- Blue part is out of 30
- Green part is out of 70

Review, Part 1

<u>Derivatives</u>

→ Remember product, quotient, chain rules

<u>Ex.</u> $\frac{d}{dx}\sin(\cos x) = \cos(\cos x)(-\sin x)$

<u>Ex.</u> If $f(x) = \sqrt{4\sin x + 2}$, find $f'(0)$.

$f'(x) = \frac{1}{2}(4\sin x + 2)^{-1/2}(4\cos x)$

$f'(0) = \frac{1}{2}(2)^{-1/2}(4) = \frac{2}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

Ex. $\frac{d}{dx} x(\ln x)^2 = x \cdot 2(\ln x)' \cdot \frac{1}{x} + (\ln x)^2 \cdot 1$

$= 2\ln x + (\ln x)^2$

$= \ln x(2 + \ln x)$

Ex. Let f and g be differentiable functions such that $f(1) = 4$, $g(1) = 3$, $f'(3) = -5$, $f'(1) = -4$, $g'(1) = -3$, $g'(3) = 2$. If $h(x) = f\big(g(x)\big)$, find $h'(1)$.

$h'(x) = f'(g(x))g'(x)$

$h'(1) = f'(g(1))g'(1)$

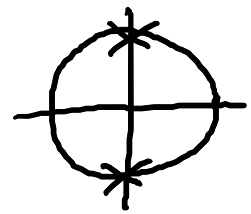
$= f'(3)\,g'(1) = (-5)(-3) = 15$

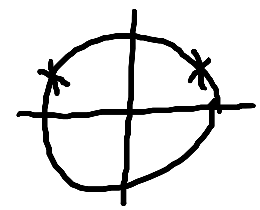
$(\sin x)^2$

Ex. Find all critical points of $f(x)=\sin^2 x - \sin x$ on $[0,2\pi]$.

$$f'(x) = 2\sin x\cos x - \cos x$$
$$= \cos x\,(2\sin x - 1) = 0$$

$\cos x = 0$ $\qquad$ $\sin x = \frac{1}{2}$





$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$

To differentiate an implicit function, we differentiate term-by-term:

- Take the derivative of x-function as usual.
- The derivative of y-function gets multiplied by y'.
- If x's and y's are in the same term, use product rule.

After differentiating, solve for y'.

Ex. $x^2 + \underbrace{4x}\underbrace{y^3} - y^4 = 17$, find the equation of the tangent line at (1,2).

$$2x + \underbrace{4x \cdot 3y^2 y' + y^3 \cdot 4} - 4y^3 y' = 0$$

$$2(1) + 4(1) \cdot 3(2)^2 y' + (2)^3 \cdot 4 - 4(2)^3 y' = 0$$

$$2 + 48y' + 32 - 32y' = 0$$

$$16y' = -34$$

$$y' = \frac{-17}{8}$$

$$\boxed{y - 2 = \frac{-17}{8}(x - 1)}$$

Ex. Let $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $f(2) = 10$, find $g'(10)$.

f

$y = x^3 + x$ $(2, 10)$

g

$x = y^3 + y$ $(10, 2)$

$1 = 3y^2 y' + 1 \cdot y'$

$1 = 3(2)^2 y' + y'$

$1 = 13y' \longrightarrow y' = \frac{1}{13}$

2013 #4

$f(0) = -8$

$\cancel{f(8)} = f'$ pos. before

$\cancel{f(1)} = f'$ pos. before

$\cancel{f(4)} =$ local max.

$f(6) = -3$

$$\int_0^8 f'(x)\,dx = f(8) - f(0)$$

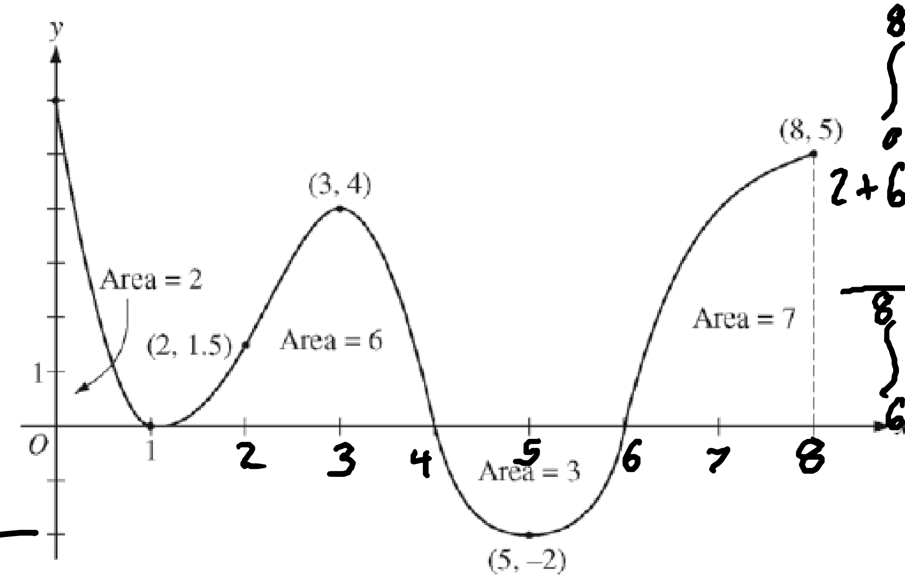
$2 + 6 - 3 + 7 = 4 - f(0)$

$f(0) = -8$

$$\int_6^8 f'(x)\,dx = f(8) - f(6)$$

$7 = 4 - f(6)$

$f(6) = -3$



Graph of f'

$g(x) = [f(x)]^3$

$g'(x) = 3[f(x)]^2 f'(x)$

$g'(3) = 3[f(3)]^2 f'(3)$

$= 3\left(-\frac{5}{2}\right)^2 (4)$

$= 75$

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer. $x = 6$, f' goes neg. to pos.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer. -8

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning. $(0, 1)$ $(3, 4)$

concave down: f' dec; increasing: f' pos.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.