Calculators Allowed

1. B 2. C 3. C 4. E 5. A 6. B

7. a) $\dfrac{dy}{dx} = \dfrac{-2x-y}{x+2y}$ b) parallel, $y' = -2$ at both points

c) $\dfrac{d^2y}{dx^2} = \dfrac{(x+2y)\left(-2-\frac{-2x-y}{x+2y}\right)-(-2x-y)\left(1+2\frac{-2x-y}{x+2y}\right)}{(x+2y)^2}$

8. 36

9. a) -9 gal/hr/hr b) 8 gal

10. a) 1.210 b) $x = 1.414$, f' goes neg. to pos.

c) $x = .642$, $x = 1.496$ d) (2, 4.061)

$$f(0)=5$$
$$f(1.414)=5+\int_0^{1.414} f'(x)\,dx = 4.307$$
$$f(1.571)=5+\int_0^{1.571} f'(x)\,dx = 4.309$$
$$f(2)=5+\int_0^{2} f'(x)\,dx = 4.061$$

11. a) 6004 b) \$104,048

12. a) conc. down, f' is dec. b) 8

$$f(1)=5$$
$$f(3)=5+\int_1^{3} f'(x)\,dx = 8$$
$$f(6)=5+\int_1^{6} f'(x)\,dx = 3$$

c) $y - 5 = 2(x - 1)$

No Calculators

1. E 2. E 3. C 4. D 5. E 6. B

7. a) -1 b) 5

$$f(0) = 10$$
$$f(4) = 10 + \int_0^4 f'(x)\,dx = 5$$
$$f(7) = 10 + \int_0^7 f'(x)\,dx = 9$$

8. a) $x = -2$, f' goes from pos. to neg.

   b) $x = 4$, f' goes from neg. to pos.

   c) $-1 < x < 1$ and $3 < x < 5$ because f' is increasing

9. a) $y = -4 + 5(x - 1)$, $f(1.2) \approx -3$, less because conc. up

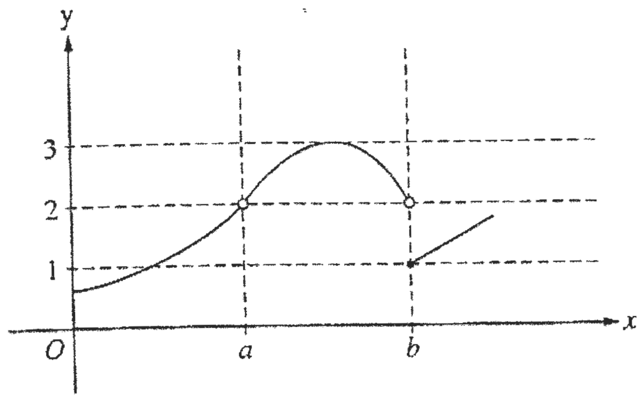
   b) $r = 6$ by MVT

Name ____________________

Period ____________________

Calculus AB – Sample Final (calculators allowed)

Show all work for free-response questions.

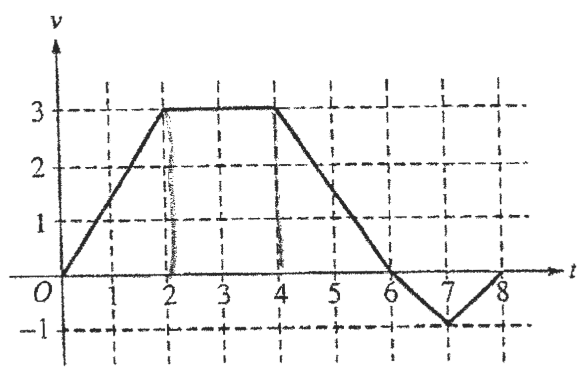


1. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

   (A) $\lim_{x\to a} f(x) = \lim_{x\to b} f(x)$ (B) $\lim_{x\to a} f(x) = 2$ (C) $\lim_{x\to b} f(x) = 2$

   (D) $\lim_{x\to b} f(x) = 1$ (E) $\lim_{x\to a} f(x)$ does not exist

Questions 2 and 3 refer to the following situation:



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

2. At what value of t does the bug change direction?

   (A) 2 (B) 4 (C) 6 (D) 7 (E) 8

3. What is the displacement of the bug's position from $t = 0$ to $t = 8$?

   (A) 14 (B) 13 (C) 11 (D) 8 (E) 6

$$\int_0^8 v(t)\,dt = \frac{1}{2}\cdot 2\cdot 3 + 2\cdot 3 + \frac{1}{2}\cdot 2\cdot 3 - \frac{1}{2}\cdot 2\cdot 1$$

