Name ___________________

Period ___________________

Calculus AB – Semester 1 Final Review

1. Exponential functions:
   (A) 100 kg. of a radioactive substance decay to 40 kg. after 10 years. Find how much remains after 35 years.
   (B) Different isotopes of the same element can have very different half-lives. The decay of plutonium-240 is described by the formula $Q = Q_0 e^{-0.00011t}$, whereas the decay of plutonium-242 is described by $Q = Q_0 e^{-0.0000081t}$. Find the half-lives of plutonium-240 and plutonium-242.
   (C) Suppose prices are increasing by 0.1% per day. By what percent do prices increase in one year?
2. Sketch the following graphs by hand (label two points on your graph):
   (A) $y = 3^x$
   (B) $y = \left(\frac{1}{2}\right)^x$
   (C) $y = \ln x$
   (D) $y = |\ln x|$
3. Evaluate each limit.
   (A) $\lim_{x\to\infty} (\ln x)$
   (B) $\lim_{x\to 1} (\ln x)$
   (C) $\lim_{x\to 0^+} (\ln x)$
4. Evaluate each limit.
   (A) $\lim_{x\to -\infty} e^x$
   (B) $\lim_{x\to\infty} e^{\ln x^{-2}}$
   (C) $\lim_{x\to 0} \frac{5}{x^2}$
5. Evaluate each limit below, without using L'hospital's Rule:
   (A) $\lim_{x\to\infty} \frac{10x^3}{e^x}$
   (B) $\lim_{x\to\infty} \frac{x^{-2}}{x^{-5}}$
   (C) $\lim_{x\to\infty} \frac{\sqrt{x}}{\ln x}$
6. Find the exact value of each expression:
   (A) $\log_2 8$
   (B) $\log_{16} 2$
   (C) $e^{\ln 7}$
   (D) $2^{\log_2 3 + \log_2 5}$
   (E) $e^{3\ln 2}$
7. The position of a car is given by the values in the table:

t (seconds)	0	1	2	3	4	5
s (feet)	0	10	32	70	119	178

   (A) Find the average velocity for the time period beginning when $t = 2$ and lasting: 3 sec., 2 sec., 1 sec.
   (B) Use the information from part (A) and other calculations to approximate the instantaneous velocity when $t = 2$.

8. Interpreting the derivative:
   (A) A yam has just been taken out of the oven and is cooling off before being eaten. The temperature, T, of the yam (measured in degrees Fahrenheit) is a function of how long it has been out of the oven, t (measured in minutes). Thus, we have $T = f(t)$. Is $f'(t)$ positive or negative? What are the units for $f'(t)$?
   (B) An economist is interested in how the price of a certain commodity affects its sales. Suppose that at a price of p dollars, a quantity q of the commodity is sold. If $q = f(p)$, explain in economic terms the meaning of the statements $f(10) = 240{,}000$ and $f'(10) = -29{,}000$.
9. Calculate derivative:
   (A) $y = (x+2)^8(x+3)^6$
   (B) $y = \left(x + \frac{1}{x^2}\right)^5$
   (C) If $f(x) = \frac{1}{(2x-1)^{\sqrt{7}}}$, find $f''(1)$.
10. Calculate derivative:
    (A) $y = \cos(3x^2 + 5)$
    (B) $y = \sin^{-1}(e^x)$
    (C) $y = (x + \sin x)^\pi$
11. Find the equation of the line tangent to the given curve at the given point.
    (A) $y = \tan x$, $\left(\frac{\pi}{3}, \sqrt{3}\right)$
    (B) $f(x) = x \sin x$, $x = \frac{\pi}{2}$
12. Calculate derivative:
    (A) $y = \csc(x - \sin x)$
    (B) $y = \ln(\sec^2 x)$
    (C) If $f(x) = 2^x$, find $f'''(x)$.
13. Find $\frac{dy}{dx}$:
    (A) $x^2y + xy^2 = 3x$
    (B) $\sin(ax) + \cos(by) = xy$ (a and b are constants)
14. Find all local max./min. points and inflection points for $y = x - 2\ln x$, $x > 0$.
15. Find all local max./min. points and inflection points for $y = x^2e^{5x}$.
16. Find the absolute max./min. values of $y = \frac{x}{x^2+1}$ on the interval $[0,2]$.
17. The table below gives values for the functions f and g, as well as their derivatives.

