Calculators Allowed

1. C
2. B
3. E
4. $f(x) = 4x^3 - 9x^2 + 4$

No Calculators

1. D 2. C 3. D 4. E 5. A

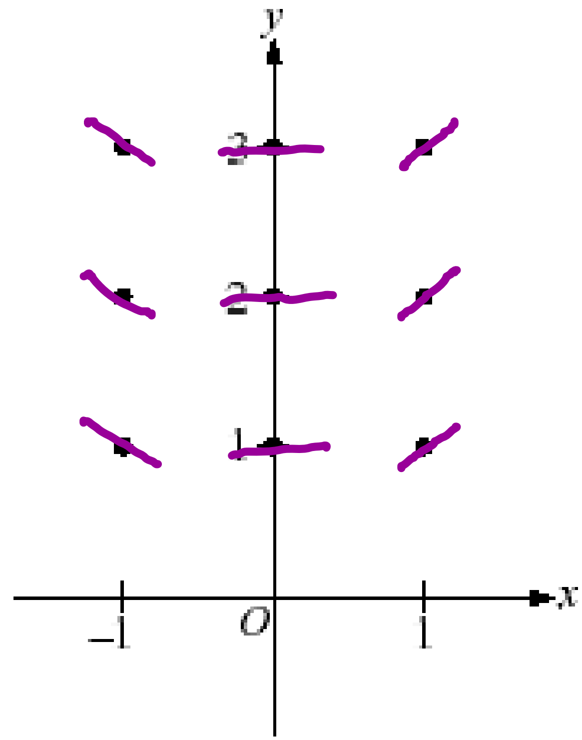
6. a. 1 b. $\frac{\pi}{6}$

7. a. $v(t) = 3t^2 - 18t + 24$

b. $x(t) = t^3 - 9t^2 + 24t + 4$

c. $t = 2$, $t = 4$

8. a. →



b. $y = 3e^{x^2/4}$

9. a. $g(-6) = -10$, $g(3) = 2\pi - 1$ b. 2

c. -2 is neither (no change in sign of f)

2 is local max (f goes from pos. to neg.)

d. -4, -2, 0 (slope of f changes signs)

Name ___________________

Period ___________________

Calculus BC – Chapter 5B Sample Test (calculators allowed)

Show all work for free-response questions.

1. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$ $F(1) + \int_1^9 \frac{(\ln x)^3}{x}\,dx$

(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1640.250

2. Find the derivative of the function $\frac{d}{dx}\int_x^{x^9} \ln t\,dt$. $= \ln(x^9)\cdot 9x^8 - \ln x$
$= 9\ln(x)\cdot 9x^8 - \ln x$
$= 81x^8 \ln x - \ln x$

(A) $x(x^8 - 1)\ln x$

(B) $(81x^8 - 1)\ln x$

(C) $8\ln x$

(D) $\frac{9}{x}$

3. $\int_0^x \sin t\,dt = -\cos t\Big|_0^x = -\cos x - (-\cos 0)$

(A) $\sin x$ (B) $-\cos x$ (C) $\cos x$ (D) $\cos x - 1$ (E) $1 - \cos x$

4. Let $f(x)$ be the function that is defined for all real numbers x and that has the following properties:

(i) $f''(x) = 24x - 18$ (ii) $f'(1) = -6$ (iii) $f(2) = 0$

Find an expression for $f(x)$.

$f'(x) = 12x^2 - 18x + C$
$f'(1) = 12 - 18 + C = -6$
$C = 0$
$f'(x) = 12x^2 - 18x$
$f(x) = 4x^3 - 9x^2 + D$
$f(2) = 4(2)^3 - 9(2)^2 + D = 0$
$D = 4$

$f(x) = 4x^3 - 9x^2 + 4$

Name ____________________

Period ____________________

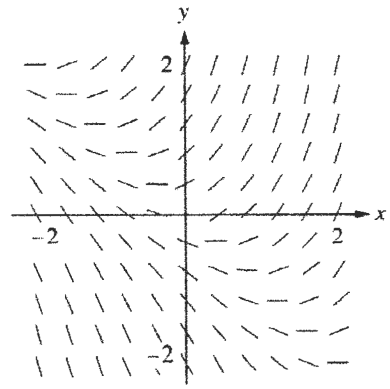
Calculus BC – Chapter 5B Sample Test (no calculators)

Show all work for free-response questions.

$f(x) = x^3 + C$
$f(-1) = -1 + C = 2 \rightarrow C = 3$

1. If $f'(x) = 3x^2$ and $f(-1) = 2$, then $\int_0^2 f(x)dx =$ $\int_0^2 (x^3+3)dx = \frac{1}{4}x^4 + 3x \Big|_0^2$
$= (\frac{1}{4}2^4 + 3\cdot 2) - 0 = 4 + 6 = 10$

(A) $\frac{8}{3}$ (B) 4 (C) 7 (D) 10 (E) 28



2. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$

(D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

3. The temperature of a solid at time $t \geq 0$ is modeled by the nonconstant function H and increases according to the differential equation $\frac{dH}{dt} = 2H + 1$, where $H(t)$ is measured in degrees Fahrenheit and t is measured in hours. Which of the following much be true?

