Name $\qquad$
Period $\qquad$

## Calculus AB -- Chapter 1/2 Sample Test (calculators allowed)

Show all work for free-response questions.

1. $\lim _{x \rightarrow \infty} \frac{x^{3}-2 x^{2}+3 x-4}{4 x^{3}-3 x^{2}+2 x-1}=$
(A) 4
(B) 1
(C) $\frac{1}{4}$
(D) 0
(E) -1
2. If $y=7$ is a horizontal asymptote of a rational function $f$, then which of the following must be true?
(A) $\lim _{x \rightarrow 7} f(x)=\infty$
(B) $\lim _{x \rightarrow \infty} f(x)=7$
(C) $\lim _{x \rightarrow 0} f(x)=7$
(D) $\lim _{x \rightarrow 7} f(x)=0$
(E) $\lim _{x \rightarrow-\infty} f(x)=-7$
3. A bacteria population starts with 30 bacteria and doubles every 6 hours.
a) Assuming exponential growth, find a model for $P(t)$, the population of the bacteria after $t$ hours.
b) What is the size of the population after 15 hours? Round to the nearest whole number.
4. A sample of an isotope has mass 8 g . If the half-life of the isotope is 15 hours, find the time required for the mass to be reduced to 0.1 g .
5. Evaluate the limits
a) $\lim _{x \rightarrow 0} \frac{\tan 6 x-6 x}{x^{3}}$
b) $\lim _{x \rightarrow 2} \frac{\sqrt{x}-\sqrt{2}}{x-2}$

6. The graph of a function $f$ is shown above. Evaluate each of the following limits.
i. $\lim _{x \rightarrow 2^{+}} f(x)=$
ii. $\quad \lim _{x \rightarrow 1^{-}} f(x)=$
iii. $\lim _{x \rightarrow 1^{+}} f(x)=$
iv. $\lim _{x \rightarrow 2^{-}} f(x)=$
v. $\lim _{x \rightarrow 0} f(x)=$
$\qquad$
Period $\qquad$

## Calculus AB -- Chapter 1/2 Sample Test (no calculators)

Show all work for free-response questions.
1.


The graph of a function $f$ is shown above. Which of the statements about $f$ is true?
(A) $f$ is continuous at $x=a$.
(B) $f$ is discontinuous at $x=a$ because $\lim _{x \rightarrow a} f(x)$ does not exist.
(C) $f$ is discontinuous at $x=a$ because $f(a)$ does not exist.
(D) $f$ is discontinuous at $x=a$ because $\lim _{x \rightarrow a} f(x) \neq f(a)$.
2. Find (and simplify) the equation for the function $(f \circ g)(x)$ if $f(x)=\frac{x-1}{x}$ and $g(x)=\frac{x}{x+2}$.
3. For the function $f(x)=x^{2}-8 x+11$, evaluate $(f \circ f)(7)$.
4. For the function $f(x)=\left\{\begin{array}{cc}1-2 \sin x & x \leq 0 \\ e^{-4 x} & x>0\end{array}\right.$, evaluate $\lim _{x \rightarrow 0} f(x)$.
5. A rectangular box has a square base, no lid, and volume $7 \mathrm{~m}^{3}$. Express the surface area of the box as a function $S(x)$ of the length $x$ of a side of the base.
6. Solve for $x$.
a) $\log _{10}(x+1)=4$
b) $2^{x}=5 e^{x+1}$
c) $4^{3 x-1}=\frac{1}{\sqrt{2}}$
7. Consider the function $y=\frac{x^{2}-1}{x^{2}-2 x-3}$. Find the following, showing all appropriate limits.
a) Find the equation of all horizontal asymptotes.
b) Find the equation of all vertical asymptotes.
c) Find the coordinates of all removable discontinuities.
8. Evaluate $\sin \left(\sin ^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$.
9. Express $4 \ln 2-\frac{2}{3} \ln (x+4)$ as a single logarithm.
10. Evaluate $\lim _{x \rightarrow-3} \frac{x^{2}+4 x+3}{x+3}$.
11. Determine what can be said about the graph of the function if the limit is true.
a) $\lim _{x \rightarrow 7} f(x)=\infty$
b) $\lim _{x \rightarrow 7} g(x)=g(7)$
c) $\lim _{x \rightarrow \infty} h(x)=7$
12. How would you define $f(2)$ in order to make $f(x)=\frac{x^{2}-3 x+2}{x-2}$ continuous at $x=2$ ?
13. Consider the function $f(x)=\left\{\begin{array}{cc}c x+9 & x \leq 2 \\ c x^{2}-3 & x>2\end{array}\right.$. Find the value of $c$ so that $f(x)$ is continuous for all real numbers. Show all work, including limits, that lead you to your answer.

14. The graph of the functions $f$ is shown above on the interval $0 \leq x \leq 5$. Evaluate $\lim _{x \rightarrow 3} e^{f(x)-1}$ or explain why the limit doesn't exist.

| $x$ | 0.995 | 0.996 | 0.997 | 0.998 | 0.999 | 1 | 1.001 | 1.002 | 1.003 | 1.004 | 1.005 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 2.631 | 2.628 | 2.626 | 2.623 | 2.621 | Undefined | 2.615 | 2.613 | 2.610 | 2.607 | 2.605 |

15. Values of $f$ at selected values of $x$ are given in the table above.
a) Based on values from the table, what is a reasonable estimate for $\lim _{x \rightarrow 1} f(x)$ ?
b) The function is undefined at $x=1$. What type of discontinuity does the function have at this point? Give a reason for your answer.
