Name _____

Period _____

Calculus AB – Chapter 5B Sample Test (calculators allowed)

Show all work for free-response questions.

1. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

   (A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1640.250

2. Find the derivative of the function $\int_x^{x^9} \ln t \, dt$.

   (A) $x(x^8 - 1)\ln x$

   (B) $(81x^8 - 1)\ln x$

   (C) $8\ln x$

   (D) $\frac{9}{x}$

3. $\int_0^x \sin t \, dt$

   (A) $\sin x$ (B) $-\cos x$ (C) $\cos x$ (D) $\cos x - 1$ (E) $1 - \cos x$

4. Let $f(x)$ be the function that is defined for all real numbers x and that has the following properties:

   (i) $f''(x) = 24x - 18$ (ii) $f'(1) = -6$ (iii) $f(2) = 0$

   Find an expression for $f(x)$.

Name _____

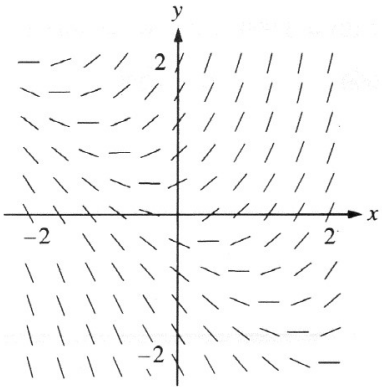
Period _____

Calculus AB – Chapter 5B Sample Test (no calculators)

Show all work for free-response questions.

1. If $f'(x) = 3x^2$ and $f(-1) = 2$, then $\int_0^2 f(x) =$

(A) $\frac{8}{3}$ (B) 4 (C) 7 (D) 10 (E) 28



2. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$
(D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

3. The temperature of a solid at time $t \geq 0$ is modeled by the nonconstant function H and increases according to the differential equation $\frac{dH}{dt} = 2H + 1$, where $H(t)$ is measured in degrees Fahrenheit and t is measured in hours. Which of the following much be true?

(A) $H = H^2 + t + C$

(B) $\ln|2H + 1| = \frac{t}{2} + C$

(C) $\ln|2H + 1| = t + C$

(D) $\ln|2H + 1| = 2t + C$

4. If $f(x)$ is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)dx =$

(A) $2F(3) - 2F(1)$ (B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$ (C) $2F(6) - 2F(2)$

(D) $F(6) - F(2)$ (E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

5. The equation $y = 2e^{6x} - 5$ is a particular solution to which of the following differential equations?

(A) $y' - 6y - 30 = 0$

(B) $2y' - 12y + 5 = 0$

(C) $y'' - 5y' - 6y = 0$

(D) $y'' - 2y' + y + 5 = 0$

6. Let R be the region bounded by the graph $y = \cos x$, the x-axis, and the y-axis.
a) Find the area of the region R.

b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.

7. A particle moves along the x-axis so that its acceleration at any time x is given by $a(t) = 6t - 18$. At time $t = 0$, the velocity of the particle is $v(0) = 24$, and at time $t = 1$, its position is $x(1) = 20$.

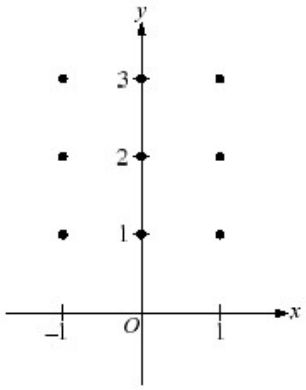
   a) Write an expression for the velocity $v(t)$ of the particle at any time t.

   b) Write an expression for the position $x(t)$ of the particle at any time t.

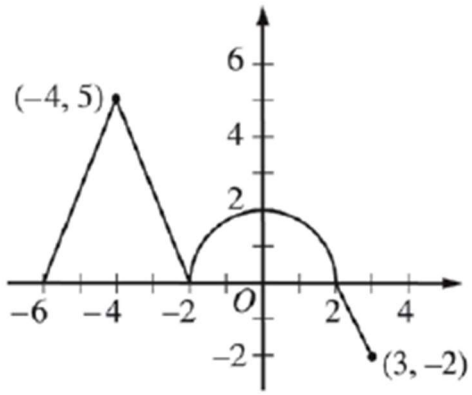
   c) For what values of t is the particle at rest?

8. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

   a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.



   b) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $y(0) = 3$.



Graph of f

5. The graph of the continuous function f, consisting of three line segments and a semicircle, is shown above. Let g be the function given by $g(x) = \int_{-2}^{x} f(t)dt$.

   a) Find $g(-6)$ and $g(3)$.

   b) Find $g'(0)$.

   c) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a horizontal tangent line. Determine whether g has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

   d) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a point of inflection. Explain your reasoning.