$\frac{1}{2H+1} dH = dt$

$\frac{1}{2} \ln|2H+1| = t + C$

$\ln|2H+1| = 2t + C$

(A) $H = H^2 + t + C$

(B) $\ln|2H + 1| = \frac{t}{2} + C$

(C) $\ln|2H + 1| = t + C$

(D) $\ln|2H + 1| = 2t + C$

4. If $f(x)$ is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then

$$\int_1^3 f(2x)dx = \frac{1}{2}F(2x)\Big|_1^3 = \frac{1}{2}F(6) - \frac{1}{2}F(2)$$

(A) $2F(3) - 2F(1)$ (B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$ (C) $2F(6) - 2F(2)$

(D) $F(6) - F(2)$ (E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

5. The equation $y = 2e^{6x} - 5$ is a particular solution to which of the following differential equations? → $y' = 12e^{6x}$ → $y'' = 72e^{6x}$

(A) $y' - 6y - 30 = 0$ → $12e^{6x} - 6(2e^{6x} - 5) - 30 = 0$

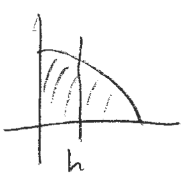
(B) $2y' - 12y + 5 = 0$ $12e^{6x} - 12e^{6x} + 30 - 30 = 0$ ✓

(C) $y'' - 5y' - 6y = 0$

(D) $y'' - 2y' + y + 5 = 0$

6. Let R be the region bounded by the graph $y = \cos x$, the x-axis, and the y-axis ~~line $x = \frac{\pi}{2}$~~.

a) Find the area of the region R.



$$\int_0^{\pi/2} \cos x\,dx = \sin x\Big|_0^{\pi/2} = \sin\frac{\pi}{2} - \sin 0 = 1$$

b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.

$$\int_0^h \cos x\,dx = \sin x\Big|_0^h = \sin h - \sin 0 = \sin h = \frac{1}{2}$$

$$h = \frac{\pi}{6}$$

7. A particle moves along the x-axis so that its acceleration at any time x is given by $a(t) = 6t - 18$. At time $t = 0$, the velocity of the particle is $v(0) = 24$, and at time $t = 1$, its position is $x(1) = 20$.

a) Write an expression for the velocity $v(t)$ of the particle at any time t.

$v(t) = 3t^2 - 18t + C$

$v(0) = 0 + 0 + C = 24$

$v(t) = 3t^2 - 18t + 24$

b) Write an expression for the position $x(t)$ of the particle at any time t.

$x(t) = t^3 - 9t^2 + 24t + D$

$x(1) = 1 - 9 + 24 + D = 20$

$D = 4$

$x(t) = t^3 - 9t^2 + 24t + 4$

c) For what values of t is the particle at rest?

$v(t) = 3t^2 - 18t + 24 = 0$

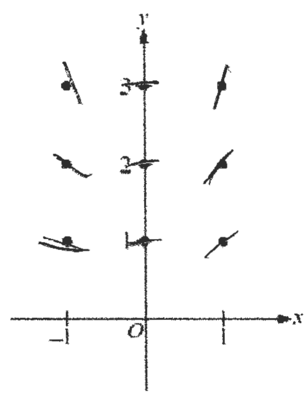
$3(t^2 - 6t + 8) = 0$

$3(t-2)(t-4) = 0$

$t = 2, t = 4$

8. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.



b) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $y(0) = 3$.

$\frac{dy}{dx} = \frac{xy}{2}$

$\frac{1}{y} dy = \frac{1}{2} x\, dx$

$\ln|y| = \frac{1}{4}x^2 + C$

$|y| = e^{\frac{1}{4}x^2 + C}$

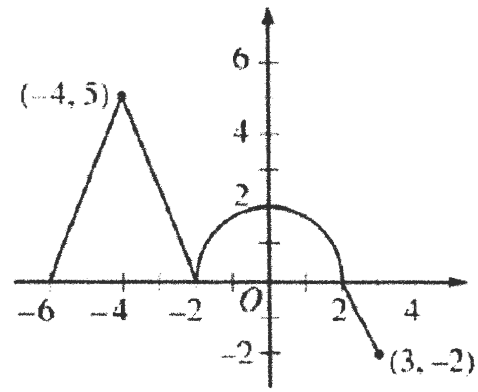
$y = De^{\frac{1}{4}x^2}$

$3 = De^0$

$D = 3$

$\boxed{y = 3e^{\frac{1}{4}x^2}}$

Calculus BC -- Chapter 5B Sample Test (no calculators)



Graph of f

9. The graph of the continuous function f, consisting of three line segments and a semicircle, is shown above. Let g be the function given by $g(x) = \int_{-2}^{x} f(t)dt$.

   a) Find $g(-6)$ and $g(3)$.

   $g(-6) = \int_{-2}^{-6} f(t)dt = -\int_{-6}^{-2} f(t)dt = -\left(\frac{1}{2}\cdot 4\cdot 5\right) = -10$

   $g(3) = \int_{-2}^{3} f(t)dt = \frac{1}{2}\pi(2)^2 - \frac{1}{2}\cdot 1\cdot 2 = 2\pi - 1$

   b) Find $g'(0)$.

   $g'(x) = f(x)$

   $g'(0) = f(0) = 2$

   c) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a horizontal tangent line. Determine whether g has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

   $g' = 0 \rightarrow x = -2$ neither, f doesn't change signs

   $x = 2$ local max., f goes pos. to neg.

   d) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a point of inflection. Explain your reasoning.

   $g'' = f'$

   $f' = 0$ or undef. at $\underbrace{x = -4, -2, 0}$, $\cancel{2}$

   inf. pt. because slope of f changes signs