p. 270: 1-10

1. (a) $s = f(t) = t^3 - 8t^2 + 24t$ ft $\Rightarrow$ $v(t) = 3t^2 - 16t + 24$ ft/s

   (b) $v(1) = 3(1)^2 - 16(1) + 24 = 11$ ft/s

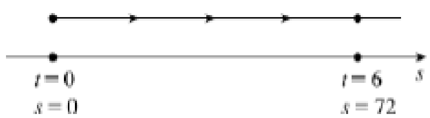
   (c) The particle is at rest when $v(t) = 3t^2 - 16t + 24 = 0 \Rightarrow$

   $$t = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(3)(24)}}{2(3)} = \frac{16 \pm \sqrt{-32}}{6}.$$ The negative discriminant indicates that v is never 0 and that the particle never rests.

   (d) From parts (b) and (c), we see that $v(t) > 0$ for all t, so the particle is always moving in the positive direction.

   (e) The total distance traveled during the first 6 seconds (since the particle doesn't change direction) is $f(6) - f(0) - 72 - 0 = 72$ ft.

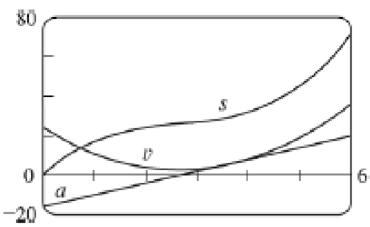
   (f)



   (g) $v(t) = 3t^2 - 16t + 24 \Rightarrow a(t) = v'(t) = 6t - 16$ ft/s^2

   $a(1) = 6(1) - 16 = -10$ ft/s^2

   (h)



   (i) The particle is speeding up when v and a have the same sign. The velocity v is always positive and a is positive when $6t - 16 > 0 \Rightarrow t > \frac{8}{3}$, so the particle is speeding up when $t > \frac{8}{3}$. It is slowing down when v and a have opposite signs; that is when $0 \le t < \frac{8}{3}$.

2. (a) $s = f(t) = \dfrac{9t}{t^2 + 9}$ ft $\Rightarrow$ $v(t) = f'(t) = \dfrac{(t^2+9)(9) - 9t(2t)}{(t^2+9)^2} = \dfrac{-9t^2 + 81}{(t^2+9)^2} = \dfrac{-9(t^2-9)}{(t^2+9)^2}$ ft/s

   (b) $v(1) = \dfrac{-9(1-9)}{(1+9)^2} = \dfrac{72}{100} = 0.72$ ft/s

   (c) The particle is at rest when $v(t) = \dfrac{-9(t^2-9)}{(t^2+9)^2} = 0 \Rightarrow t^2 - 9 = 0 \Rightarrow t = 3$ s [since $t \ge 0$].

   (d) The particle is moving in the positive direction when $v(t) > 0$.

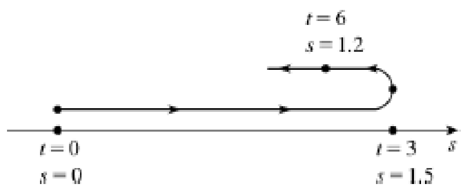
   $$\frac{-9(t^2-9)}{(t^2+9)^2} \ge 0 \Rightarrow -9(t^2-9) \ge 0 \Rightarrow t^2 - 9 < 0 \Rightarrow t^2 < 9 \Rightarrow 0 \le t < 3.$$

   (e) Since the particle is moving in both the positive and negative directions, we need to calculate the distance traveled in the intervals $[0,3]$ and $[3,6]$, respectively.

   $|f(3) - f(0)| = \left|\frac{27}{18} - 0\right| = \frac{3}{2}$; $|f(6) - f(3)| = \left|\frac{54}{45} - \frac{27}{18}\right| = \frac{3}{10}$.

   The total distance traveled is $\frac{3}{2} + \frac{3}{10} = \frac{9}{5}$ or 1.8 ft.

