Calculators Allowed

1. C 2. B

3. a) $P(t) = 30e^{0.116t}$ b) 170

4. 94.829 hrs. 5. a) 72 b) .354

6. i. ∞ ii. 2 iii. 1 iv. $-\infty$ v. 1

No Calculators

1. D 2. $(f \circ g)(x) = \frac{-2}{x}$ 3. -5 4. 1

5. $S(x) = x^2 + \frac{28}{x}$ 6. a) 9999 b) $\frac{\ln 5+1}{\ln 2-1}$ c) $\frac{1}{4}$

7. a) $y = 1$ b) $x = 3$ c) $\left(-1, \frac{1}{2}\right)$

8. $\frac{-\sqrt{3}}{2}$ 9. $\ln\left(\frac{16}{(x+4)^{2/3}}\right)$ 10. -2

11. a) vert. asympt. of $x = 7$

    b) funct. is cont. at $x = 7$

    c) horiz. asympt. of $y = 7$

12. $f(2) = 1$ 13. $c = 6$ 14. e

15. a) 2.62 b) removable discont. because $\lim_{x \to 1} f(x)$ is finite

KEY

Name _____

Period _____

Calculus AB -- Chapter 1/2 Sample Test (calculators allowed)

Show all work for free-response questions.

1. $\lim_{x\to\infty}\dfrac{x^3-2x^2+3x-4}{4x^3-3x^2+2x-1}=$ $\lim_{x\to\infty}\dfrac{x^3}{4x^3}=\dfrac{1}{4}$

(A) 4 (B) 1 (C) $\frac{1}{4}$ (D) 0 (E) -1

2. If $y=7$ is a horizontal asymptote of a rational function f, then which of the following must be true?

(A) $\lim_{x\to 7} f(x)=\infty$ (B) $\lim_{x\to\infty} f(x)=7$ (C) $\lim_{x\to 0} f(x)=7$

(D) $\lim_{x\to 7} f(x)=0$ (E) $\lim_{x\to-\infty} f(x)=-7$

3. If a bacteria population starts with 30 bacteria and doubles every 6 hours.

a) Assuming exponential growth, find a model for $P(t)$, the population of the bacteria after t hours.

$P(t)=30e^{kt}$

$60=30e^{6k}$

$2=e^{6k}$

$\ln 2=6k$

$k=\dfrac{\ln 2}{6}=.116$

$P(t)=30e^{.116t}$

b) What is the size of the population after 15 hours?

$P(15)=30e^{.116(15)}$

$=169.706$

4. A sample of an isotope has mass 8g. If the half-life of the isotope is 15 hours, find the time required for the mass to be reduced to 0.1g.

$P=8e^{kt}$

$4=8e^{k(15)}$

$\frac{1}{2}=e^{15k}$

$\ln\frac{1}{2}=15k$

$k=\dfrac{\ln 1/2}{15}=-.046$

$P=8e^{-.046t}$

$.1=8e^{-.046t}$

$\frac{.1}{8}=e^{-.046t}$

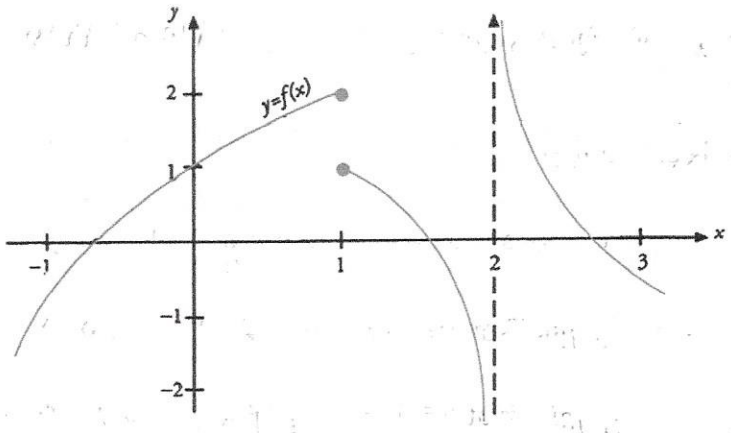
$\ln(\frac{.1}{8})=-.046t$

$t=\dfrac{\ln(.1/8)}{-.046}=94.829$ hrs.