4. If $f(x)=\dfrac{e^{2x}}{2x}$, then $f'(x)=$

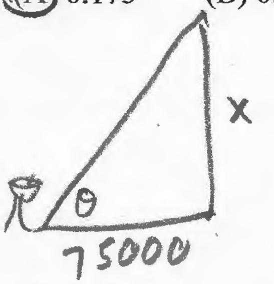
$$\frac{2x\cdot e^{2x}\cdot 2-e^{2x}\cdot 2}{(2x)^2}=\frac{2e^{2x}(2x-1)}{4x^2}$$

(A) 1 (B) $\dfrac{e^{2x}(1-2x)}{2x^2}$ (C) e^{2x}

(D) $\dfrac{e^{2x}(2x+1)}{x^2}$ (E) $\dfrac{e^{2x}(2x-1)}{2x^2}$

5. A missile rises vertically from a point on the ground 75,000 feet from a radar station. If the missile is rising at the rate of 16,500 feet per minute at the instant when it is 38,000 feet high, what is the rate of change, in radians per minute, of the missile's angle of elevation from the radar station at this instant?

(A) 0.175 (B) 0.219 (C) 0.227 (D) 0.469 (E) 0.507



$\tan\theta=\dfrac{x}{75000}$

$\theta=\tan^{-1}\left(\dfrac{x}{75000}\right)$

$\dfrac{d\theta}{dt}=\dfrac{1}{1+\left(\frac{x}{75000}\right)^2}\cdot\dfrac{1}{75000}\dfrac{dx}{dt}$

$x=38000$

$\dfrac{dx}{dt}=16500$

$\dfrac{d\theta}{dt}=?$

6. The function $P(t)$ models the population of the world, in billions of people, where t is the number of years since January 1, 2010. Which of the following is the best interpretation of the statement $P'(1)=0.076$?

(A) On February 1, 2010, the population of the world is increasing at a rate of 0.076 billion people per year.

(B) On January 1, 2011, the population of the world is increasing at a rate of 0.076 billion people per year.

(C) On January 1, 2011, the population of the world is 0.076 billion people.

(D) From January 1, 2010 to January 1, 2011, the population of the world was increasing at an average rate of 0.076 billion people per year.

(E) When the population of the world is 1 billion people, the population of the world was increasing at a rate of 0.076 billion people per year.

7. Consider the curve defined by $x^2 + xy + y^2 = 27$.

a) Write an expression for the slope of the curve at any point (x, y).

$$2x + xy' + y\cdot 1 + 2yy' = 0$$
$$y'(x+2y) = -2x - y$$
$$y' = \frac{-2x-y}{x+2y}$$

b) Find the x-intercepts of the curve. Determine whether the tangent lines to the curve at the x-intercepts are parallel. Show the analysis that leads to your conclusion.

x-int. $\to y=0$

$x^2 + x(0) + 0^2 = 27$

$x = \pm\sqrt{27}$

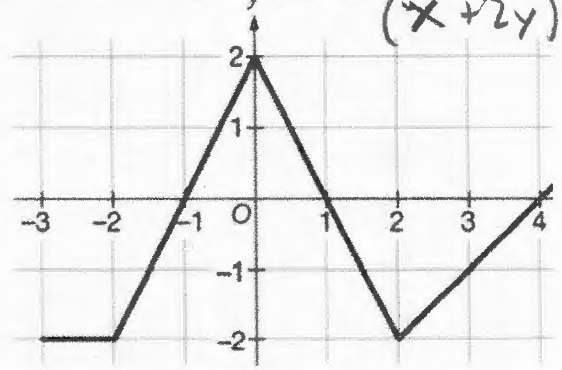
$(\sqrt{27}, 0) \to \frac{dy}{dx} = \frac{-2\sqrt{27} + 0}{\sqrt{27} + 2(0)} = -2$

$(-\sqrt{27}, 0) \to \frac{dy}{dx} = \frac{-2(-\sqrt{27}) - 0}{-\sqrt{27} + 2(0)} = -2$

same slope, so parallel

c) Find an expression for $\frac{d^2y}{dx^2}$ in terms of x and y. **DO NOT SIMPLIFY.**

$$\frac{d^2y}{dx^2} = \frac{(x+2y)(-2-y') - (-2x-y)(1+2y')}{(x+2y)^2}$$
$$= \frac{(x+2y)\left(-2 - \frac{-2x-y}{x+2y}\right) - (-2x-y)\left(1 + 2\frac{-2x-y}{x+2y}\right)}{(x+2y)^2}$$