(f)

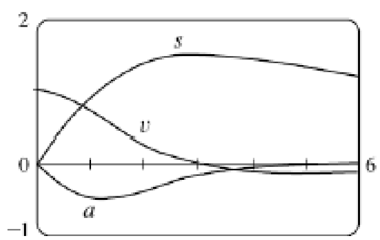


(g) $v(t) = -9\dfrac{(t^2-9)}{(t^2+9)^2} \Rightarrow a(t) = v'(t) = -9\dfrac{(t^2+9)^2(2t)-(t^2-9)2(t^2+9)(2t)}{\left[(t^2+9)^2\right]^2}$

$$= -9\frac{2t(t^2+9)\left[(t^2+9)-2(t^2-9)\right]}{\left[(t^2+9)^2\right]^2} = \frac{18t(t^2-27)}{(t^2+9)^3} \text{ ft/s}^2$$

$$a(1) = \frac{18t(-26)}{10^3} = -0.468 \text{ ft/s}^2$$

(h)



(i) The particle is speeding up when v and a have the same sign. The acceleration a is always negative for $0 < t < \sqrt{27} \approx 5.2$ s, so from the figure in part (h), we see that v and a are both negative for $3 < t < 3\sqrt{3}$. The particle is slowing down when v and a have opposite signs. This occurs when $0 < t < 3$ and when $t < 3\sqrt{3}$.

3. (a) $s = f(t) = \sin\left(\dfrac{\pi t}{2}\right)$ ft $\Rightarrow$ $v(t) = f'(t) = \dfrac{\pi}{2}\cdot\cos\left(\dfrac{\pi t}{2}\right)$ ft/s

(b) $v(1) = \dfrac{\pi}{2}\cos\left(\dfrac{\pi t}{2}\right) = \dfrac{\pi}{2}(0) = 0$ ft/s

(c) The particle is at rest when $v(t) = \dfrac{\pi}{2}\cos\left(\dfrac{\pi t}{2}\right) = 0 \Rightarrow \cos\left(\dfrac{\pi t}{2}\right) = 0 \Rightarrow \dfrac{\pi t}{2} = \dfrac{\pi}{2} + n\pi \Rightarrow$

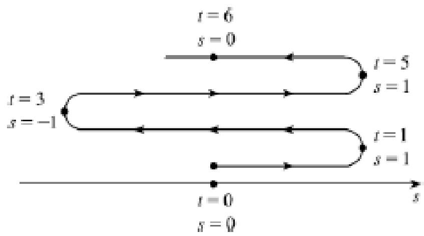
$\dfrac{\pi t}{2} = \dfrac{\pi}{2} + n\pi \Rightarrow t = 1 + 2n$, where n is a non-negative integer since $t \geq 0$.

(d) The particle is moving in the positive direction when $v(t) > 0$. From part (c), we see that v changes sign at every positive odd integer. The velocity is positive when $0 < t < 1$, $3 < t < 5$, $7 < t < 9$, and so on.

(e) The velocity changes sign at $t = 1$, 3, and 5 in the interval $[0, 6]$. The total distance traveled during the first 6 seconds is $|f(1)-f(0)| + |f(3)-f(1)| + |f(5)-f(3)| + |f(6)-f(5)|$

$$= |1-0| + |-1-1| + |1-(-1)| + |0-1| = 1+2+2+1 = 6 \text{ ft.}$$

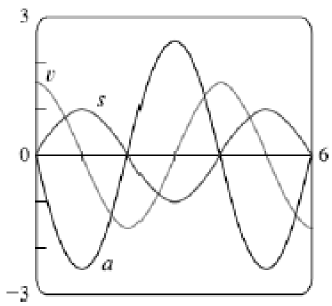
(f)



(g) $v(t)=\frac{\pi}{2}\cos\left(\frac{\pi t}{2}\right) \Rightarrow a(t)=v'(t)=\frac{\pi}{2}\left[-\sin\left(\frac{\pi t}{2}\right)\cdot\frac{\pi}{2}\right]=-\frac{\pi^2}{4}\sin\left(\frac{\pi t}{2}\right)$ ft/s^2

$a(1)=-\frac{\pi^2}{4}\sin\left(\frac{\pi}{2}\right)=-\frac{\pi^2}{4}$ ft/s^2

(h)



(i) The particle is speeding up when v and a have the same sign. From the figure in part (h), we see that v and a are both positive when $3<t<4$ and both negative when $1<t<2$ and $5<t<6$. Thus, the particle is speeding up $1<t<2$, $3<t<4$, and $5<t<6$. The particle is slowing down when v and a have opposite signs. This occurs when $0<t<1$, $2<t<3$, and $4<t<5$.

