Calculators Allowed

1. B
2. When the temp is 23ºF, the cost is decreasing at a rate of \$0.17 per ºF. Units are \$/ºF.
3. a. $\lim_{h \to 0} \frac{[2^{1+h}+(1+h)]-[2^1+1]}{h} = 2.386$

   b. $f'(x) = 2^x \ln 2 + 1 \Rightarrow f'(1) = 2 \ln 2 + 1$
4. a) (0,35) and (45,50) because $v(t)$ is inc.

   b) 1.44 ft/sec^2 c) -2.1 ft/sec^2 (uses $35 < t < 45$)

No Calculators

1. C 2. A 3. C 4. D

5. E 6. D 7. $y - 1 = 2(x + 1)$ 8. $5x^{-1/2} + 6e^x$

9. a. (-5,-3) and (1,4) because f' is positive

   b. (-4,-1) and (2,5) because f' is decreasing

Name ________________

Period ________________

Calculus AB -- Chapter 3A Sample Test (calculators allowed)

Show all work for free-response questions.

1. The position, in ft, of a particle moving along the x-axis is given by the function $x(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$.

   (A) 20.086 ft/sec (B) 26.447 ft/sec (C) 32.809 ft/sec

   (D) 40.671 ft/sec (E) 79.342 ft/sec

   $$\frac{x(3)-x(0)}{3-0} = \frac{(e^3+3e^3)-(e^0+0e^0)}{3}$$

2. Suppose that $C = f(T)$ is the monthly cost, in dollars, to heat my house when the temperature outside is T degrees Fahrenheit. What does $f'(23) = -0.17$ mean? What are the units on $f'(23)$? $\frac{\$}{{}^\circ F}$

   When the temp. is 23°F, the cost is dec. at a rate of \$.17 per °F.

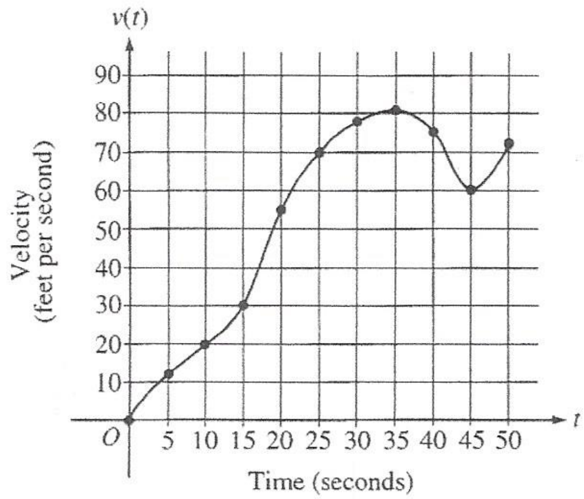
3. Consider the function $f(x) = 2^x + x$.

   a) Estimate $f'(1)$ using the definition of derivative.

   $$f'(1) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0}\frac{[2^{1+h}+1+h]-[2^1+1]}{h}$$
   $$= 2.386$$

   b) Find the exact value of $f'(1)$ using derivative rules.

   $$f'(x) = 2^x \ln 2 + 1$$
   $$f'(1) = 2\ln 2 + 1$$



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

4. The graph of the velocity, $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

   a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

   $(0, 35)$ because v is increasing
   $(45, 50)$

   b) Find the average acceleration of the car over the interval $0 \le t \le 50$. Indicate units of measure.

   $$\frac{v(50)-v(0)}{50-0} = \frac{72-0}{50-0} = 1.44 \text{ ft/sec}^2$$

   c) Find one approximation for the acceleration of the car at $t = 40$. Show the computations you used to arrive at your answer and indicate units of measure.

   $$\frac{v(45)-v(35)}{45-35} = \frac{60-81}{10} = -2.1 \text{ ft/sec}^2$$

Name ____________________

Period ____________________

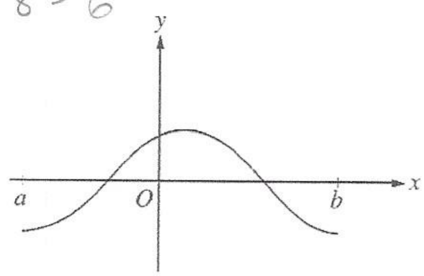
Calculus AB -- Chapter 3A Sample Test (no calculators)

Show all work for free-response questions.