8. The graph of the function f on the closed interval $-3 \le x \le 4$ consists of four line segments, as shown in the figure above. It is known that $\int_{-3}^{-1} g(x)\,dx = -4.8$ and $\int_{-3}^{4} g(x)\,dx = 11.2$. Find the value of $\int_{-1}^{4} (2g(x) - 4f(x))\,dx$.

$$\int_{-1}^{4} g(x)\,dx = \int_{-3}^{4} g(x)\,dx - \int_{-3}^{-1} g(x)\,dx$$
$$= 11.2 - (-4.8)$$
$$= 16$$

$$= 2\int_{-1}^{4} g(x)\,dx - 4\int_{-1}^{4} f(x)\,dx$$
$$= 2(16) - 4\left(\tfrac{1}{2}\cdot 2\cdot 2 - \tfrac{1}{2}\cdot 3\cdot 2\right)$$
$$= 32 + 4 = 36$$

Calculus AB – Sample Final (calculators allowed)

t (hours)	0	$\frac{1}{3}$	$\frac{2}{3}$	1
$R(t)$ (gallons per hour)	11	8	5	0

9. The rate at which water leaks from a container is modeled by the twice-differentiable function R, where $R(t)$ is measured in gallons per hour and t is measured in hours for $0 \le t \le 1$. Values of $R(t)$ are given in the table above for selected values of t.

a) Use the data in the table to find an approximation for $R'\left(\frac{1}{2}\right)$. Show the computations that lead to your answer and indicate units of measure.

$$R'\left(\tfrac{1}{2}\right) \approx \frac{R\left(\frac{2}{3}\right) - R\left(\frac{1}{3}\right)}{\frac{2}{3} - \frac{1}{3}} = \frac{5-8}{1/3} = -9 \text{ gal/hr./hr.}$$

b) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate $\int_0^1 R(t)\,dt$. Show the computations that lead to your answer and indicate units of measure.

$$\tfrac{1}{3}R(0) + \tfrac{1}{3}R\left(\tfrac{1}{3}\right) + \tfrac{1}{3}R\left(\tfrac{2}{3}\right) = 8 \text{ gal}$$

10. The derivative of a function f is given by $f'(x) = \left(x^3 - 2x\right)\cos x$ for $0 \le x \le 2$.

a) Find $f''(1.3)$. $= 1.210$

b) Find the x-coordinate of the relative minimum of $f(x)$. Show the analysis that leads to your conclusion.

$f'(x) = 0$

$x = 1.414, 1.571$

$x = 1.414$, f' goes neg. to pos.

c) Find the x-coordinate of every point of inflection on the graph of $f(x)$. Justify your answer.

$f''(x) = (x^3 - 2x)(-\sin x) + (3x^2 - 2)\cos x = 0$

$x = .642, 1.496$ ← both inf. pts. because f'' changes sign

d) If $f(0) = 5$, find the coordinates of the point at which $f(x)$ attains an absolute minimum. Justify your answer.

$f(0) = 5$

$f(1.414) = 5 + \int_0^{1.414} f'(x)\,dx = 4.307$

$f(1.571)$ = local max, f' goes pos. to neg

$f(2) = 5 + \int_0^{2} f'(x)\,dx = 4.061$

min. at $(2, 4.061)$

11. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t)=\frac{15600}{t^2-24t+160}$$

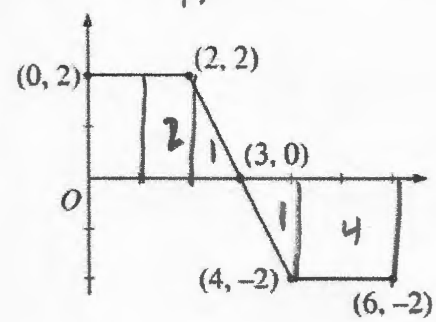
$E(t)$ is measured in people per hour and time t is measured in hours after midnight. The function is valid for $9\le t\le 23$, the hours during which the park is open. At time $t=9$, there are no people in the park.

a) Write an integral expression to determine the number of people who have entered the park by 5:00pm ($t=17$)? Evaluate the integral, rounding your answer to the nearest whole number.