4. (a) $s=f(t0=t^2e^{-t}$ (in feet) $\Rightarrow v(t)=f'(t)=t^2(-e^{-t})+e^{-t}(2t)=te^{-t}(2-t)$ (in ft/s)

(b) $v(1)=(1)e^{-1}(2-1)=e^{-1}$ ft/s.

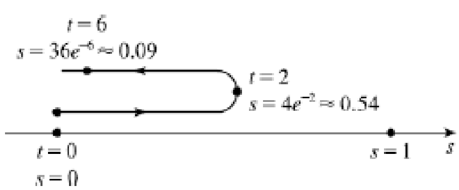
(c) The particle is at rest when $v(t)=0 \Leftrightarrow t=0$ or 2 s.

(d) The particle is moving in the positive directions when $v(t)>0 \Leftrightarrow te^{-t}(2-t)>0 \Leftrightarrow$ $t(2-t)>0 \Leftrightarrow 0<t<2$.

(e) The velocity changes sign at $t=2$ in the interval $[0,6]$. The total distance traveled during the first 6 seconds is $|f(2)-f(0)|+|f(6)-f(2)|=|4e^{-2}-0|+|36e^{-6}-4e^{-2}|=4e^{-2}+4e^{-2}-36e^{-6}$

$$=8e^{-2}-36e^{-6}\approx 0.99 \text{ ft.}$$

(f)

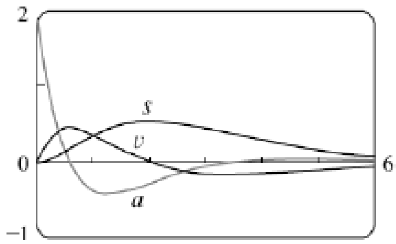


(g) $v(t)=(2t-t^2)e^{-t} \Rightarrow$

$a(t)=v'(t)=(2t-t^2)(-e^{-t})+e^{-t}(2-2t)=e^{-t}\left[-(2t-t^2)+(2-2t)\right]=e^{-t}\left[t^2-4t+2\right]$ ft/s^2.

$a(1)=e^{-1}[1-4+2]=-e^{-1}$ ft/s^2.

(h)



(i) $a(t)=0 \Leftrightarrow t^2-4t+2=0\ \left[e^{-t}\neq 0\right] \Leftrightarrow\ t=\dfrac{4\pm\sqrt{8}}{2}=2\pm\sqrt{2}\approx 0.6$ and 3.4. The particle is speeding up when v and a have the same sign. Using the previous information and the figure in part (h), we see that v and a are both positive when $0<t<2-\sqrt{2}$ anD both negative when $2<t<2+\sqrt{2}$. The particle is slowing down when v and a have opposite signs. This occurs when $2-\sqrt{2}<t<2$ and $t>2+\sqrt{2}$.

5. (a) From the figure, the velocity v is positive on the interval $(0,2)$ and negative on the interval $(2,3)$. The acceleration a is positive/negative when the slope of the tangent line is positive/negative, so the acceleration is positive on the interval $(0,1)$ and negative on the interval $(1,3)$. The particle is speeding up when v and a have the same sign, that is, on the interval $(0,1)$ when $v>0$ and $a>0$, and on the interval $(2,3)$ when $v<0$ and $a<0$. The particle is slowing down when v and a have opposite signs, that is, on the interval $(1,2)$ when $v>0$ and $a<0$.

(b) The velocity is positive on $(0,3)$ and $v<0$ on $(3,4)$. The acceleration is positive on $(1,2)$ and $a<0$ on $(0,1)$ and $(2,4)$. The particle is speeding up on $(1,2)$ $[v>0,\ a>0]$ and on $(3,4)$ $[v<0,\ a<0]$. The particle is slowing down on $(0,1)$ and $(2,3)$ $[v>0,\ a<0]$.

6. (a) The velocity v is positive when s is increasing, that is, on the intervals $(0,1)$ and $(3,4)$; and it is negative when s is decreasing, that is, on the interval $(1,3)$. The acceleration a is positive when the graph of s is concave upward (v is increasing), that is, on the interval $(2,4)$; and it is negative when the graph of s is concave downward (v is decreasing), that is, on the interval $(0,2)$. The particle is speeding up on the interval $(1,2)$ and on $(3,4)$. The particle is slowing down on the interval $(0,1)$ $[v<0,\ a>0]$ and on $(2,3)$ $[v<0,\ a>0]$.

