<u>5.3</u>

p. 428: 10-17, 32, 34, 40-41, 43-53 odd, 58-62, 64-65, 67-68

10. (a) $\int_0^{10} f(x)dx \approx R_5 = [f(2)+f(4)+f(6)+f(8)+f(10)]\Delta x$

$$= [-1+0+(-2)+2+4](2) = 3(2) = 6$$

(b) $\int_0^{10} f(x)dx \approx L_5 = [f(0)+f(2)+f(4)+f(6)+f(8)]\Delta x$

$$= [3+(-1)+0+(-2)+2](2) = 2(2) = 4$$

(c) $\int_0^{10} f(x)dx \approx M_5 = [f(1)+f(3)+f(5)+f(7)+f(9)]\Delta x$

$$= [0+(-1)+(-1)+0+3](2) = 1(2) = 2$$

11. (a) $\int_{-2}^{4} g(x)dx \approx R_6 = [g(-1)+g(0)+g(1)+g(2)+g(3)+g(4)]\Delta x$

$$= [-\tfrac{3}{2}+0+\tfrac{3}{2}+\tfrac{1}{2}+(-1)+\tfrac{1}{2}](1) = 0$$

(b) $\int_{-2}^{4} g(x)dx \approx L_6 = [g(-2)+g(-1)+g(0)+g(1)+g(2)+g(3)]\Delta x$

$$= [0+(-\tfrac{3}{2})+0+\tfrac{3}{2}+\tfrac{1}{2}+(-1)](1) = -\tfrac{1}{2}$$

(c) $\int_{-2}^{4} g(x)dx \approx M_6 = [g(-\tfrac{3}{2})+g(-\tfrac{1}{2})+g(\tfrac{1}{2})+g(\tfrac{3}{2})+g(\tfrac{5}{2})+g(\tfrac{7}{2})]\Delta x$

$$= [-1+(-1)+1+1+0+(-\tfrac{1}{2})](1) = -\tfrac{1}{2}$$

12. Since f is increasing, $L_5 \le \int_{10}^{30} f(x)dx \le R_5$.

Lower estimate $= L_5 = \sum_{i=1}^{5} f(x_{i-1})\Delta x = 4[f(10)+f(14)+f(18)+f(22)+f(26)]$

$$= 4[-12+(-6)+(-2)+1+3] = 4(-16) = -64$$

Upper estimate $= R_5 = \sum_{i=1}^{5} f(x_i)\Delta x = 4[f(14)+f(18)+f(22)+f(26)+f(30)]$

$$= 4[-6+(-2)+1+3+8] = 4(4) = 16$$

13. (a) Using the right endpoints to approximate $\int_3^9 f(x)dx$, we have

$$\sum_{i=1}^{3} f(x_i)\Delta x = 2[f(5)+f(7)+f(9)] = 2(-0.6+0.9+1.8) = 4.2$$

Since f is *increasing*, using *right* endpoints gives us an *overestimate*.

(b) Using the left endpoints to approximate $\int_3^9 f(x)dx$, we have

$$\sum_{i=1}^{3} f(x_{i-1})\Delta x = 2[f(3)+f(5)+f(7)] = 2(-3.4-0.6+0.9) = -6.2$$

Since f is *increasing*, using *left* endpoints gives us an *underestimate*.

(c) Using the midpoint of each interval to approximate $\int_3^9 f(x)dx$, we have

$$\sum_{i=1}^{3} f(\overline{x}_i)\Delta x = 2[f(4)+f(8)+f(8)] = 2(-2.1+0.3+1.4) = -0.8$$

We cannot say anything about the midpoint estimate compared to the exact value of the integral.

14. $\Delta x = (8-0)/4 = 2$, so the endpoints are 0, 2, 4, ,6, and 8, and the midpoints are 1, 3, 5, and 7. The Midpoint Rule gives

$$\int_0^8 \sin\sqrt{x}dx \approx \sum_{i=1}^{4} f(\bar{x}_i)\Delta x = 2\left(\sin\sqrt{1}+\sin\sqrt{3}+\sin\sqrt{5}+\sin\sqrt{7}\right) \approx 2(3.0910) = 6.1820.$$

15. $\Delta x = (1-0)/5 = \frac{1}{5}$, so the endpoints are $0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$ and 1, and the midpoints are $\frac{1}{10}, \frac{3}{10}, \frac{5}{10}, \frac{7}{10}$, and $\frac{9}{10}$. The Midpoint Rule gives

$$\int_0^1 \sqrt{x^3+1}dx \approx \sum_{i=1}^{5} f(\bar{x}_i)\Delta x = \tfrac{1}{5}\left(\sqrt{\left(\tfrac{1}{10}\right)^3+1}+\sqrt{\left(\tfrac{3}{10}\right)^3+1}+\sqrt{\left(\tfrac{5}{10}\right)^3+1}+\sqrt{\left(\tfrac{7}{10}\right)^3+1}+\sqrt{\left(\tfrac{9}{10}\right)^3+1}\right) \approx 1.1097.$$