$\int_9^{17} E(t)dt = 6004$

b) The price of admission to the park is \$15 until 5:00pm ($t=17$). After 5:00pm, the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

$15\int_9^{17} E(t)dt + 11\int_{17}^{23} E(t)dt = \$104,048$



Graph of f'

12. The function f is defined on the closed interval $[0,6]$ with $f(1)=5$. The graph of f', the first derivative of f, consists of three line segments and is shown above.

a) Determine whether the graph of f is concave up, concave down, or neither at the point where $x=2.5$. Explain your reasoning.

conc. down, f' is dec.

b) What is the maximum value of $f(x)$ for $1\le x\le 6$?

$f(1)=5$

$f(3)=5+\int_1^3 f'(x)dx = 5+3=8$

$f(6)=5+\int_1^6 f'(x)dx = 5+(3-5)=3$

max. value $=8$

c) Write an equation for the line tangent to the graph of f when $x=1$.

$(1,5)$

$m=f'(1)=2$

$y-5=2(x-1)$

Name ____________________

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Calculus AB – Sample Final (no calculators)

Show all work for free-response questions.

1. If $y=\left(x^3+1\right)^2$, then $\frac{dy}{dx}=$ $2\left(x^3+1\right)\cdot 3x^2$

   (A) $\left(3x^2\right)^2$ (B) $2\left(x^3+1\right)$ (C) $2\left(3x^2+1\right)$

   (D) $3x^2\left(x^3+1\right)$ (E) $6x^2\left(x^3+1\right)$

2. If $y=x^2\sin 2x$, then $\frac{dy}{dx}=$ $x^2\cdot\cos 2x\cdot 2+\sin 2x\cdot 2x=2x\left(x\cos 2x+\sin 2x\right)$

   (A) $2x\cos 2x$ (B) $4x\cos 2x$ (C) $2x\left(\sin 2x+\cos 2x\right)$

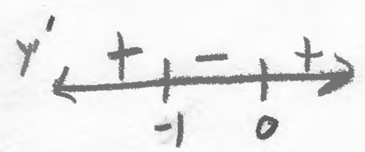
   (D) $2x\left(\sin 2x-x\cos 2x\right)$ (E) $2x\left(\sin 2x+x\cos 2x\right)$

3. What are all the values of x for which the graph of $y=6x^2+\frac{x}{2}+3+\frac{6}{x}$ is concave downward?

   $6x^{-1}$

   (A) $x<-1$ (B) $x<0$ (C) $-1<x<0$

   (D) $0<x<1$ (E) $x>-1$

   $y'=12x+\frac{1}{2}-6x^{-2}$

   $y''=12+12x^{-3}$

   $=12+\frac{12}{x^3}$

   $=\frac{12x^3+12}{x^3}=0$

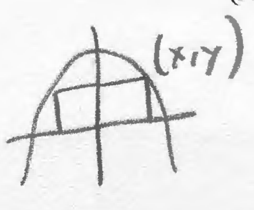
   $12x^3+12=0$

   $x^3=-1$

   $x=-1$

4. What is the area of the largest rectangle with lower base on the x-axis and upper vertices on the curve $y=12-x^2$?

   (A) 8 (B) 12 (C) 16 (D) 32 (E) 48

   $A=2xy=2x\left(12-x^2\right)=24x-2x^3$

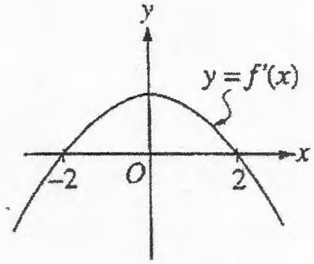
   $A'=24-6x^2=0$

   $24=6x^2$

   $x^2=4\rightarrow x=2$

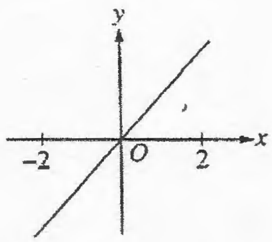
   $y=12-2^2=8$

   $A=2(2)(8)$

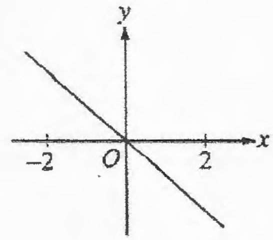


5. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

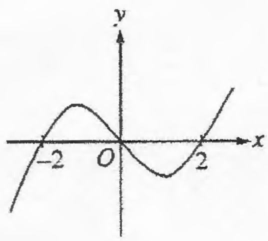
(A)