(b) The velocity v is positive on $(3,4)$ and negative on $(0,3)$. The acceleration a is positive on $(0,1)$ and $(2,4)$ and negative on $(1,2)$. The particle is speeding up on the interval $(1,2)$ $[v<0,\ a<0]$ and on $(3,4)$ $[v>0,\ a>0]$ The particle is slowing down on the interval $(0,1)$ $[v<0,\ a>0]$ and on $(2,3)$ $[v<0,\ a>0]$.

7. (a) $h(t)=2+24.5t-4.9t^2 \Rightarrow v(t)=h'(t)=24.5-9.8t$. The velocity two seconds is $v(2)=24.5-9.8(2)=4.9$ m/s and after four seconds is $v(4)=24.5-9.8(4)=-14.7$ m/s.

(b) The projectile reaches its maximum height when the velocity is zero.

$v(t)=0 \Leftrightarrow 24.5-9.8t=0 \Leftrightarrow t=\dfrac{24.5}{9.8}=2.5$ s.

(c) The maximum height occurs when $t = 2.5$ s. $h(2.5)=2+24.5(2.5)-4.9(2.5)^2=32.625$ m.

(d) The projectile hits the ground when $h=0 \Leftrightarrow 2+24.5t-4.9t^2=0 \Leftrightarrow$

$t=\dfrac{-24.5\pm\sqrt{24.5^2-4(-4.9)(2)}}{2(-4.9)} \Rightarrow t=t_f\approx 5.080$ s [since $t\geq 0$].

(e) The projectile hits the ground when $t=t_f$. Its velocity is $v(t_f)=24.5-9.8t_f\approx -25.3$ m/s [downward].

8. (a) At maximum height the velocity of the ball is 0 ft/s.

$v(t)=s'(t)=80-32t=0 \Leftrightarrow 32t=80 \Leftrightarrow t=\frac{5}{2}$. So the maximum height is

$s\left(\frac{5}{2}\right)=80\left(\frac{5}{2}\right)-16\left(\frac{5}{2}\right)^2=200-100=100$ ft.

(b) $s(t)=80t-16t^2=96 \Leftrightarrow 16t^2-80t+96=0 \Leftrightarrow 16\left(t^2-5t+6\right)=0 \Leftrightarrow 16(t-3)(t-2)=0.$

So the ball has a height of 96 ft on the way up at $t = 2$ and on the way down at $t = 3$. At these times the velocities are $v(2)=80-32(2)=16$ ft/s and $v(3)=80-32(3)=-16$ ft/s, respectively.

9. (a) $h(t)=15t-1.86t^2 \Rightarrow v(t)=h'(t)=15-3.72t$. The velocity after 2 seconds is $v(2)=15-3.72(2)=7.56$ m/s.

(b) $25=h \Leftrightarrow 1.86t^2-15t+25=0 \Leftrightarrow t=\dfrac{15\pm\sqrt{15^2-4(1.86)(25)}}{2(1.86)} \Leftrightarrow t=t_1\approx 2.353$ or

$t=t_2\approx 5.711$s. The velocities are $v(t_1)=15-3.27t_1\approx 6.24$ m/s [upward] and

$v(t_2)=15-3.27t_2\approx -6.245$ m/s or 6.245 m/s downward.

10. (a) $s(t)=t^4-4t^3-20t^2+20t \;\; s \Rightarrow v(t)=s'(t)=4t^3-12t^2-40t+20$

$v=20 \Leftrightarrow 4t^3-12t^2-40t+20=20 \Leftrightarrow 4t^3-12t^2-40t=0 \Leftrightarrow 4t\left(t^2-3t-10\right)=0 \Leftrightarrow$

$4t(t-5)(t+2)=0 \Leftrightarrow t=0$ s or $t=5$ s $[\text{for } t\geq 0]$.

(b) $a(t)=v'(t)=12t^2-24t-40$. $a=0 \Leftrightarrow 12t^2-24t-40=0 \Leftrightarrow 4\left(3t^2-6t-10\right)=0 \Leftrightarrow$

$t=\dfrac{6\pm\sqrt{6^2-4(3)(-10)}}{2(3)}=1\pm\dfrac{1}{3}\sqrt{39}$; so $t\approx 3.082$ s $[\text{for } t\geq 0]$. At this time, the acceleration changes from negative to positive and the velocity attains its minimum value.