1. The tangent line to $y = f(x)$ at $(8,10)$ passes through the point $(6,-30)$. Find $f'(8)$.

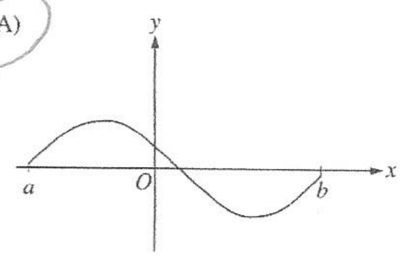
   (A) 40 (B) 30 (C) 20 (D) 45 (E) -20

   $f'(8) = \frac{10-(-30)}{8-6}$

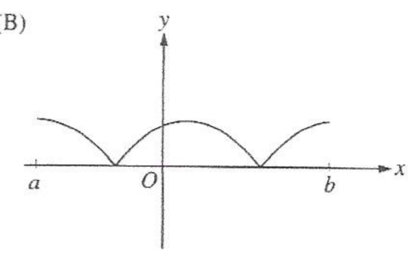


2. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

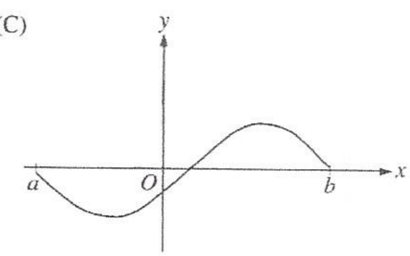
(A)



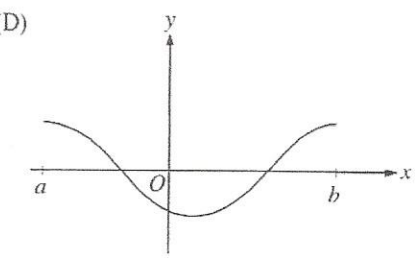
(B)



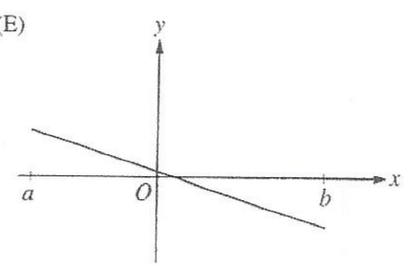
(C)



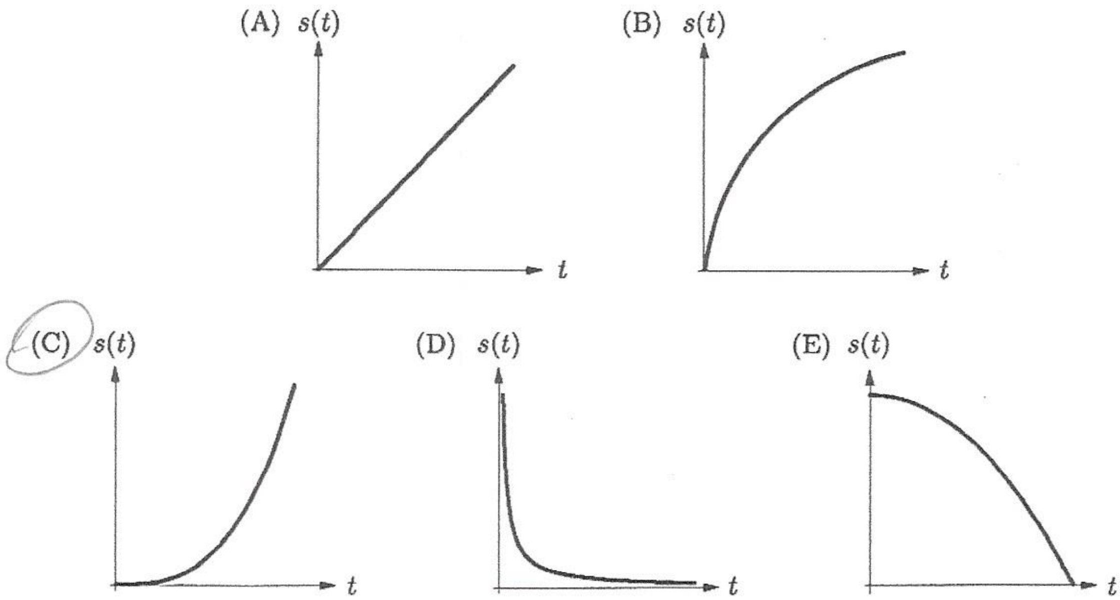
(D)



