Calculators Allowed

1. B 2. C

3. a. 9.408

b. $\dfrac{\pi}{2}\displaystyle\int_0^1 \left(\frac{e^x-(x-1)^2}{2}\right)^2 dx$

4. a. 0.082

b. $\displaystyle\int_{-.715}^{0} \sqrt{1+(e^x)^2}+\sqrt{1+(-2x)^2}\,dx$

5. a. 8.997

b. $\pi\displaystyle\int_0^{1.488}\left[(1+(4-2x))^2-\left(1+\frac{x^3}{1+x^2}\right)^2\right]dx$

No Calculators

1. D 2. A 3. 1 4. $50-\frac{16\sqrt{2}}{\pi}$

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Calculus BC – Chapter 6 Sample Test (calculators allowed)

Show all work for free-response questions.

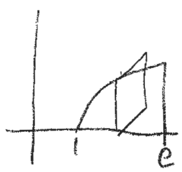
1\. Let R be the region enclosed by the graph of $y = 1 + \ln(\cos^4 x)$, the x-axis, and the vertical lines $x = -\frac{2}{3}$ and $x = \frac{2}{3}$. The closest integer approximation of the area of R is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

$$\int_{-2/3}^{2/3} 1 + \ln(\cos^4 x)\, dx$$

2\. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the vertical line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

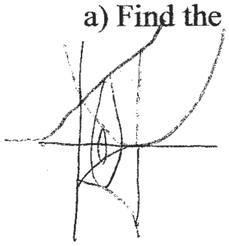
(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) $\frac{1}{3}(e^3 - 1)$



$$\int_1^e (\sqrt{\ln x})^2\, dx$$

3\. Let R be the region enclosed by the graphs of $y = e^x$, $y = (x-1)^2$, and the vertical line $x = 1$.

a) Find the volume of the solid generated when R is revolved about the x-axis.

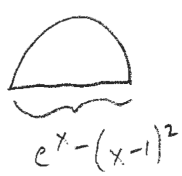


$R = e^x$

$r = (x-1)^2$

$$\pi \int_0^1 (e^x)^2 - [(x-1)^2]^2\, dx = 2.995\pi$$
$$= 9.408$$

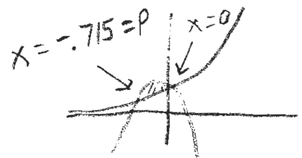
b) The base of a solid is the region R. Each cross section of the solid perpendicular to the x-axis is a semicircle. Write an expression involving one or more integrals that gives the volume of the solid. Do not evaluate.



$$\int_0^1 \frac{\pi}{2}\left(\frac{e^x - (x-1)^2}{2}\right)^2 dx$$

4. Let R be the region bounded by the graphs of $y = e^x$ and $y = -x^2 + 1$.

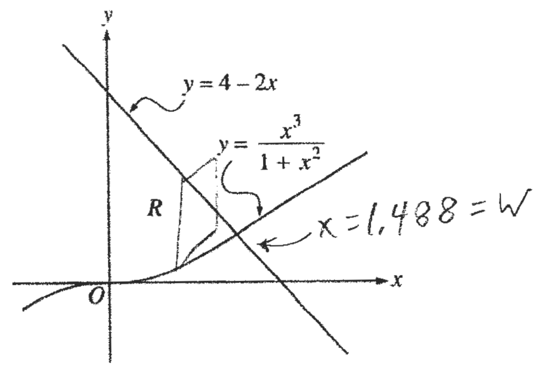
a) Find the area of R.



$$\int_P^0 (-x^2+1) - e^x \, dx = .082$$

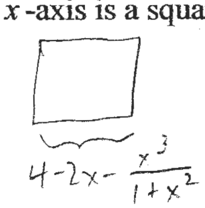
b) Write an expression involving one or more integrals that gives the length of the boundary of the region R. Do not evaluate.

$$\int_P^0 \sqrt{1 + (e^x)^2}\, dx + \int_P^0 \sqrt{1 + (-2x)^2}\, dx$$



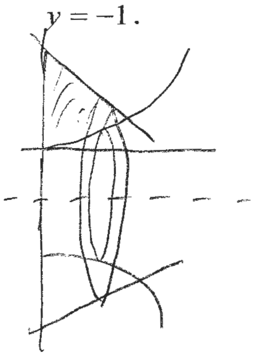
5. Let R be the region bounded by the y-axis and the graphs of $y = \dfrac{x^3}{1+x^2}$ and $y = 4 - 2x$, as shown in the figure above.

a) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.



$$V = \int_0^W \left((4-2x) - \frac{x^3}{1+x^2}\right)^2 dx = 8.997$$

b) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when region R is revolved about the horizontal line $y = -1$.



