Review, Part 4

Piecewise Functions

→ Equation depends on your value of x

Ex. Let $g(x) = \begin{cases} x^4 - 3 & x < 1 \\ 2 & 1 \le x \le 3 \\ x - 1 & x > 3 \end{cases}$

a. Find $g(-2)$. $= (-2)^4 - 3$

$= 16 - 3$

$= \boxed{13}$

Piecewise Functions

→ Equation depends on your value of x

Ex. Let $g(x) = \begin{cases} x^4 - 3 & x < 1 \\ 2 & 1 \le x \le 3 \\ x - 1 & x > 3 \end{cases}$

b. Find $\int_{-4}^{4} g(x)dx = \int_{-4}^{1} (x^4-3)\,dx + \int_{1}^{3} 2\,dx + \int_{3}^{4} (x-1)\,dx$

$= \frac{1}{5}x^5 - 3x \Big|_{-4}^{1} + 2x \Big|_{1}^{3} + \frac{1}{2}x^2 - x \Big|_{3}^{4}$

$= \left(\frac{1}{5} - 3\right) - \left(\frac{1}{5}(-4)^5 + 12\right) + 6 - 2 + \left(8 - 4\right) - \left(\frac{9}{2} - 3\right)$

Piecewise Functions

→ Equation depends on your value of x

Ex. Let $g(x) = \begin{cases} x^4 - 3 & x < 1 \\ 2 & 1 \le x \le 3 \\ x - 1 & x > 3 \end{cases}$

c Find $\lim\limits_{x\to 3} g(x) = 2$

$$\lim_{x\to 3^+} g(x) = \lim_{x\to 3^+} (x-1) = 2$$

$$\lim_{x\to 3^-} g(x) = \lim_{x\to 3^-} 2 = 2$$

SAMPLE A

Consider the differential equation $\frac{dy}{dx} = 3x + 2y + 1$.

(a) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

y''

$$y' = 3x + 2y + 1$$

$$y'' = 3 + 2y'$$

$$y'' = 3 + 2(3x + 2y + 1)$$

$\lim_{x \to 2^+}$ means that x approaches 2 from the right (larger than 2)

$\lim_{x \to 2^-}$ means that x approaches 2 from the left (smaller than 2)

x	10	10.9	10.99	10.999	11.001	11.01	11.1	12
$f(x)$	29	31.7	31.97	31.997	32.003	32.03	32.3	35

11

The table above gives values of the function f at selected values of x. Which of the following conclusions is supported by the data in the table?

(A) $\lim_{x \to 11} f(x) = 32$

(B) $\lim_{x \to 11} f(x) = \infty$

(C) $\lim_{x \to 32} f(x) = 11$

(D) $\lim_{x \to 32} f(x) = \infty$

2 2

x	1	1.9	1.99	1.999	1.9999	2.0001	2.001	2.01	2.1	3
$f(x)$	-4	-1.399	-1.040	-1.004	-1.000	6.001	6.012	6.121	7.261	25

The table above gives values of the function f at selected values of x. Which of the following conclusions is supported by the data in the table?

-1 6

(A) $\lim_{x\to 2} f(x) = -1$

(B) $\lim_{x\to 2} f(x) = 6$

(C) $\lim_{x\to 2^-} f(x) = -1$ and $\lim_{x\to 2^+} f(x) = 6$

(D) $\lim_{x\to 2^-} f(x) = 6$ and $\lim_{x\to 2^+} f(x) = -1$

Def. A function $f(x)$ is continuous on an interval if, for all points c on the interval:

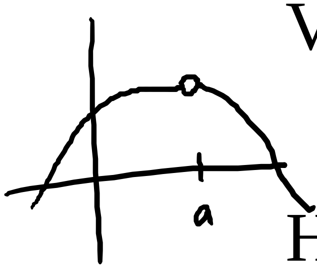
i. $\lim_{x \to c} f(x)$ exists

ii. $f(c)$ exists

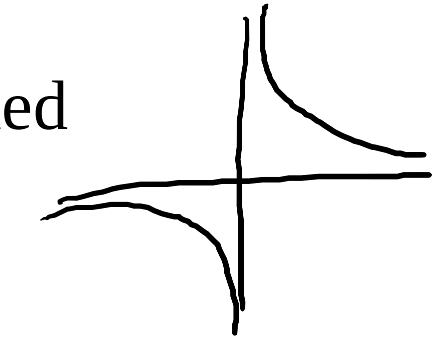
iii. $\lim_{x \to c} f(x) = f(c)$

Def. A function $f(x)$ is differentiable on an interval if $f(x)$ and $f'(x)$ are continuous on the interval.

<u>Asymptotes</u>



Vertical: any point where function is undefined
(and can't be removed) → $\lim_{x \to a} f(x) = \infty$



Horizontal: $\lim_{x \to \infty} f(x) = L$

<u>Ex.</u> Find all asymptotes of $y = \frac{x}{x^2-4}$

<u>v.a.</u> $x^2 - 4 = 0$

$x = \pm 2$

$\lim_{x \to 2} \frac{x}{x^2-4} = \frac{2}{0} = \infty$

$\lim_{x \to -2} \frac{x}{x^2-4} = \frac{-2}{0} = \infty$

$\boxed{x = 2}$

$\boxed{x = -2}$

<u>h.a.</u> $\lim_{x \to \infty} \frac{x}{x^2-4} = \lim_{x \to \infty} \frac{x}{x^2}$

$= \lim_{x \to \infty} \frac{1}{x} = 0$

$\boxed{y = 0}$

Summary

For a horizontal asymptote,

$$x \to \infty \text{ and } f(x) \to \text{finite}$$

For a vertical asymptote,

$$x \to \text{finite and } f(x) \to \infty$$

<u>Tangent Line</u>

$y = mx + b$ [slope = derivative at the point]

$y - y_1 = m(x - x_1)$ [slope = derivative at the point]

$y = f(a) + f'(a)(x - a)$ [tangent line at $x = a$]

<u>Linear Approximation</u>

<u>Ex.</u> Let f be a function such that $f(3) = 2$ and $f'(3) = 5$. Approximate a zero of f.

$$f(x) \approx f(3) + f'(3)(x-3)$$

$$f(x) \approx 2 + 5(x-3) = 0$$

$$5x - 13 = 0$$

$$\boxed{x = \frac{13}{5}}$$

2013 #2

A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

a) $|v(t)| = 2$

$t = 3.128, 3.473$

b) $s(5) = s(0) + \int_0^5 v(t)\,dt \approx -9.207$

$s(t) = s(0) + \int_0^t v(x)\,dx$

c) $v(t) = 0$

$t = .536, 3.318$

$v(t)$ changes signs at these times

d) $v(4) = -11.476$

$a(4) = -22.296$

speed inc. because $v(4)$ and $a(4)$ are same sign