Spring Break Assignment

ALL WORK MUST GO ON A SEPARATE SHEET OF PAPER. IF YOU DON'T KNOW HOW TO DO A PROBLEM, COPY THE ENTIRE PROBLEM AND ALL CHOICES ONTO YOUR PAPER.

No Calculators Allowed:

2. The graph of $y = 3x^2 - x^3$ has a relative maximum at

(A) $(0, 0)$ only

(B) $(1, 2)$ only

(C) $(2, 4)$ only

(D) $(4, -16)$ only

(E) $(0, 0)$ and $(2, 4)$

3. If $x > 0$, $\int x^{-\frac{1}{3}}\, dx =$

(A) $\frac{2}{3}x^{\frac{2}{3}} + C$ (B) $-\frac{1}{3}x^{-\frac{4}{3}} + C$ (C) $-\frac{3}{2}x^{\frac{2}{3}} + C$

(D) $\frac{3}{2}x^{\frac{2}{3}} + C$ (E) $-\frac{3}{4}x^{-\frac{4}{3}} + C$

4. If $f(x) = e^{\sin x}$, how many zeros does $f'(x)$ have on the closed interval $[0, 2\pi]$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

5. $\lim_{x \to \infty} \dfrac{10^8x^5 + 10^6x^4 + 10^4x^2}{10^9x^6 + 10^7x^5 + 10^5x^3} =$

(A) 0 (B) 1 (C) -1 (D) $\frac{1}{10}$ (E) $-\frac{1}{10}$

6. The graph of which function has $y = -1$ as an asymptote ?

(A) $y = e^{-x}$ (B) $y = \dfrac{-x}{1 - x}$ (C) $y = \ln(x + 1)$

(D) $y = \dfrac{x}{x + 1}$ (E) $y = \dfrac{x}{1 - x}$

7. If $f(x) = \sqrt{4\sin x + 2}$, then $f'(0) =$

(A) -2

(B) 0

(C) $\sqrt{2}$

(D) $\frac{\sqrt{2}}{2}$

(E) 1

8. $\frac{d}{dx}\left(e^{3\ln x}\right) =$

(A) $e^{3\ln x}$ (B) $\frac{e^{3\ln x}}{x}$ (C) x^3 (D) $3x^2$ (E) 3

9. The equation of the tangent line to the curve $x^2 + y^2 = 169$ at the point $(5, -12)$ is

(A) $5y - 12x = -120$

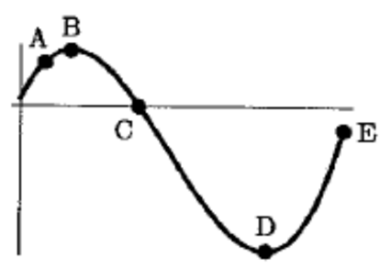
(B) $5x - 12y = 119$

(C) $5x - 12y = 169$

(D) $12x + 5y = 0$

(E) $12x + 5y = 169$

10.



The figure above shows the graph of the velocity of a moving object as a function of time. At which of the marked points is the speed the greatest?

(A) A

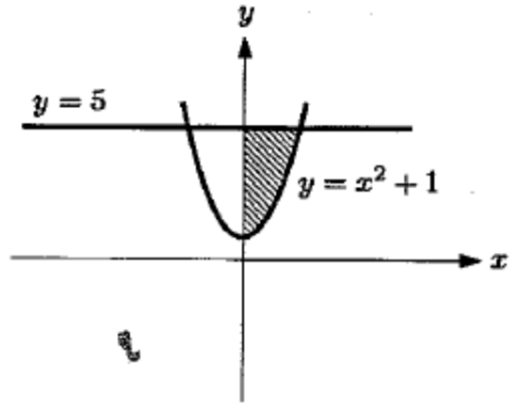
(B) B

(C) C

(D) D

(E) E

11.



For the figure above, the area of the shaded region is

(A) $\frac{14}{3}$

(B) $\frac{16}{3}$

(C) $\frac{28}{3}$

(D) $\frac{32}{3}$

(E) $\frac{65}{3}$

13. If the graph of $f(x) = 2x^2 + \frac{k}{x}$ has a point of inflection at $x = -1$, then the value of k is

(A) 1

(B) -1

(C) 2

(D) -2

(E) 0

14. $\int \sin(3x + 4)\, dx =$

(A) $-\frac{1}{3}\cos(3x + 4) + C$

(B) $-\cos(3x + 4) + C$

(C) $-3\cos(3x + 4) + C$

(D) $\cos(3x + 4) + C$

(E) $\frac{1}{3}\cos(3x + 4) + C$

15. For what values of x is the graph of $y = \frac{2}{4-x}$ concave downward ?

(A) No values of x

(B) $x < 4$

(C) $x > -4$

(D) $x < -4$

(E) $x > 4$

16. A particle moves along the x–axis in such a way that its position at time t is given by $x(t) = \frac{1-t}{1+t}$. What is the acceleration of the particle at time $t = 0$?