5. Evaluate the limits

a) $\lim_{x\to 0} \frac{\tan 6x - 6x}{x^3}$ = 72

b) $\lim_{x\to 2} \frac{\sqrt{x}-\sqrt{2}}{x-2}$ = .354



6. The graph of a function f is shown above. Evaluate each of the following limits.

i. $\lim_{x\to 2^+} f(x) =$ ∞

ii. $\lim_{x\to 1^-} f(x) =$ 2

iii. $\lim_{x\to 1^+} f(x) =$ 1

iv. $\lim_{x\to 2^-} f(x) =$ $-\infty$

v. $\lim_{x\to 0} f(x) =$ 1

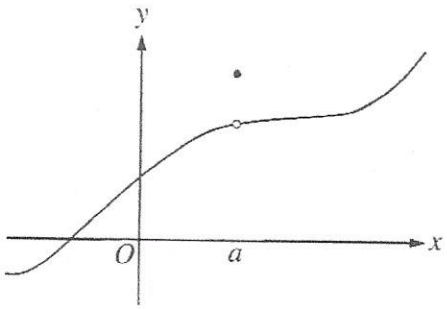
Name _____

Period _____

Calculus AB -- Chapter 1/2 Sample Test (no calculators)

Show all work for free-response questions.

1.



The graph of a function f is shown above. Which of the statements about f is true?

(A) f is continuous at $x = a$.

(B) f is discontinuous at $x = a$ because $\lim_{x \to a} f(x)$ does not exist.

(C) f is discontinuous at $x = a$ because $f(a)$ does not exist.

(D) f is discontinuous at $x = a$ because $\lim_{x \to a} f(x) \neq f(a)$.

2. Find (and simplify) the equation for the function $(f \circ g)(x)$ if $f(x) = \frac{x-1}{x}$ and $g(x) = \frac{x}{x+2}$.

$f(g(x)) = f\left(\frac{x}{x+2}\right) = \frac{\left(\frac{x}{x+2} - 1\right)(x+2)}{\frac{x}{x+2}(x+2)} = \frac{x-(x+2)}{x} = \frac{-2}{x}$

3. For the function $f(x) = x^2 - 8x + 11$, evaluate $(f \circ f)(7)$.

$f(f(7)) = f(4) = 4^2 - 8(4) + 11 = -5$

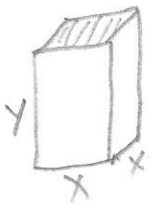
$f(7) = 7^2 - 8(7) + 11 = 4$

4. For the function $f(x) = \begin{cases} 1 - 2\sin x & x \leq 0 \\ e^{-4x} & x > 0 \end{cases}$, evaluate $\lim_{x \to 0} f(x)$.

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} e^{-4x} = 1$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 1 - 2\sin x = 1$

So $\lim_{x \to 0} f(x) = 1$



5. A rectangular box has a square base, no lid, and volume 7 m^3. Express the surface area of the box as a function $S(x)$ of the length x of a side of the base.

$x^2y = 7$

$y = \frac{7}{x^2}$

$S = x^2 + 4xy$

$= x^2 + 4x\left(\frac{7}{x^2}\right)$

$S(x) = x^2 + \frac{28}{x}$

6. Solve for x.

a) $\log_{10}(x+1) = 4$

$x + 1 = 10^4$

$x = 9999$

b) $2^x = 5e^{x+1}$

$\ln(2^x) = \ln(5e^{x+1})$

$x\ln 2 = \ln 5 + \ln(e^{x+1})$

$x\ln 2 = \ln 5 + x + 1$

$x\ln 2 - x = \ln 5 + 1$

$x(\ln 2 - 1) = \ln 5 + 1$

$x = \frac{\ln 5 + 1}{\ln 2 - 1}$

c) $4^{3x-1} = \frac{1}{\sqrt{2}}$

$(2^2)^{3x-1} = 2^{-1/2}$

$2^{6x-2} = 2^{-1/2}$

$6x - 2 = -\frac{1}{2}$

$6x = \frac{3}{2}$

$x = \frac{1}{4}$

7. Consider the function $y = \frac{x^2-1}{x^2-2x-3}$. Find the following, showing all appropriate limits.

a) Find the equation of all horizontal asymptotes.

