Name _____

Period _____

Calculus AB -- Chapter 3B Sample Test (Calculators Allowed)

Show all work for free-response questions.

1. If f and g are twice differentiable functions and if $h(x) = f(x)g(x)$, find $h''(x)$.

2. A particle moves along the x-axis such that its position, for $t \geq 0$, is given by $x(t) = 2t^3 - 21t^2 + 36t$.

   a. Find all times when the particle changes directions. Justify your answer.

   b. Is the particle moving left or right at $t = 5$? Justify your answer.

   c. Find $a(5)$.

   d. Is the speed of the particle increasing or decreasing at $t = 5$? Justify your answer.

Name _____

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Calculus AB -- Chapter 3B Sample Test (No Calculators)

Show all work for free-response questions.

1. If $f(x)=\sqrt{4\sin x+2}$, then $f'(0)=$

(A) -2 (B) 0 (C) $\sqrt{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) 1

2. If $x^2+xy=10$, then when $x=2$, $\frac{dy}{dx}=$

(A) $-\frac{7}{2}$ (B) -2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

3. Let f be the function defined by $f(x)=x^3+x$. If $g(x)=f^{-1}(x)$ and $g(2)=1$, what is the value of $g'(2)$?

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$

(D) 4 (E) 13

4. Let f be the function given by $f(x)=2xe^x$. The graph of f is concave down when

(A) $x<-2$ (B) $x>-2$ (C) $x<-1$

(D) $x>-1$ (E) $x<0$

5. $\frac{d}{dx}\left(e^{3\ln x}\right)=$

(A) $e^{3\ln x}$ (B) $\dfrac{e^{3\ln x}}{x}$ (C) x^3 (D) $3x^2$ (E) 3

6. A particle moves along the y-axis such that its position is given by $y(t) = (t^2 - 3)e^{-t}$. What are all values of t for which the particle is moving upward?

(A) There are no values (B) $t < -1$ and $t > 3$ (C) $-3 < t < 1$

(D) $-1 < t < 3$ (E) All values of t

7. What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point $(3,2)$?

(A) 0 (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

8. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only (B) $t = 3$ only (C) $t = \dfrac{7}{2}$ only

(D) $t = 3$ and $t = \dfrac{7}{2}$ (E) $t = 3$ and $t = 4$

9. Find the equation of the line tangent to the graph of $y = \frac{e^{-7x}}{x^7+1}$ at the point where $x = 0$.

(A) $y = -7x + 1$ (B) $y = 7x + 1$

(C) $y = x + 1$ (D) $y = -7x$

10. Find the derivative of the function $f(x) = \frac{1+\cos 3x}{1-\cos 3x}$.

(A) $f'(x) = \frac{6\sin 3x}{(1-\cos 3x)^2}$ (B) $f'(x) = \frac{-6\sin}{(1-\cos\ \ )^2}$

(C) $f'(x) = \frac{-2\sin}{(1-\cos 3x)^2}$ (D) $f'(x) = \frac{2\sin 3x}{(1-\cos 3x)^2}$

x	2	3	4
$f(x)$	1	2	6
$f'(x)$	4	5	3

11. The table above gives values of the differentiable function f and its derivative at selected values of x. If g is the inverse function of f, which of the following is the equation of the line tangent to the graph of g at the point where $x = 2$?

(A) $y = -\frac{1}{5}(x-2)+3$ (B) $y = -\frac{1}{4}(x-2)+1$

(C) $y = \frac{1}{5}(x-2)+3$ (D) $y = 4(x-2)+1$

12. For any real number x, $\lim_{h\to 0} \frac{\sin(2(x+h))-\sin\ (2x)}{h} =$

(A) 0 (B) 1 (C) $\cos(2x)$ (D) $2\cos(2x)$

13. Consider the function $f(x) = \sin^{-1}\left(\frac{x}{2}\right)$.

a. Find the equation of the tangent line at $x = 1$.

b. Use your answer from Part a to approximate the value of $f(1.2)$.

14. A particle moves along the x-axis in such a way that its position at time t is given by $x(t)=\dfrac{1-t}{1+t}$.

a. What is the acceleration of the particle at time $t=0$?

b. Is the speed of the particle increasing or decreasing at time $t=0$? Justify your answer.

15. Let $f(x)$ and $g(x)$ be functions with values given in the table. Use the information to answer the questions that follow.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	1	-1	2	5
1	-1	2	4	0
2	7	3	11	0.5

a. If $H(x)=e^{f(x)}+\pi x$, find $H'(0)$.

b. If $J(x)=[f(x)]^2$, find $J'(1)$.

c. If $K(x)=f(g(x))$, find $K'(0)$.

16. Consider the curve defined by $4x^2 + 3y^2 + 6y = 9$.

a. Find $\frac{dy}{dx}$ in terms of x and y.

b. Find $\frac{d^2y}{dx^2}$ in terms of x and y.

c. Find all values of x at which the curve has a vertical tangent line.