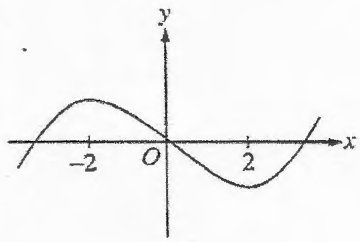
(B)



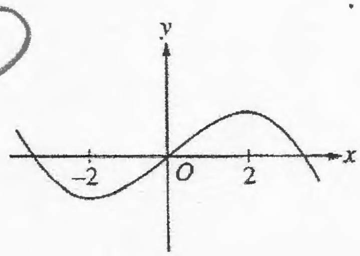
(C)



(D)



(E)



6. The function g is continuous on the closed interval $[1,4]$ with $g(1) = 5$ and $g(4) = 8$. Of the following conditions, which would guarantee that there is a number c in the interval $(1,4)$ where $g'(c) = 1$? → MVT

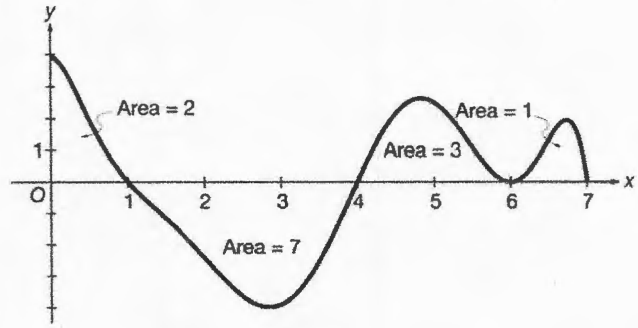
$$\frac{g(4)-g(1)}{4-1} = \frac{8-5}{3} = 1$$

(A) g is increasing on the closed interval $[1,4]$

(B) g is differentiable on the open interval $(1,4)$

(C) g has a maximum value on the closed interval $[1,4]$

(D) The graph of g has at least one horizontal tangent in the open interval $(1,4)$



Graph of f'

7. The figure above shows the graph of f', the derivative of a differentiable function f, on the closed interval $0 \le x \le 7$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f($[redacted]$) = 10$.

   a. Find the value of $\int_0^7 f'(x)\,dx$. $= 2 - 7 + 3 + 1 = -1$

   b. Find the absolute minimum value of f on the closed interval $0 \le x \le 7$. Justify your answers.

$f(7) \to f'$ pos. before

$f(1) \to f'$ goes pos. to neg, so local max.

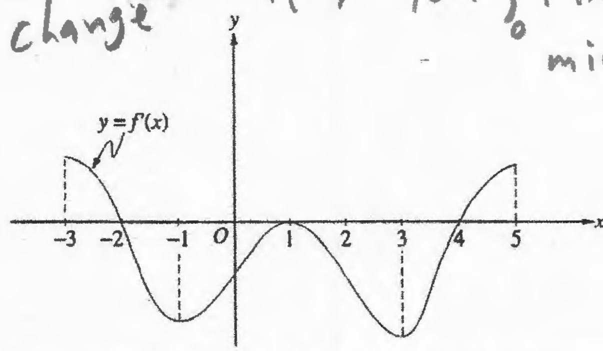
$f(6) \to f'$ doesn't change signs

$f(0) = 10$

$f(4) = 10 + \int_0^4 f'(x)\,dx = 10 + (2-7) = 5$

$f(7) = 10 + \int_0^7 f'(x)\,dx = 10 + (-1) = 9$

min value $= 5$



8. The figure above shows the graph of f', the derivative of a function f. The domain of f is $-3 < x < 5$.

   a. For what values of x does f have a local maximum? Why?

   $x = -2$, f' goes pos. to neg.

   b. For what values of x does f have a local minimum? Why?

   $x = 4$, f' goes neg. to pos.

   c. On what intervals is the graph of f concave upward? Use f' to justify your answer.

   $-1 < x < 1$, $3 < x < 5 \Rightarrow f'$ has pos. slope

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

9. Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

   a. Write the equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

   $(1, -4)$
   slope $= 5$

   $y = -4 + 5(x-1)$
   $f(1.2) \approx -4 + 5(1.2-1)$
   $= -3$

   less, f is concave up

   b. Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

   f twice diff. ⇒ f′ diff. ⇒ MVT applies to f′

   $$f''(c) = \frac{f'(.5) - f'(0)}{.5 - 0} = \frac{3-0}{.5} = 6 \Rightarrow r = 6$$