Calculators Allowed

1. $fg'' + 2f'g' + f''g$
2. a. $t = 1, 6$; v changes signs   b. left; $v(5) < 0$

   c. 18   d. dec.; $v(5)$ and $a(5)$ diff. signs

No Calculators

1. C	2. A	3. B	4. A	5. D	6. D
7. B	8. E	9. A	10. B	11. C	12. D

13. a. $y = \frac{\pi}{6} + \frac{1}{\sqrt{3}}(x-1)$ b. $\frac{\pi}{6} + \frac{.2}{\sqrt{3}}$

14. a. 4   b. dec.; $v(0)$ and $a(0)$ diff. signs

15. a. $\pi - e$ b. -4   c. 15

16. a. $\frac{-4x}{3y+3}$ b. $\frac{-12y-12-\frac{48x^2}{3y+3}}{(3y+3)^2}$

Name ___________________

Period ___________________

Calculus AB -- Chapter 3B Sample Test (Calculators Allowed)

Show all work for free-response questions.

1. If f and g are twice differentiable functions and if $h(x) = f(x)g(x)$, find $h''(x)$.

$h'(x) = f(x)g'(x) + g(x)f'(x)$

$h''(x) = f(x) \cdot g''(x) + g'(x)f'(x) + g(x)f''(x) + f'(x)g'(x)$

$= f(x)g''(x) + 2f'(x)g'(x) + f''(x)g(x)$

2. A particle moves along the x-axis such that its position, for $t \geq 0$, is given by $x(t) = 2t^3 - 21t^2 + \overset{36}{\cancel{13}}t$.

a. Find all times when the particle is at rest.

$v(t) = 6t^2 - 42t + 36$

$= 6(t^2 - 7t + 6)$

$= 6(t-1)(t-6) = 0$

$t = 1,\ t = 6$

+ − + v

1 6

v(t) changes signs at both points

b. Is the particle moving left or right at $t = 5$? Justify your answer.

$v(5) = 6(5-1)(5-6) = -24$

moving left because $v(5) < 0$

c. Find $a(5)$. $a(t) = 12t - 42$

$a(5) = 12(5) - 42 = 18$

d. Is the speed of the particle increasing or decreasing at $t = 5$? Justify your answer.

decreasing, v(5) & a(5) diff. signs

Name ____________________

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Calculus AB -- Chapter 3B Sample Test (No Calculators)

Show all work for free-response questions.

1. If $f(x)=\sqrt{4\sin x+2}$, then $f'(0)=$

(A) -2 (B) 0 (C) $\sqrt{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) 1

$f'(x)=\frac{1}{2}(4\sin x+2)^{-1/2}\cdot 4\cos x$

$f'(0)=\frac{1}{2}(2)^{-1/2}\cdot 4=\frac{2}{\sqrt{2}}=\sqrt{2}$

2. If $x^2+xy=10$, then when $x=2$, $\frac{dy}{dx}=$

$2^2+2y=10$
$2y=6$
$y=3$

(A) $-\frac{7}{2}$ (B) -2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

$2x+xy'+y=0$
$2(2)+2y'+3=0$
$2y'=-7$
$y'=-\frac{7}{2}$

3. Let f be the function defined by $f(x)=x^3+x$. If $g(x)=f^{-1}(x)$ and $g(2)=1$, what is the value of $g'(2)$?

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$

(D) 4 (E) 13

g
$x=y^3+y \rightarrow 1=3y^2y'+y'$
$(2,1)$ $\quad 1=3(1)^2y'+y'$
$1=4y' \rightarrow y'=\frac{1}{4}$

4. Let f be the function given by $f(x)=2xe^x$. The graph of f is concave down when

(A) $x<-2$ (B) $x>-2$ (C) $x<-1$

(D) $x>-1$ (E) $x<0$

$-\quad +\quad f''$
-2

$f'(x)=2xe^x+2e^x=2e^x(x+1)$

$f''(x)=2e^x\cdot 1+(x+1)2e^x=2e^x(x+2)=0$

$x=-2$

5. $\frac{d}{dx}\left(e^{3\ln x}\right) = \frac{d}{dx} e^{\ln(x^3)} = \frac{d}{dx} x^3 = 3x^2$

