# Review, Part 3

Riemann Sums

→ LHS, RHS, Midpoint, Trapezoid

$A = \frac{1}{2}(h_1 + h_2)b$

→ Don't forget to write the $f(2) + f(3)$ step

2013 #3

| $t$ (minutes) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $C(t)$ (ounces) | 0 | 5.3 | 8.8 | 11.2 | 12.8 | 13.8 | 14.5 |

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time $t$, $0 \le t \le 6$, is given by a differentiable function $C$, where $t$ is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

$\frac{C(4)-C(3)}{4-3} = \frac{12.8-11.2}{1} = 1.6$ oz/min.

(b) Is there a time $t$, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

$\frac{C(4)-C(2)}{4-2} = \frac{12.8-8.8}{2} = 2$ yes by MVT because $C$ is cont. and diff.

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

$\frac{1}{6}\int_0^6 C(t)\,dt = \frac{1}{6}[2C(1) + 2C(3) + 2C(5)] = \frac{1}{6}[2\cdot 5.3 + 2\cdot 11.2 + 2\cdot 13.8]$

Ave. amount of coffee in cup, in oz, from $t=0$ min. to $t=6$ min.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

$B'(t) = -16e^{-.4t}(-.4)$

$B'(5) = -16e^{-.4(5)}(-.4)$

# Fundamental Theorem of Calculus

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

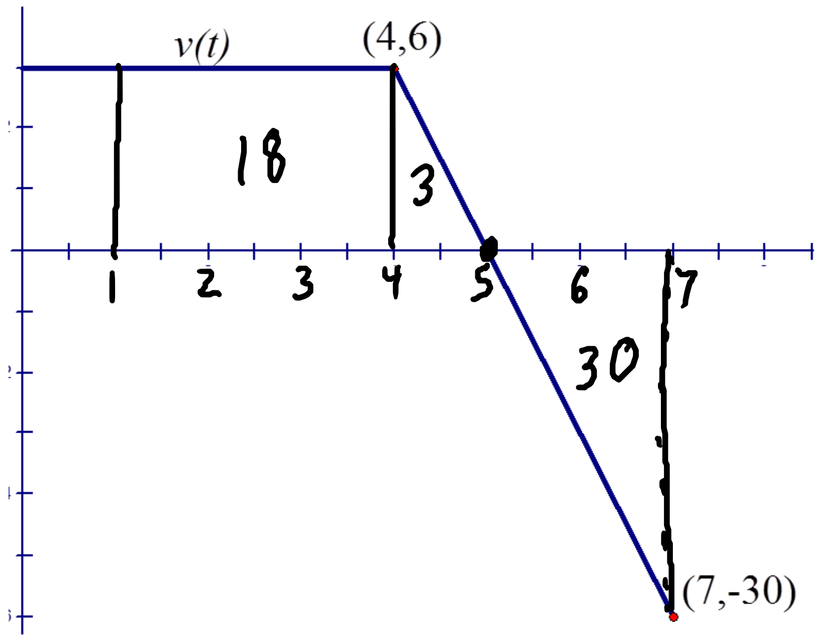
The integral of the rate of change gives the total change.

Ex. For a particle moving along the $x$-axis, you are given the graph of the velocity below. Assume $x(1) = 10$.

a) Find the total distance travelled on [1,7]. $\int_1^7 |v(t)|\,dt = 21 + 30 = 51$

b) Find $x(7)$. $= x(1) + \int_1^7 v(t)\,dt = 10 + 21 - 30 = 1$

c) When is the particle farthest to the left on [1,7]?




$x(1) = 10$

$x(5) = x(1) + \int_1^5 v(t)\,dt = 10 + 21 = 31$

$x(7) = 1$

$\boxed{t = 7}$

# Average Value

$$\begin{pmatrix}\text{ave. value of}\\ f(x)\text{ on }[a,b]\end{pmatrix} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Ex. Find the average value of $f(x) = x^2\sqrt{x^3+1}$ on $[0, 2]$.

$$\frac{1}{2-0}\int_0^2 x^2\sqrt{x^3+1}\,dx = \frac{1}{2}\int_{*}^{**} u^{1/2}\cdot\frac{1}{3}\,du = \frac{1}{6}\,\frac{2}{3}\,u^{3/2}\Big|_{*}^{**} = \frac{1}{9}\left(x^3+1\right)^{3/2}\Big|_0^2$$

$$= \frac{1}{9}(9)^{3/2} - \frac{1}{9}(1)^{3/2} = \frac{1}{9}(27) - \frac{1}{9} = \boxed{\frac{26}{9}}$$

$$\boxed{\begin{aligned} u &= x^3+1\\ du &= 3x^2\,dx\\ \tfrac{1}{3}\,du &= x^2\,dx \end{aligned}}$$