(A) $-\frac{3}{5}$

(B) -4

(C) 4

(D) 2

(E) -2

17. Suppose $f(x) = \ln 3x$ and $f^{-1}(x)$ denotes the inverse of f. Then $\int f^{-1}(x)\, dx =$

(A) $3e^{x} + C$

(B) $\frac{1}{3}e^{x} + C$

(C) $\frac{1}{x} + C$

(D) $\frac{1}{3x} + C$

(E) $\frac{1}{3}e^{3x} + C$

18. If $y = x(\ln x)^2$, then $\frac{dy}{dx} =$

(A) $3(\ln x)^2$

(B) $(\ln x)(2x + \ln x)$

(C) $(\ln x)(2 + \ln x)$

(D) $(\ln x)(2 + x\ln x)$

(E) $(\ln x)(1 + \ln x)$

20. A particle moves on the x-axis so that at any time t its velocity $v(t) = \sin 2t$ subject to the condition $x(0) = 0$ where $x(t)$ is the position function. Which of the following is an expression for $x(t)$?

(A) $\cos 2t + \frac{1}{2}$

(B) $-\frac{1}{2}\sin 2t + \frac{1}{2}$

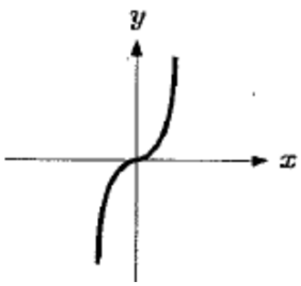
(C) $-\frac{1}{2}\cos 2t$

(D) $-\frac{1}{2}\cos 2t + \frac{1}{2}$

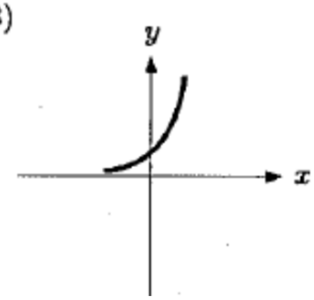
(E) $-\frac{1}{2}\cos 2t - \frac{1}{2}$

21. If, for all values of x, $f'(x) < 0$ and $f''(x) > 0$, which of the following curves could be part of the graph of f ?

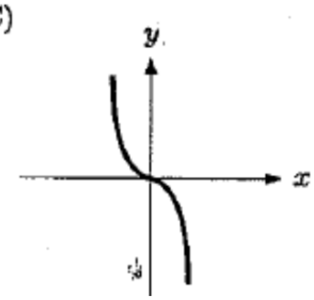
(A)



(B)



(C)



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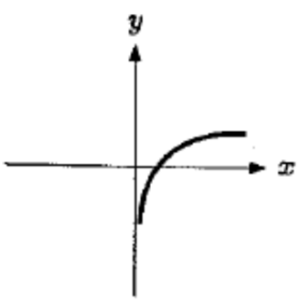
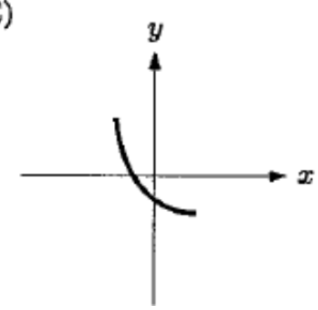


(E)



22. If $\frac{dy}{dx} = xy^2$ and $x = 1$ when $y = 1$, then $y =$

(A) x^2

(B) $\frac{-2}{x^2 - 3}$

(C) $x^2 - 3$

(D) $\frac{2}{x^2 + 1}$

(E) $\frac{x^2 - 3}{2}$

24. The maximum value of $f(x) = 2x^3 - 9x^2 + 12x - 1$ on $[-1, 2]$ is

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

25. For which pair of functions $f(x)$ and $g(x)$ below, will the $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$?

