p. 12: 17-18, 23-27, 39-42, 74, 76-77, 79, 82

17. (a) $f(-4)=-2;\quad g(3)=4$
(b) $f(x)=g(x)$ when $x=-2$ and $x=2$.
(c) $f(x)=-1$ when $x\approx-3.4$.
(d) f is decreasing on the interval $\{0\le x\le 4\}$.
(e) The domain of f is $\{-4\le x\le 4\}$. The range of f is $\{-2\le y\le 3\}$.
(f) The domain of g is $\{-4\le x\le 4\}$. The range of g is $\{0.5\le y\le 4\}$.

18. (a) $f(2)=12$ (b) $f(2)=16$ (c) $f(a)=3a^2-a+2$
(d) $f(-a)=3a^2+a+2$ (e) $f(a+1)=3a^2+5a+4$ (f) $2f(x)=6a^2-2a+4$
(g) $f(2a)=12a^2-2a+2$ (h) $f(a^2)=3a^4-a^2+2$
(i) $[f(a)]^2=(3a^2-a+2)^2=9a^4-6a^3+13a^2-4a+4$
(j) $f(a+h)=3(a+h)^2-(a+h)+2=3a^2+3h^2+6ah-a-h+2$

23. The domain of $f(x)=\dfrac{x+4}{x^2-9}$ is $\{x\in\mathbb{R}\mid x\ne-3,3\}$.

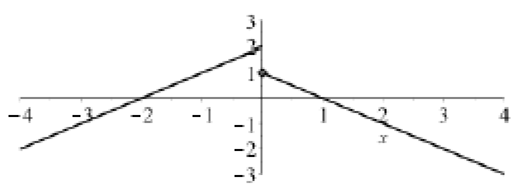
24. The domain of $f(x)=\dfrac{2x^3-5}{x^2+x-6}$ is $\{x\in\mathbb{R}\mid x\ne-3,2\}$.

25. The domain of $f(t)=\sqrt[3]{2t-1}$ is all real numbers.

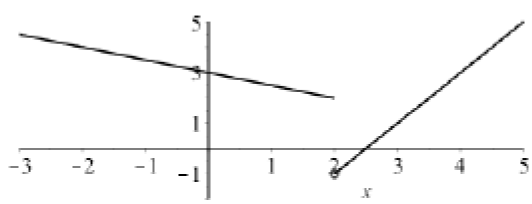
26. The domain of $g(t)=\sqrt{3-t}-\sqrt{2-t}$ is $\{t\le 2\}$.

27. The domain of $h(x)=-\dfrac{1}{\sqrt[4]{x^2-5x}}$ is $(-\infty,0)\cup(5,\infty)$.

39. $f(x)=\begin{cases}x+2 & \text{if } x<0\\ 1-x & \text{if } x\ge 0\end{cases}$;
$f(-3)=-1;\quad f(0)=1;\quad f(2)=-1.$

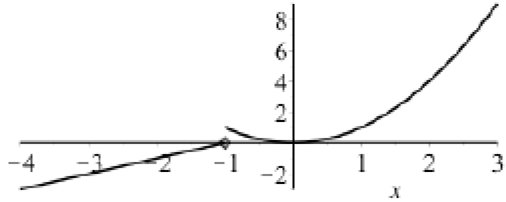


40. $f(x)=\begin{cases}3-\frac{1}{2}x & \text{if } x<2\\ 2x-5 & \text{if } x\ge 2\end{cases}$;
$f(-3)=\frac{9}{2};\quad f(0)=3;\quad f(2)=-1.$



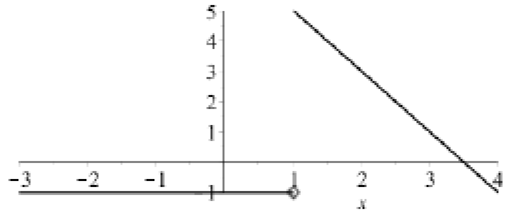
41. $f(x)=\begin{cases} x+1, & \text{if } x\le -1 \\ x^2, & \text{if } x>-1 \end{cases}$

$f(-3)=-2;\ \ f(0)=0;\ \ f(2)=4.$



42. $f(x)=\begin{cases} -1 & \text{if } x\le 1 \\ 7-2x & \text{if } x>1 \end{cases};$

$f(-3)=-1;\ \ f(0)=-1;\ \ f(2)=3.$



74. For $0\le x\le 3$ the graph is a line with slope -1 and y-intercept 3. For $3<x\le 5$ the graph is a line with slope 2 that passes through the point (3, 0).