$R = 1 + 4 - 2x$

$r = 1 + \dfrac{x^3}{1+x^2}$

$$V = \pi \int_0^W \left(1+4-2x\right)^2 - \left(1 + \frac{x^3}{1+x^2}\right)^2 dx$$

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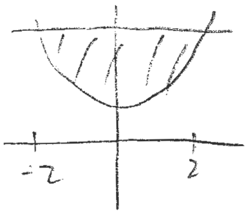
Calculus BC – Chapter 6 Sample Test (no calculators)

Show all work for free-response questions.

1. The area of the region enclosed by the graph of $y = x^2 + 1$ and the horizontal line $y = 5$ is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$ (D) $\frac{32}{3}$ (E) 8π



$x^2+1=5$

$x^2=4$

$x=\pm 2$

$A=\int_{-2}^{2} 5-(x^2+1)\,dx = \int_{-2}^{2}(4-x^2)\,dx = 4x-\frac{1}{3}x^3\Big|_{-2}^{2}$

$=(8-\frac{8}{3})-(-8+\frac{8}{3}) = 16-\frac{16}{3} = \frac{32}{3}$

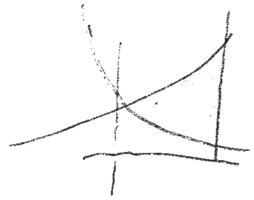
2. Find the area of the region bounded by $y = e^x$, $y = e^{-x}$, and the vertical line $x = 1$.

(A) $e+\frac{1}{e}-2$

(B) $e-\frac{1}{e}$

(C) $e+\frac{1}{e}$

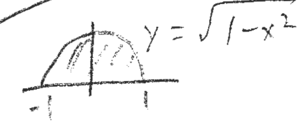
(D) $2e-2$



$\int_0^1 (e^x-e^{-x})\,dx = e^x+e^{-x}\Big|_0^1$

$=(e^1+e^{-1})-(1+1)$

$= e+\frac{1}{e}-2$



3. Find the average value of $f(x) = 1+\sqrt{1-x^2}-\frac{1}{1+x^2}$ from $x = -1$ to $x = 1$.

$\frac{1}{1-(-1)}\int_{-1}^{1}\left(1+\sqrt{1-x^2}-\frac{1}{1+x^2}\right)dx = \frac{1}{2}\left(\int_{-1}^{1} 1\,dx + \int_{-1}^{1}\sqrt{1-x^2}\,dx - \int_{-1}^{1}\frac{1}{1+x^2}\,dx\right)$

$=\frac{1}{2}\left(x\Big|_{-1}^{1} + \frac{1}{2}\pi(1)^2 - \tan^{-1}x\Big|_{-1}^{1}\right)$

$=\frac{1}{2}\left(2+\frac{\pi}{2}-\left(\frac{\pi}{4}-\left(-\frac{\pi}{4}\right)\right)\right) = \boxed{1}$

4. On a certain day, the temperature, in degrees Fahrenheit, in a small town t hours after midnight ($t = 0$) is modeled by the function $g(t) = 50 - 8\sin\left(\frac{\pi t}{12}\right)$. What is the average temperature of the town between 3am ($t = 3$) and 6am ($t = 6$), in degrees Fahrenheit?

$\frac{1}{6-3}\int_{3}^{6} 50-8\sin\left(\frac{\pi t}{12}\right)dt = \frac{1}{3}\left[50t+8\cdot\frac{12}{\pi}\cos\left(\frac{\pi t}{12}\right)\right]_3^6$

$=\frac{1}{3}\left[\left(50\cdot 6+\frac{96}{\pi}\cos\frac{\pi}{2}\right)-\left(50\cdot 3+\frac{96}{\pi}\cos\frac{\pi}{4}\right)\right] = \frac{1}{3}\left(300+0-150-\frac{96}{\pi}\cdot\frac{\sqrt{2}}{2}\right)$

$=\frac{1}{3}\left(150-\frac{48\sqrt{2}}{\pi}\right) \longrightarrow = \boxed{50-\frac{16\sqrt{2}}{\pi}}$