Definite Integral ↔ Limit of a Riemann Sum

width

Concept: $\displaystyle \int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{k=1}^{n}\big(f(a+k\Delta x)\Delta x\big) = \lim_{n\to\infty}\sum_{k=1}^{n}\left(f\!\left(a+k\,\frac{b-a}{n}\right)\frac{b-a}{n}\right)$

where n is the number of subdivisions.

right-hand heights

Example: $\displaystyle \int_1^4 x^5\,dx = \lim_{n\to\infty}\sum_{k=1}^{n}\left(\left(1+k\cdot\frac{3}{n}\right)^5\frac{3}{n}\right)$

Write each Riemann Sum as a definite integral and each definite integral as a right Riemann Sum. Do <u>not</u> evaluate.

1. $\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n}\left(f\!\left(2+k\cdot\frac{3}{n}\right)\frac{3}{n}\right)$

2. $\displaystyle \int_1^5 f(x)\,dx$

3. $\displaystyle \int_2^4 \sin x\,dx$

4. $\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n}\left(\cos\!\left(0+\frac{k\pi}{n}\right)\frac{\pi}{n}\right)$

5. $\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n}\left(\sqrt{3+\frac{2k}{n}}\cdot\frac{2}{n}\right)$

6. $\displaystyle \int_4^5 (x+1)^2\,dx$

7. $\displaystyle \int_0^5 (x^2+1)\,dx$

8. $\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n}\left(\left(3\!\left(\frac{k}{n}+4\right)+2\right)\frac{1}{n}\right)$

9. $\displaystyle \int_2^7 (2x^2+5x)\,dx$

10. $\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n}\left(\left(\cos\!\left(\frac{k\pi}{3n}\right)\right)\frac{\pi}{3n}\right)$

Solutions:

1. $\displaystyle \int_2^5 f(x)\,dx$

2. $\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n}\left(f\!\left(1+k\cdot\frac{4}{n}\right)\frac{4}{n}\right)$

3. $\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n}\left(\sin\!\left(2+\frac{2k}{n}\right)\frac{2}{n}\right)$

4. $\displaystyle \int_0^\pi \cos x\,dx$

5. $\displaystyle \int_3^5 \sqrt{x}\,dx$

6. $\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n}\left(\left(5+\frac{k}{n}\right)^2\frac{1}{n}\right)$

7. $\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n}\left(\left(\left(\frac{5k}{n}\right)^2+1\right)\frac{5}{n}\right)$

8. $\displaystyle \int_4^5 (3x+2)\,dx$

9. $\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n}\left(\left(2\!\left(2+\frac{5k}{n}\right)^2+5\!\left(2+\frac{5k}{n}\right)\right)\frac{5}{n}\right)$

10. $\displaystyle \int_0^{\frac{\pi}{3}} \cos x\,dx$