$, find the displacement and distance travelled from $t = 0$ to $t = 2$.

$$\text{disp.} = \int_0^2 v(t)\,dt = .855$$

$$\text{dist. trav.} = \int_0^2 |v(t)|\,dt = 1.497$$