Review, Part 2

L'Hopital's Rule

Ex. $\lim\limits_{x\to 0}\dfrac{e^x - 1}{\sin x} \overset{L}{=} \lim\limits_{x\to 0} \dfrac{e^x}{\cos x} = 1$

$\lim\limits_{x\to 0} e^x - 1 = 0$

$\lim\limits_{x\to 0} \sin x = 0$

Ex. $\lim\limits_{x\to -\infty}\dfrac{e^x}{x^3} = 0$

$\lim\limits_{x\to -\infty} e^x = e^{-\infty} = \dfrac{1}{e^{\infty}} = 0$

$\lim\limits_{x\to -\infty} x^3 = -\infty$

SAMPLE B

Consider the differential equation $\frac{dy}{dx} = y^2(2x+2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.

(a) Find $\lim_{x \to 0} \frac{f(x)+1}{\sin x}$. Show the work that leads to your answer.

(c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

a) $\lim_{x \to 0} \frac{f(x)+1}{\sin x} \stackrel{L}{=} \lim_{x \to 0} \frac{f'(x)}{\cos x} = \frac{(-1)^2(2 \cdot 0 + 2)}{1} = 2$

$\lim_{x \to 0} [f(x)+1] = 0$

$\lim_{x \to 0} \sin x = 0$

c) $\frac{dy}{dx} = y^2(2x+2)$

$\int y^{-2}\,dy = \int (2x+2)\,dx$

$-y^{-1} = x^2 + 2x + C$

$\frac{1}{y} = -x^2 - 2x + D$

$y = \frac{1}{-x^2-2x+D}$

$-1 = \frac{1}{0+D}$

$D = -1$

$\boxed{y = \frac{1}{-x^2-2x-1}}$

Critical Points

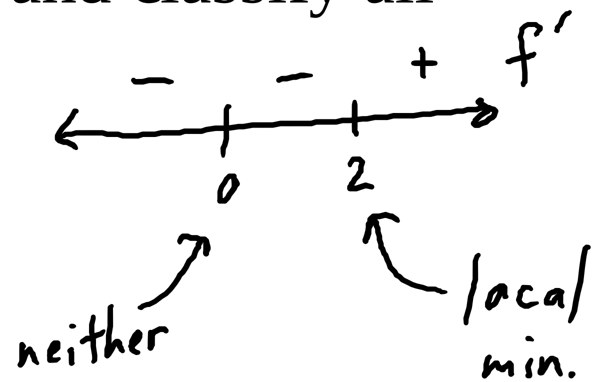
→ Where $f' = 0$ or is undefined

<u>Ex.</u> If $f(x) = x^4 - \frac{8}{3}x^3$, find and classify all critical points

$f'(x) = 4x^3 - 8x^2$

$= 4x^2(x-2) = 0$

$\boxed{x = 0 \quad \vert \quad x = 2}$



→ Inflection points are where the concavity of f changes

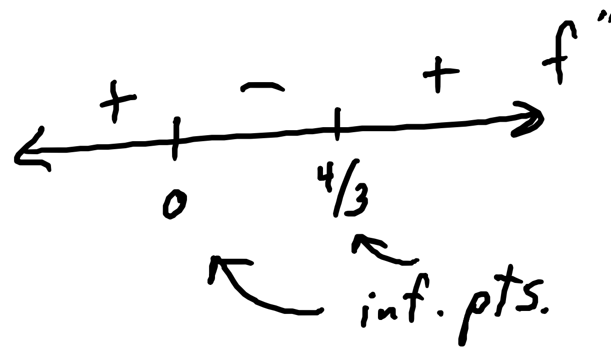
<u>Ex.</u> If $f(x) = x^4 - \frac{8}{3}x^3$, find all inflection points.
Where is f concave up?

$f'(x) = 4x^3 - 8x^2$

$f''(x) = 12x^2 - 16x$

$= 4x(3x-4) = 0$

$x = 0 \qquad x = \frac{4}{3}$



$\boxed{(-\infty, 0) \quad (\tfrac{4}{3}, \infty)}$

Absolute max/min

→ Check all critical points and endpoints

Ex. If the velocity of a particle is given by $\cancel{v(t) = t^3 - 3t^2 + 12t + 4}$, find its maximum acceleration on the interval $0 \le t \le 3$.

$a(t) = 3t^2 - 6t + 12$

$a'(t) = 6t - 6 = 0$

$t = 1$

$a(0) = 12$

$a(1) = 9$

$a(3) = 21$

$\boxed{\text{max. accel. is } 21}$

2013 #1

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

$G(t)$: ton/hr.

$\frac{dG}{dt} = \frac{\text{tons/hr.}}{\text{hrs.}}$

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer. → 635.376

a) $G'(5) = -24.588$ tons/hr² (5 ← hrs.) At $t=5$ hrs., the rate at which gravel enters is decreasing at a rate of 24.588 tons/hr.²

b) $\int_0^8 G(t)\,dt = 825.551$

c) $G(5) = 98.141 < 100$
decreasing because rate in is less than rate out at $t=5$

d) $A(t) = 500 + \int_0^t (G(x)-100)\,dx$ $\quad A(0) = 500$

$A'(t) = G(t) - 100 = 0$ $\quad A(4.923) = 635.376$

$t = 4.923$ $\quad A(8) = 525.551$

Ex. A particle moves along a curve defined by the equation $2x^2 + 3y^2 - 4xy = 36$. At time $t = 1$, the particle is at the point $(2, -2)$ and the rate of change of the y-coordinate is 4. Find the rate of change of the x-coordinate at $t = 1$.

$\frac{dy}{dt} = 4 \qquad x = 2 \qquad y = -2$

$$2x^2 + 3y^2 - 4xy = 36$$

$$4x\frac{dx}{dt} + 6y\frac{dy}{dt} \underline{-4x \cdot \frac{dy}{dt} + y\left(-4\frac{dx}{dt}\right)} = 0$$

$$4(2)\frac{dx}{dt} + 6(-2)(4) - 4(2)(4) - 4(-2)\frac{dx}{dt} = 0$$

$$8\frac{dx}{dt} - 48 - 32 + 8\frac{dx}{dt} = 0$$

$$16\frac{dx}{dt} = 80$$

$$\frac{dx}{dt} = 5$$

Ex. Let f be a twice-differentiable function such that $f(0) = -13$ and $f(7) = 15$. Must there exist a value of c, for $0 < c < 7$, such that $f(c) = 0$? Justify your answer.

$f(0) < 0$ $\quad \therefore f(c) = 0$ on interval by IVT

$f(7) > 0$ $\quad \Rightarrow f$ is cont. because it is twice-diff.

- You need to state that f is continuous to use IVT
- You need to state how you know that f is continuous
- You need to state that the desired value is between the two given values