Thus the function is $f(x)=\begin{cases} -x+3, & \text{if } 0\le x\le 3 \\ 2x-6, & \text{if } 3<x\le 5 \end{cases}.$

76. Let the length and width of the rectangle be l and w. Then the perimeter is $2l+2w=20$ and the area is $A=lw$. Solving the first equation for w in terms of l gives $w=\dfrac{20-2l}{2}=10-l$. Thus $A(l)=l(10-l)=10l-l^2$. Since the length must be positive, the domain of A is $0<l<10$. If we further restrict l to be larger than w, the $5<l<10$ would be the domain.

77. Let the length and width of the rectangle be l and w. Then the area is $lw=16$ so that $w=16/l$. The perimenter is $P=2l+2w$, so $P(l)=2l+2(16/l)=2l+32/l$, and the domain of P is $l>0$ since the lengths must be positive. If we further require l to be larger than w, then the domain would be $l>4$.

79. Let the length, width, and height of the closed rectangular box be denoted by l, w, and h respectively. The length is twice the width, so $l=2w$. The volume V of the box is $V=lwh$. Since $V=8$, we have $8=(2w)wh\Rightarrow 8=2w^2h\Rightarrow h=\dfrac{8}{2w^2}=\dfrac{4}{w^2}$, so $h=f(w)=\dfrac{4}{w^2}$.

82. The height of the box is x and the length and width are $l=20-2x$, $w=12-2x$. Then $V=lwx$ so $V(x)=(20-2x)(12-2x)x=4(10-x)(6-x)x=4x(60-16x+x^2)=4x^3-64x^2+240x$. Because the sides must have positive lengths, $l>0\Leftrightarrow 20-2x<0\Leftrightarrow x<10$; $w>0\Leftrightarrow x<6$; and $x>0$. Combining these restrictions indicates the domain is $0<x<6$.

p. 25: 32-33, 35, 43a

32. The linear function such that $f(3)=11$ and $f(7)=9$ must have slope $=\dfrac{19-11}{7-3}=2$. An equation for this line is $y=2(x-3)+11=2x+5$.

33. If $f(x)=3x+5$ then $\dfrac{f(b)-f(a)}{b-a}=\dfrac{(3b+5)-(3a+5)}{b-a}=\dfrac{3b-3a}{b-a}=3\left(\dfrac{b-a}{b-a}\right)=3$. This will always be the value because the rate of change (i.e. "slope") of a linear function is constant.

35. The domain of $f(x)=2(x-3)^2+5$ is all real numbers. The range is $\{y\in\mathbb{R}\mid y\ge 5\}$.

43. (a) A quadratic function whose graph has vertex $(0,3)$ and goes through the point $(4,2)$ is $f(x)=2(x-3)^2$.

p. 58: 33, 39-43 odd, 56-57, 61-62

33. (a) To find the equation of the graph that results from shifting the graph of $y=e^x$ 2 units downward, we subtract 2 from the original function to get $y=e^x-2$.

(b) To find the equation of the graph that results from shifting the graph of $y=e^x$ two units to the right, we replace x with $x-2$ in the original function to get $y=e^{(x-2)}$.

(c) To find the equation of the graph that results from reflecting the graph of $y=e^x$ about the x-axis, we multiply the original function by –1 to get $y=-e^x$.

(d) To find the equation of the graph that results from reflecting the graph of $y=e^x$ about the y-axis, we replace x with $-x$ in the original function to get $y=e^{-x}$.

(e) To find the equation of the graph that results from reflecting the graph of $y=e^x$ about the x-axis and then about the y-axis, we first multiply the original function by –1 (to get $y=-e^x$) and then replace x with $-x$ in this equation to get $y=-e^{-x}$.