	$f(x)$	$g(x)$
(A)	e^x	x^2
(B)	e^x	$\ln x$
(C)	$\ln x$	e^x
(D)	x	$\ln x$
(E)	3^x	2^x

26. Let $f(x)$ be the function defined by $f(x) = \begin{cases} x, & x \le 0 \\ x+1, & x > 0 \end{cases}$.

The value of $\int_{-2}^{1} x f(x)\, dx =$

(A) $\frac{3}{2}$

(B) $\frac{5}{2}$

(C) $\frac{7}{2}$

(D) $\frac{11}{2}$

(E) 3

27. The average value of the function $f(x) = \cos(\frac{1}{2}x)$ on the closed interval $[-4, 0]$ is

(A) $-\frac{1}{2}\sin(2)$

(B) $-\frac{1}{4}\sin(2)$

(C) $\frac{1}{2}\cos(2)$

(D) $\frac{1}{4}\sin(2)$

(E) $\frac{1}{2}\sin(2)$

Calculators Allowed:

29. The volume of the solid formed by revolving the region bounded by the graph of $y = (x-3)^2$ and the coordinate axes about the <u>x–axis</u> is given by which of the following integrals?

(A) $\pi \int_0^3 (x-3)^2\, dx$

(B) $\pi \int_0^3 (x-3)^4\, dx$

(C) $2\pi \int_0^3 (x-3)^2\, dx$

(D) $2\pi \int_0^3 x(x-3)^2\, dx$

(E) $2\pi \int_0^3 x(x-3)^4\, dx$

30. $\lim_{x \to -3} \frac{x^2 + 3x}{\sqrt{x^2 + 6x + 9}}$ is

(A) -3 (B) -1 (C) 1 (D) 3 (E) nonexistent

31. The cost C of producing x items is given by $C(x) = 20,000 + 5(x - 60)^2$. The revenue R obtained by selling x items is given by $R(x) = 15,000 + 130x$. The revenue will exceed the cost for all x such that

(A) $0 < x < 46$

(B) $x > 46$

(C) $x < 100$

(D) $46 < x < 100$

(E) $x > 100$

32.

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	20	19.5	18	15.5	12	7.5	2	−4.5	−12	−20.5	−30

Some values of a continuous function are given in the table above. The Trapezoidal Rule approximation for $\int_0^{10} f(x)\,dx$ is

(A) 30.825

(B) 32.500

(C) 33.325

(D) 33.333

(E) 35.825

33. Let $R(t)$ represent the rate at which water is leaking out of a tank, where t is measured in hours. Which of the following expressions represents the total amount of water that leaks out in the first three hours?

(A) $R(3) - R(0)$

(B) $\dfrac{R(3) - R(0)}{3 - 0}$

(C) $\int_0^3 R(t)\,dt$

(D) $\int_0^3 R'(t)\,dt$

(E) $\frac{1}{3}\int_0^3 R(t)\,dt$

34. Let f and g be differentiable functions such that

$$f(1) = 4,\ g(1) = 3,\ f'(3) = -5$$

$$f'(1) = -4,\ g'(1) = -3,\ g'(3) = 2$$

If $h(x) = f(g(x))$, then $h'(1) =$

(A) −9

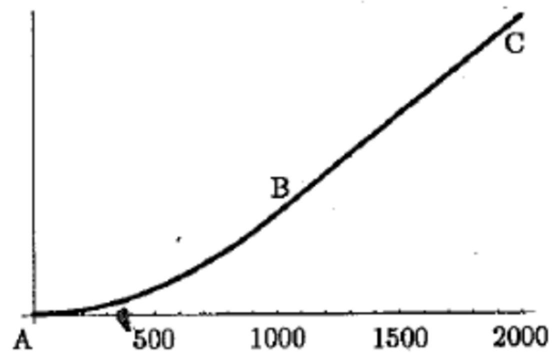
(B) 15

(C) 0

(D) −5

(E) −12

35.



The figure above shows a road running in the shape of a parabola from the bottom of a hill at A to point B. At B it changes to a line and continues on to C. The equation of the road is

$$R(x) = \begin{cases} ax^2, & \text{from A to B} \\ bx + c, & \text{from B to C} \end{cases}$$

B is 1000 feet horizontally from A and 100 feet higher. Since the road is smooth, $R'(x)$ is continuous. What is the value of b?