Remember:

$$\begin{pmatrix}\text{ave. value of} \\ f(x) \text{ on } [a, b]\end{pmatrix} = \frac{1}{b-a}\int_a^b f(x)dx$$

$$\begin{pmatrix}\text{ave. rate of change} \\ \text{from } t = a \text{ to } t = b\end{pmatrix} = \frac{f(b) - f(a)}{b-a}$$

2013 #6

Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of $f$ at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

a) $\left.\frac{dy}{dx}\right|_{(1,0)} = e^0(3\cdot 1^2 - 6\cdot 1) = -3$

$y - 0 = -3(x-1)$

$f(1.2) \approx -3(1.2-1) = -.6$

$$\boxed{y = -\ln(-x^3 + 3x^2 - 1)}$$

b) $\frac{dy}{dx} = e^y(3x^2 - 6x)$

$\int e^{-y}\,dy = \int (3x^2 - 6x)\,dx$

$-e^{-y} = x^3 - 3x^2 + C$

$e^{-y} = -x^3 + 3x^2 + D$

$-y = \ln(-x^3 + 3x^2 + D)$

$y = -\ln(-x^3 + 3x^2 + D)$

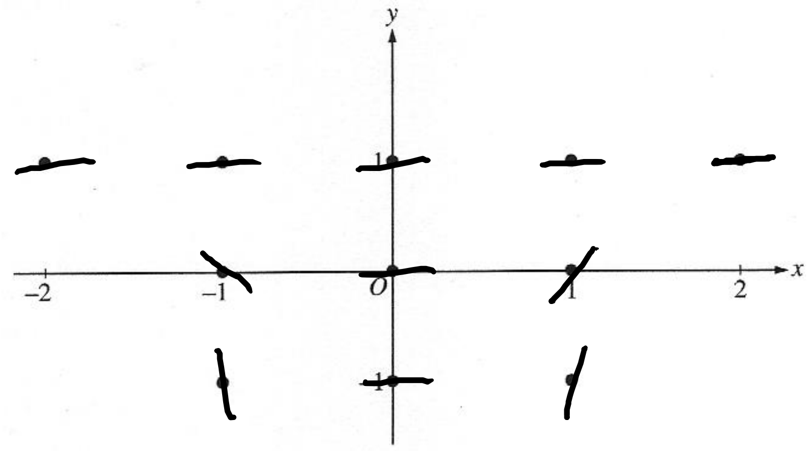
$0 = -\ln(-1 + 3 + D)$

$e^0 = 2 + D \longrightarrow D = -1$

# Slope Fields

Ex. Consider $\frac{dy}{dx} = x(y-1)^2$

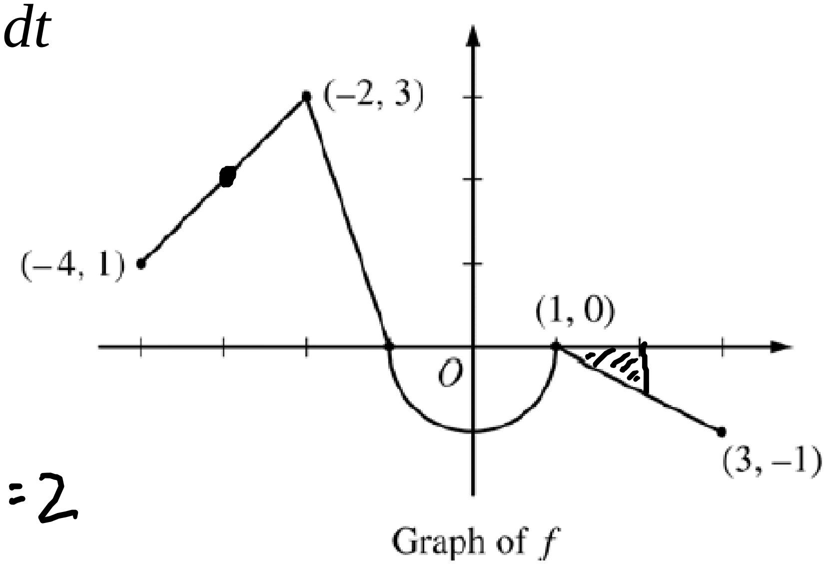
Sketch a slope field at the points indicated



Ex. Let $f$ be the continuous function whose graph is shown.

Let $g$ be the function $g(x) = \int_1^x f(t)\,dt$

a) Find $g(2)$. $= \int_1^2 f(t)\,dt = -\frac{1}{2}(1)\left(\frac{1}{2}\right) = -\frac{1}{4}$



Graph of $f$

b) Find $g'(-3)$.

$g'(x) = f(x)$

$g'(-3) = f(-3) = 2$

c) Find $g''(-3)$.

$g''(x) = f'(x)$

$g''(-3) = f'(-3) = 1$