$\lim_{x\to\infty} \frac{x^2-1}{x^2-2x-3} = \lim_{x\to\infty} \frac{x^2}{x^2} = 1$

$y = 1$

b) Find the equation of all vertical asymptotes.

$x = 3$

$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{(x+1)(x-1)}{(x-3)(x+1)} = \frac{2}{0} = \text{infinite}$

$y = \frac{(x+1)(x-1)}{(x-3)(x+1)}$

c) Find the coordinates of all removable discontinuities.

$\lim_{x\to -1} f(x) = \lim_{x\to -1} \frac{(x+1)(x-1)}{(x-3)(x+1)} = \frac{-2}{-4} = \frac{1}{2}$

Hole at $\left(-1, \frac{1}{2}\right)$

8. Evaluate $\sin\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$. $= \sin\left(-\frac{\pi}{3}\right) = \boxed{-\frac{\sqrt{3}}{2}}$

9. Express $4\ln 2 - \frac{2}{3}\ln(x+4)$ as a single logarithm.

$\ln 2^4 - \ln(x+4)^{2/3}$

$\boxed{\ln\left(\frac{16}{(x+4)^{2/3}}\right)}$

10. Evaluate $\lim_{x\to -3} \frac{x^2+4x+3}{x+3}$.

$= \lim_{x\to -3} \frac{(x+1)(x+3)}{x+3} = \lim_{x\to -3} (x+1) = -2$

11. Determine what can be said about the graph of the function if the limit is true.

a) $\lim_{x\to 7} f(x) = \infty$ → vert asymp. at $x=7$

b) $\lim_{x\to 7} g(x) = g(7)$ → function cont. at $x=7$

c) $\lim_{x\to \infty} h(x) = 7$ → horiz. asymp. of $y=7$

12. How would you define $f(2)$ in order to make $f(x) = \frac{x^2-3x+2}{x-2}$ continuous at $x = 2$?

$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{(x-1)(x-2)}{x-2} = \lim_{x\to 2} (x-1) = 1$

$\boxed{f(2) = 1}$

13. Consider the function $f(x) = \begin{cases} cx+9 & x \le 2 \\ cx^2-3 & x > 2 \end{cases}$. Find the value of c so that $f(x)$ is continuous for all real numbers. Show all work, including limits, that lead you to your answer.

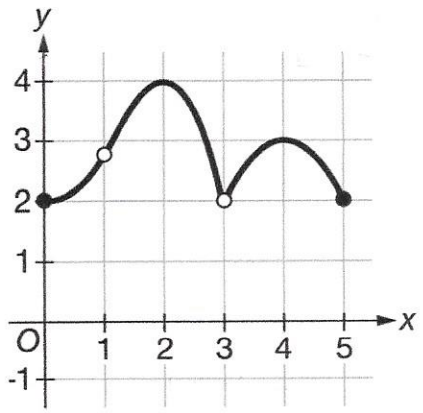
$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (cx^2-3) = 4c-3$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (cx+9) = 2c+9$

$4c-3 = 2c+9$

$2c = 12$

$c = 6$



Graph of f

$\lim_{x \to 3} f(x) = 2$

14. The graph of the functions f is shown above on the interval $0 \leq x \leq 5$. Evaluate $\lim_{x \to 3} e^{f(x)-1}$ or explain why the limit doesn't exist.

$= e^{2-1} = e$

x	0.995	0.996	0.997	0.998	0.999	1	1.001	1.002	1.003	1.004	1.005
$f(x)$	2.631	2.628	2.626	2.623	2.621	Undefined	2.615	2.613	2.610	2.607	2.605

15. Values of f at selected values of x are given in the table above.

a) Based on values from the table, what is a reasonable estimate for $\lim_{x \to 1} f(x)$?

2.62

b) The function is undefined at $x = 1$. What type of discontinuity does the function have at this point?

removable because $\lim_{x \to 1} f(x)$ is finite