(A) 0.2

(B) 0.02

(C) 0.002

(D) 0.0002

(E) 0.00002

36. The shortest distance from the curve $xy = 4$ to the origin is

(A) 2

(B) 4

(C) $\sqrt{2}$

(D) $2\sqrt{2}$

(E) $\frac{1}{2}\sqrt{2}$

37.

x	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
$f(x)$	2.018	2.008	2.002	2	2.002	2.008	2.018
$g(x)$	1	1	1	2	2	2	2
$h(x)$	1.971	1.987	1.997	undefined	1.997	1.987	1.971

The table above gives the values of three functions, f, g, and h near $x = 0$. Based on the values given, for which of the functions does it appear that the limit as x approaches zero is 2?

(A) f only

(B) g only

(C) h only

(D) f and h only

(E) f, g, and h

38. Suppose that $f(x)$ is an <u>even</u> function and let $\int_0^1 f(x)\,dx = 5$ and $\int_0^7 f(x)\,dx = 1$.

What is $\int_{-7}^{-1} f(x)\,dx$?

(A) -5

(B) -4

(C) 0

(D) 4

(E) 5

39. The area of the region enclosed by the graphs of $y = e^{(x^2)} - 2$ and $y = \sqrt{4 - x^2}$ is

(A) 2.525

(B) 4.049

(C) 4.328

(D) 5.050

(E) 6.289

40. If $f(x) = |(x^2 - 12)(x^2 + 4)|$, how many numbers in the interval $-2 \le x \le 3$ satisfy the conclusion of the Mean Value Theorem?

(A) None

(B) One

(C) Two

(D) Three

(E) Four

41. If f and g are differentiable functions, then $\int_0^{g(x)} f'(t)\,dt =$

(A) $f(g(x))$

(B) $g(f(x))$

(C) $g(f(x)) - g(f(0))$

(D) $f(g(x)) - f(0)$

(E) $f(g(x)) - f(g(0))$

42. The amount $A(t)$ of a certain item produced in a factory is given by

$$A(t) = 4000 + 48(t - 3) - 4(t - 3)^3$$

where t is the number of hours of production since the beginning of the workday at 8:00 am. At what time is the rate of production increasing most rapidly?

(A) 8:00 am

(B) 10:00 am

(C) 11:00 am

(D) 12:00 noon

(E) 1:00 pm

43. At how many points on the curve $y = 4x^5 - 3x^4 + 15x^2 + 6$ will the line tangent to the curve pass through the origin?

(A) One

(B) Two

(C) Three

(D) Four

(E) Five

44. A population grows according to the equation $P(t) = 6000 - 5500e^{-0.159t}$ for $t \geq 0$, t measured in years. This population will approach a limiting value as time goes on. During which year will the population reach half of this limiting value?

(A) Second

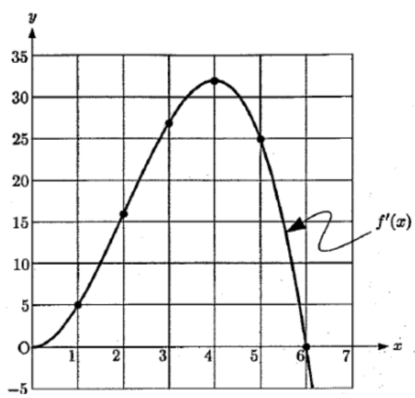
(B) Third

(C) Fourth

(D) Eighth

(E) Twenty-ninth

45.



Let f be a differentiable function for all x. The graph of $f'(x)$ is shown above. If $f(2) = 10$, which of the following best approximates the maximum value of $f(x)$?

(A) 30

(B) 50

(C) 70

(D) 90

(E) 110

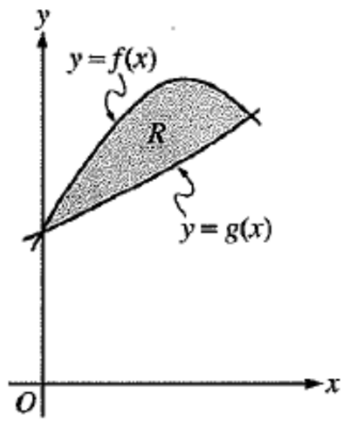
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the x-axis.

   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

2. A water tank at Camp Newton holds 1200 gallons of water at time $t = 0$. During the time interval $0 \le t \le 18$ hours, water is pumped into the tank at the rate

$$W(t) = 95\sqrt{t}\sin^2\left(\frac{t}{6}\right) \text{ gallons per hour.}$$

During the same time interval, water is removed from the tank at the rate

$$R(t) = 275\sin^2\left(\frac{t}{3}\right) \text{ gallons per hour.}$$

(a) Is the amount of water in the tank increasing at time $t = 15$? Why or why not?