(E)



3. Which graph best represents the position of a particle, $s(t)$, as a function of time, if the particle's velocity and acceleration are both positive?



$$f(x) = \begin{cases} x+2 & \text{if } x \le 3 \\ 4x-7 & \text{if } x > 3 \end{cases}$$

$f'(x) = \begin{cases} 1 & x<3 \\ 4 & x>3 \end{cases}$

4. Let f be the function given above. Which of the following statements is true about f?

I. $\lim_{x\to 3} f(x)$ exists.

II. f is continuous at $x = 3$.

III. f is differentiable at $x = 3$.

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} 4x-7 = 5$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} x+2 = 5$

(A) None (B) I only (C) II only

(D) I and II only (E) I, II, and III

5. Let g be a twice-differentiable function with $g'(x) > 0$ and $g''(x) > 0$ for all real numbers x, such that $g(4) = 12$ and $g(5) = 18$. Of the following, which is a possible value for $g(6)$?

(A) 15 (B) 18 (C) 21 (D) 24 (E) 27

6. The height above the ground of a passenger on a Ferris wheel t minutes after the ride begins is modeled by the differentiable function H, where $H(t)$ is measured in meters. Which of the following is an interpretation of the statement $H'(7.5) = 15.708$?

(A) The Ferris wheel is turning at a rate of 15.708 meters per minute when the passenger is 7.5 meters above the ground.

(B) The Ferris wheel is turning at a rate of 15.708 meters per minute 7.5 minutes after the ride begins.

(C) The passenger's height above the ground is increasing by 15.708 meters per minute when the passenger is 7.5 meters above the ground.

(D) The passenger's height above the ground is increasing by 15.708 meters per minute 7.5 minutes after the ride begins.

(E) The passenger is 15.708 meters above the ground 7.5 minutes after the ride begins.

7. Find the equation of the tangent line to $y = \frac{1}{x^2}$ $= x^{-2}$ at the point where $x = -1$.

$y' = -2x^{-3}$

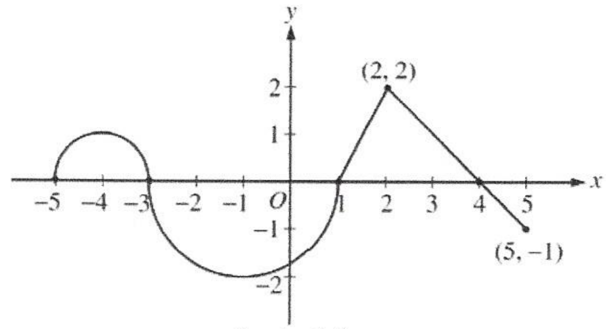
$y(-1) = \frac{1}{(-1)^2} = 1$

$y'(-1) = \frac{-2}{(-1)^3} = 2 \leftarrow$ slope

$y - 1 = 2(x+1)$

8. $\frac{d}{dx}\left(10\sqrt{x} + 6e^x\right) =$ $\frac{d}{dx}\left(10x^{1/2} + 6e^x\right) = 5x^{-1/2} + 6e^x$

$= \frac{5}{\sqrt{x}} + 6e^x$



Graph of f'

9. The graph of f', the derivative of the function f, is shown in the figure above. The graph of f' consists of two line segments and two semicircles.

   a) Find all intervals on which the graph of f is increasing. Give a reason for your answer, using the graph of f' to justify your answer.

   $(-5, -3)$ and $(1, 4)$
   f' is positive

   b) Find all intervals on which the graph of f is concave down. Give a reason for your answer, using the graph of f' to justify your answer.

   $(-4, -1)$ and $(2, 5)$
   f' is decreasing