x	-1	0	1	2	3
f	3	3	1	0	1
g	1	2	2.5	3	4
f'	-3	-2	-1.5	-1	1
g'	2	3	2	2.5	3

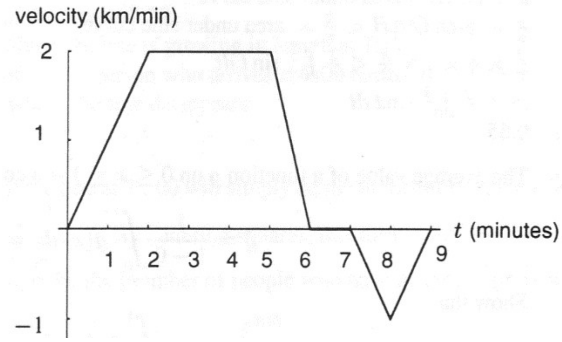
    (A) Find $\frac{d}{dx}f(x)g(x)$ and $\frac{d}{dx}\frac{f(x)}{g(x)}$ at $x = -1$.
    (B) Find $\frac{d}{dx}f(g(x))$ and $\frac{d}{dx}g(f(x))$ at $x = 0$.
    (C) Find the slope of the curve $x^2 + 3y^2 = 7$ at the point $(-2,1)$.
18. The volume of a cube is increasing at a rate of 10 cm^3/min. How fast is the surface area increasing when the length of an edge is 30 cm.?
19. A balloon is rising at a constant rate of 5 ft./sec. A boy is cycling along a straight road at a speed of 15 ft./sec. When he passes under the balloon it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 sec. later?

20. The angle of elevation of the sun is decreasing at a rate of 0.25 rad./hr. How fast is the shadow cast by a 400-ft-tall building increasing when the angle of elevation is $\frac{\pi}{6}$?
21. A 10 ft. ladder is leaning up against the side of a building, but slipping so that the top of the ladder is descending at a rate of 3 ft./sec. What is the rate at which the base of the ladder moves away from the building when the top is 6 ft. from the ground?
22. Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.
23. The top and bottom margins of a poster are each 6 cm. and the side margins are each 4 cm. If the area of printed material on the poster is 384 cm^2, find the dimensions of the entire poster with the smallest area.
24. Limits:
    (A) $\lim_{x\to 0} \frac{\sin(a+2x)-2\sin(a+x)+\sin a}{x^2}$
    (B) $\lim_{x\to\infty} \frac{x}{\ln(1+2e^x)}$
    (C) $\lim_{x\to\pi} \frac{x-\pi}{\tan x}$
25. Area under a curve:
    (A) Find the area under one arch of the curve $y = \sin x$.
    (B) Assume $f(x)$ is a positive function on the interval [2,5]. If $\int_2^5 (2f(x)+3)dx = 17$, find the area under the curve $f(x)$ on the interval [2,5].
26. Area under a curve:
    (A) Find the area bounded by $y = x^2 - 9$ and the x-axis.
    (B) Without a calculator, compute $\int_{-1}^{1} \sqrt{1-x^2}dx$.
27. Riemann Sums:
    (A) The value of $\int_0^2 (x^2 - x)dx$ is estimated using 4 subintervals. Find the estimates using left- and right-hand sums.
    (B) Consider the function with values given in the table below. Approximate $\int_0^6 f(x)dx$using left- and right-hand Riemann sums with 6 subintervals.

x	0	1	2	3	4	5	6
$f(x)$	1	2.5	3	1	-2	-3.5	-4

28. Theorems about integrals:
    (A) Suppose $\int_0^2 f(x)dx = A$ and $\int_0^7 f(x)dx = B$. What is $\int_2^7 f(x)dx$?
    (B) Suppose $\int_4^1 g(x)dx = 5$, find $\int_1^4 [3g(x)+2]dx$.
    (C) Suppose $h(x)$ is an even function and $\int_0^3 h(x)dx = M$, find $\int_{-3}^{3} h(x)dx$.

29. A car is moving along a straight road from points A to B, starting from A at time $t = 0$. Below is the velocity (positive direction is from A to B) plotted against time.



(A) How many kilometers away from A is the car at time $t = 2,5,6,7,9$?
(B) What do you think is happening between $t = 6$ and $t = 7$?
(C) Explain what the car is doing at $t = 7$?

30. Fundamental Theorem of Calculus
(A) The rate at which the world's oil is being consumed is continuously increasing. Suppose the rate (in billions of barrels per year) is given by the function $r = 32e^{0.05}$, where t is measured in years since 1990. Find the total quantity of oil used between 1990 and 1995.
(B) Assume that $r(t)$ represents the rate at which a country's debt is growing, where t is years since 1990. In terms of debt, explain the meaning of $\int_0^{10} r(t)dt$.