39. $9^x=\left(\frac{1}{27}\right)^{x-2} \Leftrightarrow \left(3^2\right)^x=\left(3^{-\frac{1}{3}}\right)^{x-2} \Leftrightarrow 3^{2x}=3^{-3(x-2)} \Leftrightarrow 2x=-3x+6 \Leftrightarrow 5x=6 \Leftrightarrow x=\frac{6}{5}$

41. $8^x=\left(\sqrt{2}\right)^{2x^2+4} \Leftrightarrow 8^x=2^{\frac{1}{2}(2x^2+4)} \Leftrightarrow \left(2^3\right)^x=2^{x^2+2} \Leftrightarrow 2^{3x}=2^{x^2+2} \Leftrightarrow 3x=x^2+2 \Leftrightarrow x^2-3x+2=0$
$\Leftrightarrow (x-1)(x-2)=0 \Leftrightarrow x=1 \text{ or } x=2$

43.

$$3^{1/x}=27^{x^2}$$

$$3^{1/x}=\left(3^3\right)^{x^2}$$

$$3^{1/x}=3^{3x^2}$$

$$\frac{1}{x}=3x^2$$

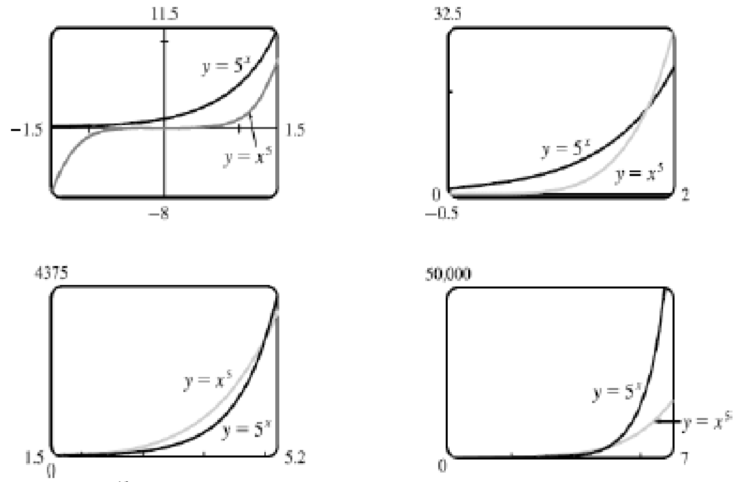
$$1=3x^3$$

$$x^3=\frac{1}{3}$$

$$x=\sqrt[3]{\frac{1}{3}}$$

56.

We see from the graphs that for x less than about 1.8, $g(x) = 5^x > f(x) = x^5$, and then near the point (1.765, 17.125), the curves intersect. Then $f(x) > g(x)$ from $x \approx 1.765$ until $x = 5$. At (5, 3125) there is another point of intersection, and for $x > 5$ we see that $g(x) > f(x)$. In fact, g increases much more rapidly than f beyond that point.



57. We graph $y = e^x$, and $y = 1{,}000{,}000{,}000$ and determine where $e^x \approx 1\times10^9$. This seems to be true at $x \approx 20.723$, so $e^x > 1\times10^9$ for $x > 20.723$.

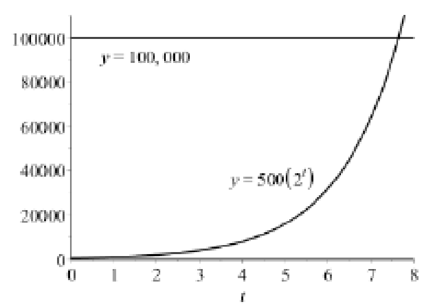
61.

(a) Three hours is 3 doubling periods (each doubling period is 1 hour) so there would be 4,000 bacteria after 3 hours.

(b) In t hours there will be t doubling periods. The initial population is 500 so at time t the number of bacteria will be $y = 500\cdot 2^t$.

(c) After 40 minutes there will be $500\cdot 2^{(40/60)} = 500\cdot 2^{(2/3)} \approx 794$ bacteria.

(d) From the graphs of $y_1 = 500\cdot 2^t$ and $y_2 = 100{,}000$ we see that the curves intersect at about t ≈ 7.64, so the population reaches 100,000 in about 7.64 hours.



62.

(a) Fifteen days is 3 half-life periods (the half-life is 5 days). So $200\left(\frac{1}{2}\right)^3 = 25$ mg.

(b) There would be $t/5$ doubling periods after t days. The initial population is 200 mg so after t days the amount of ^{210}Bi is $y = 200\left(\frac{1}{2}\right)^{t/5} = 200\cdot 2^{-t/5}$ mg.

(c) $t = 3$ weeks is 21 days $\Rightarrow$ $y = 200\cdot 2^{-21/5} \approx 10.882$ mg.

(d) We graph $y_1 = 200\cdot 2^{-t/5}$ and $y_2 = 1$. The two curves intersect at $t \approx 38.219$, so the mass will be reduced to 1 mg in about 38.219 days.