(A) $e^{3\ln x}$ (B) $\frac{e^{3\ln x}}{x}$ (C) x^3 (D) $3x^2$ (E) 3

6. A particle moves along the y-axis such that its position is given by $y(t) = (x^2 - 3)e^{-x}$. What are all values of t for which the particle is moving upward?

(A) There are no values (B) $t < -1$ and $t > 3$ (C) $-3 < t < 1$

(D) $-1 < t < 3$ (E) All values of t

$v(t) = (t^2-3)e^{-t}(-1) + e^{-t}(2t)$
$= e^{-t}(-t^2+3+2t) = -e^{-t}(t^2-2t-3)$
$= -e^{-t}(t-3)(t+1) = 0 \rightarrow t = -1, 3$

$-$ $+$ $-$ ✓
-1 3

7. What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point $(3,2)$?

(A) 0 (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

$6yy' - 4x = -2xy' - 2y$
$6(2)y' - 4(3) = -2(3)y' - 2(2) \rightarrow 18y' = 8$
$12y' - 12 = -6y' - 4$ $y' = \frac{4}{9}$

8. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only (B) $t = 3$ only (C) $t = \frac{7}{2}$ only

(D) $t = 3$ and $t = \frac{7}{2}$ (E) $t = 3$ and $t = 4$

$v(t) = 6t^2 - 42t + 72$
$= 6(t^2 - 7t + 12)$ $t = 3, 4$
$= 6(t-3)(t-4) = 0$

9. Find the equation of the line tangent to the graph of $y = \frac{e^{-7x}}{x^7+1}$ at the point where $x = 0$.

(A) $y = -7x + 1$ (B) $y = 7x$

(C) $y = 7x + 1$ (D) $y = -7x$

$y = \frac{e^0}{1} = 1$

$y' = \frac{(x^7+1)(e^{-7x})(-7) - e^{-7x}(7x^6)}{(x^7+1)^2}$

$y'(0) = \frac{1 \cdot e^0(-7) - e^0 \cdot 0}{1^2} = -7$

10. Find the derivative of the function $f(x) = \frac{1+\cos 3x}{1-\cos 3x}$.

(A) $f'(x) = \frac{6\sin 3x}{(1-\cos 3x)^2}$ (B) $f'(x) = \frac{-6\sin 3x}{(1-\cos 3x)^2}$

(C) $f'(x) = \frac{-2\sin 3x}{(1-\cos 3x)^2}$ (D) $f'(x) = \frac{2\sin 3x}{(1-\cos 3x)^2}$

$$f'(x) = \frac{(1-\cos 3x)(-3\sin 3x) - (1+\cos 3x)(3\sin 3x)}{(1-\cos 3x)^2}$$
$$= \frac{-3\sin 3x + 3\sin 3x\cos 3x - 3\sin 3x - 3\sin 3x\cos 3x}{(1-\cos 3x)^2}$$

x	2	3	4
$f(x)$	1	2	6
$f'(x)$	4	5	3

f: (2,1) (3,2) (4,6)

g: (1,2) (2,3) (6,4)

11. The table above gives values of the differentiable function f and its derivative at selected values of x. If g is the inverse function of f, which of the following is the equation of the line tangent to the graph of g at the point where $x = 2$?