(b) To the nearest whole number, how many gallons of water are in the tank at time $t = 18$?

(c) At what time t, for $0 \le t \le 18$, is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.

(d) For $t > 18$, no water is pumped into the tank, but water continues to be removed at the rate $R(t)$ until the tank becomes empty. Let k be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

3. A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by $v(t) = \ln(t^2 - 3t + 3)$. The particle is at position $x = 8$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 4$.

(b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left?

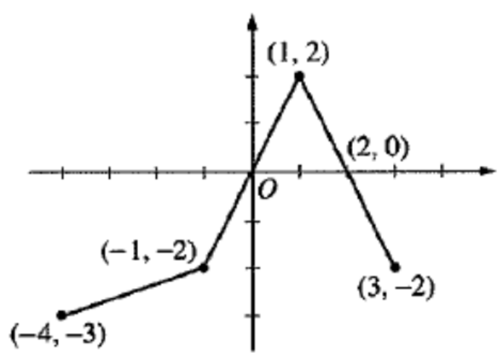
(c) Find the position of the particle at time $t = 2$.

(d) Find the average speed of the particle over the interval $0 \le t \le 2$.

2005 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. The graph of the function f above consists of three line segments.

(a) Let g be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

(b) For the function g defined in part (a), find the x-coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

(c) Let h be the function given by $h(x) = \int_{x}^{3} f(t)\,dt$. Find all values of x in the closed interval $-4 \le x \le 3$ for which $h(x) = 0$.

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

5. Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

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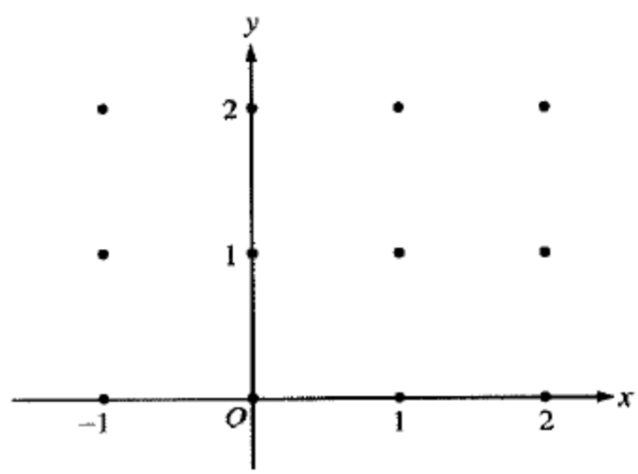
6. Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the test booklet.)**



(b) Write an equation for the line tangent to the graph of f at $x = -1$.

(c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

Multiple Choice

1. E	2. C	3. D	4. B	5. A	6. E	7. C	8. D	9. C	10. D	11. B	12. A	13. C	14. A	15. E
16. C	17. B	18. C	19. B	20. D	21. E	22. B	23. C	24. E	25. C	26. C	27. E	28. E	29. B	30. E
31. D	32. B	33. C	34. B	35. A	36. D	37. D	38. B	39. D	40. D	41. D	42. C	43. A	44. C	45. E

Problem 1

The graphs of f and g intersect in the first quadrant at $(S, T) = (1.13569, 1.76446)$.

(a) Area $= \int_0^S (f(x) - g(x))\,dx$

$= \int_0^S \left(1 + \sin(2x) - e^{x/2}\right) dx$

$= 0.429$

(b) Volume $= \pi \int_0^S \left((f(x))^2 - (g(x))^2\right) dx$

$= \pi \int_0^S \left((1 + \sin(2x))^2 - \left(e^{x/2}\right)^2\right) dx$

$= 4.266$ or 4.267

(c) Volume $= \int_0^S \frac{\pi}{2}\left(\frac{f(x) - g(x)}{2}\right)^2 dx$

$= \int_0^S \frac{\pi}{2}\left(\frac{1 + \sin(2x) - e^{x/2}}{2}\right)^2 dx$

$= 0.077$ or 0.078

Problem 2

(a) No; the amount of water is not increasing at $t = 15$ since $W(15) - R(15) = -121.09 < 0$.

(b) $1200 + \int_0^{18} (W(t) - R(t))\,dt = 1309.788$

1310 gallons

(c) $W(t) - R(t) = 0$

$t = 0, 6.4948, 12.9748$

t (hours)	gallons of water
0	1200
6.495	525
12.975	1697
18	1310

The values at the endpoints and the critical points show that the absolute minimum occurs when $t = 6.494$ or 6.495.

(d) $\int_{18}^{k} R(t)\,dt = 1310$

Problem 3

(a) $a(4) = v'(4) = \frac{5}{7}$

(b) $v(t) = 0$

$t^2 - 3t + 3 = 1$

$t^2 - 3t + 2 = 0$

$(t-2)(t-1) = 0$

$t = 1, 2$

$v(t) > 0$ for $0 < t < 1$

$v(t) < 0$ for $1 < t < 2$

$v(t) > 0$ for $2 < t < 5$

The particle changes direction when $t = 1$ and $t = 2$.
The particle travels to the left when $1 < t < 2$.

(c) $s(t) = s(0) + \int_0^t \ln\left(u^2 - 3u + 3\right) du$

$s(2) = 8 + \int_0^2 \ln\left(u^2 - 3u + 3\right) du$

$= 8.368$ or 8.369

(d) $\frac{1}{2}\int_0^2 |v(t)|\, dt = 0.370$ or 0.371

Problem 4

(a) $g(-1) = \int_{-4}^{-1} f(t)\, dt = -\frac{1}{2}(3)(5) = -\frac{15}{2}$

$g'(-1) = f(-1) = -2$

$g''(-1)$ does not exist because f is not differentiable at $x = -1$.

(b) $x = 1$

$g' = f$ changes from increasing to decreasing at $x = 1$.

(c) $x = -1, 1, 3$

(d) h is decreasing on $[0, 2]$

$h' = -f < 0$ when $f > 0$

Problem 5

(a) $2yy' = y + xy'$

$(2y - x)y' = y$

$y' = \dfrac{y}{2y - x}$

(b) $\dfrac{y}{2y - x} = \dfrac{1}{2}$

$2y = 2y - x$

$x = 0$

$y = \pm\sqrt{2}$

$(0, \sqrt{2}), (0, -\sqrt{2})$

(c) $\dfrac{y}{2y - x} = 0$

$y = 0$

The curve has no horizontal tangent since $0^2 \neq 2 + x \cdot 0$ for any x.

(d) When $y = 3$, $3^2 = 2 + 3x$ so $x = \dfrac{7}{3}$.

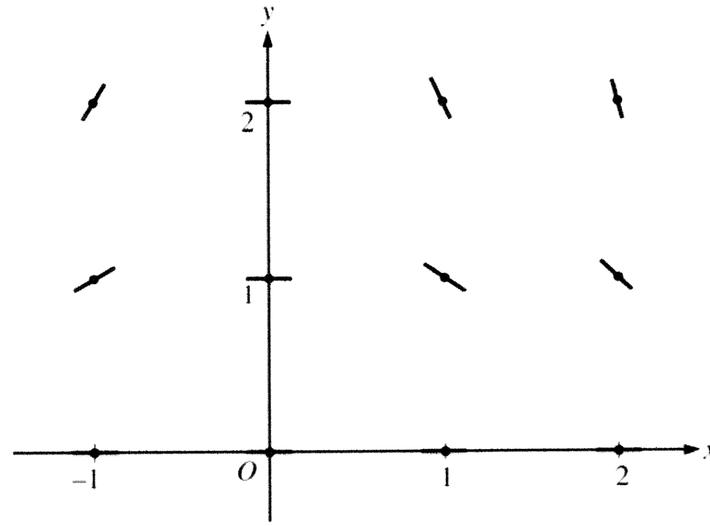
$\dfrac{dy}{dt} = \dfrac{dy}{dx} \cdot \dfrac{dx}{dt} = \dfrac{y}{2y - x} \cdot \dfrac{dx}{dt}$

At $t = 5$, $6 = \dfrac{3}{6 - \frac{7}{3}} \cdot \dfrac{dx}{dt} = \dfrac{9}{11} \cdot \dfrac{dx}{dt}$

$\left.\dfrac{dx}{dt}\right|_{t=5} = \dfrac{22}{3}$

Problem 6

(a)



(b) Slope $= \dfrac{-(-1)4}{2} = 2$

$y - 2 = 2(x + 1)$

(c) $\dfrac{1}{y^2}dy = -\dfrac{x}{2}dx$

$-\dfrac{1}{y} = -\dfrac{x^2}{4} + C$

$-\dfrac{1}{2} = -\dfrac{1}{4} + C$; $C = -\dfrac{1}{4}$

$y = \dfrac{1}{\frac{x^2}{4} + \frac{1}{4}} = \dfrac{4}{x^2 + 1}$