(A) $y = -\frac{1}{5}(x-2)+3$ (B) $y = -\frac{1}{4}(x-2)+1$

(C) $y = \frac{1}{5}(x-2)+3$ (D) $y = 4(x-2)+1$

$f:\ y = f(x)$

$g:\ x = f(y)$

$1 = f'(y)\,y'$

at $x=2 \to$ $y=3$ $\quad 1 = f'(3)\,y'$

$y' = \frac{1}{f'(3)}$

12. For any real number x, $\lim_{h\to 0} \frac{\sin(2(x+h)) - \sin(2x)}{h} = \frac{d}{dx}(\sin 2x)$

(A) 0 (B) 1 (C) $\cos(2x)$ (D) $2\cos(2x)$

13. Consider the function $f(x) = \sin^{-1}\left(\frac{x}{2}\right)$. $\quad f'(x) = \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\cdot\frac{1}{2}$

a. Find the equation of the tangent line at $x = 1$.

$y = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ $\quad f'(1) = \frac{1}{\sqrt{1-\frac{1}{4}}}\cdot\frac{1}{2} = \frac{1}{\sqrt{\frac{3}{4}}}\cdot\frac{1}{2} = \frac{1}{\sqrt{3}}$

$y = f(1) + f'(1)(x-1) \rightarrow y = \frac{\pi}{6} + \frac{1}{\sqrt{3}}(x-1)$

b. Use your answer from Part a to approximate the value of $f(1.2)$.

$f(1.2) = \frac{\pi}{6} + \frac{1}{\sqrt{3}}(1.2-1) = \frac{\pi}{6} + \frac{.2}{\sqrt{3}}$

14. A particle moves along the x-axis in such a way that its position at time t is given by $x(t)=\dfrac{1-t}{1+t}$.

a. What is the acceleration of the particle at time $t=0$?

$v(t)=\dfrac{(1+t)(-1)-(1-t)\cdot 1}{(1+t)^2}=\dfrac{-1-t-1+t}{(1+t)^2}=-2(1+t)^{-2}$

$a(t)=4(1+t)^{-3} \rightarrow a(0)=4$

b. Is the speed of the particle increasing or decreasing at time $t=0$? Justify your answer.

$v(0)=-2$ dec, $v(0)$ and $a(0)$ diff. signs

15. Let $f(x)$ and $g(x)$ be functions with values given in the table. Use the information to answer the questions that follow.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	1	-1	2	5
1	-1	2	4	0
2	7	3	11	0.5

a. If $H(x)=e^{f(x)}+\pi x$, find $H'(0)$.

$H'(x)=e^{f(x)}f'(x)+\pi$

$H'(0)=e^{f(0)}f'(0)+\pi=e^{1}(-1)+\pi=\pi-e$

b. If $J(x)=[f(x)]^2$, find $J'(1)$.

$J'(x)=2f(x)f'(x)$

$J'(1)=2f(1)f'(1)=2(-1)(2)=-4$

c. If $K(x)=f(g(x))$, find $K'(0)$.

$K'(x)=f'(g(x))g'(x)$

$K'(0)=f'(g(0))g'(0)=f'(2)g'(0)=(3)(5)=15$

16. Consider the curve defined by $4x^2 + 3y^2 + 6y = 9$.

a. Find $\frac{dy}{dx}$ in terms of x and y.

$$8x + 6yy' + 6y' = 0$$
$$y'(6y+6) = -8x$$
$$y' = \frac{-8x}{6y+6} = \frac{-4x}{3y+3}$$

b. Find $\frac{d^2y}{dx^2}$ in terms of x and y.

$$y'' = \frac{(3y+3)(-4) - (-4x)(3y')}{(3y+3)^2}$$
$$= \frac{-12y - 12 + 12x\left(\frac{-4x}{3y+3}\right)}{(3y+3)^2}$$

c. Find all values of x at which the curve has a vertical tangent line.

y' undef. $\Rightarrow$ $3y + 3 = 0$

$y = -1$

$$4x^2 + 3y^2 + 6y = 9$$
$$4x^2 + 3(-1)^2 + 6(-1) = 9$$
$$4x^2 - 3 = 9$$
$$4x^2 = 12$$
$$x^2 = 3$$
$$x = \pm\sqrt{3}$$