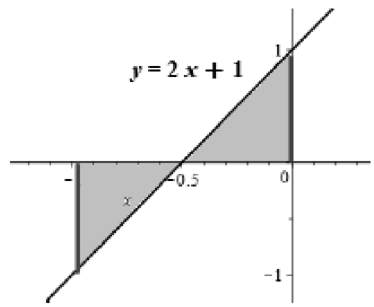
16. $\Delta x = (2-0)/5 = \frac{2}{5}$, so the endpoints are $0, \frac{2}{5}, \frac{4}{5}, \frac{6}{5}, \frac{8}{5}$ and 2, and the midpoints are $\frac{1}{5}, \frac{3}{5}, \frac{5}{5}, \frac{7}{5}$ and $\frac{9}{5}$. The Midpoint Rule gives

$$\int_0^2 \frac{x}{x+1}dx \approx \sum_{i=1}^{5} f(\bar{x}_i)\Delta x = \tfrac{2}{5}\left(\frac{\frac{1}{5}}{\frac{1}{5}+1}+\frac{\frac{3}{5}}{\frac{3}{5}+1}+\frac{1}{1+1}+\frac{\frac{7}{5}}{\frac{7}{5}+1}+\frac{\frac{9}{5}}{\frac{9}{5}+1}\right) = \tfrac{2}{5}\left(\tfrac{127}{56}\right) = \tfrac{127}{140} \approx 0.9071.$$

17. $\Delta x = (\pi-0)/4 = \frac{\pi}{4}$; the endpoints are $0, \frac{\pi}{4}, \frac{2\pi}{4}, \frac{3\pi}{4}$ and $\frac{4\pi}{4}$, and the midpoints are $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}$, and $\frac{7\pi}{8}$.

The Midpoint Rule gives $\int_0^\pi x\sin^2 x dx \approx \sum_{i=1}^{4} f(\bar{x}_i)\Delta x = \frac{\pi}{4}\left(\frac{\pi}{8}\sin^2\frac{\pi}{8}+\frac{3\pi}{8}\sin^2\frac{3\pi}{8}+\frac{5\pi}{8}\sin^2\frac{7\pi}{8}\right) \approx 2.4674.$

32. Let $f(x) = 2x+1$ on $[-1,0]$. Then $\int_{-1}^{0}(2x+1)dx$ is equal to sum of the areas of the two triangles shown. The triangles have equal area, but one is above the x-axis, and the other is below. Therefore, the integral is equal to zero. This is choice (**D**). The other 3 choices all depict represent areas entirely above the x-axis.



34. Graph the line and use the area of the trapezoid:

$$\int_a^b x dx = \frac{1}{2}(a+b)(b-a) = \frac{1}{2}(b^2-a^2)$$

40. (a) Think of $\int_0^2 f(x)\,dx$ as the area of a trapezoid with bases 1 and 3 and height 2. The area of a trapezoid is $A = \frac{1}{2}(b+B)h$, so $\int_0^2 f(x)\,dx = \frac{1}{2}(1+3)2 = 4$.

(b) $\int_0^5 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx + \int_3^5 f(x)\,dx$

trapezoid rectangle triangle

$= \frac{1}{2}(1+3)2 \; + \; 3\cdot 1 \; + \; \frac{1}{2}\cdot 2\cdot 3 = 4+3+3 = 10$

(c) $\int_5^7 f(x)\,dx$ is the negative of the area of the triangle with base 2 and height 3.

$\int_5^7 f(x)\,dx = -\frac{1}{2}\cdot 2\cdot 3 = -3$

(d) $\int_7^9 f(x)\,dx$ is the negative of the area of a trapezoid with bases 3 and 2 and height 2, so it equals $-\frac{1}{2}(b+B)h = -\frac{1}{2}(3+2)2 = -5$.

Thus, $\int_0^9 f(x)\,dx = \int_0^5 f(x)\,dx + \int_5^7 f(x)\,dx + \int_7^9 f(x)\,dx = 10 + (-3) + (-5) = 2$.

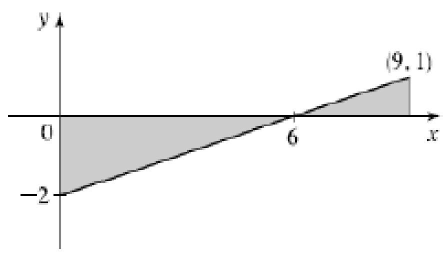
41. (a) $\int_0^2 g(x)dx = \frac{1}{2}\cdot 4\cdot 2 = 4$ [area of a triangle]

(b) $\int_2^6 g(x)dx = -\frac{1}{2}\cdot \pi \cdot 2^2 = -2\pi$ [negative of the area of a semicircle]

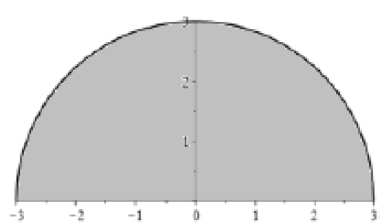
(c) $\int_6^7 g(x)dx = \frac{1}{2}\cdot 1\cdot 1 = \frac{1}{2}$ [area of a triangle]

$\int_0^7 g(x)dx = \int_0^2 g(x)dx + \int_2^6 g(x)dx + \int_6^7 g(x)dx = \frac{1}{2}4 - 2\pi + \frac{1}{2} = 4.5 - 2\pi$

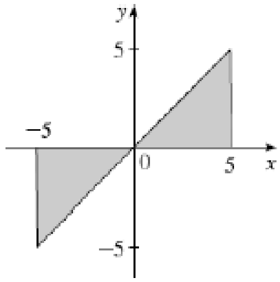
43. $\int_0^9 \left(\frac{1}{3}x - 2\right)dx$ can be interpreted as the difference of the areas of the two shaded triangles; that is, $-\frac{1}{2}(6)(2) + \frac{1}{2}(3)(1) = -6 + \frac{3}{2} = -\frac{9}{2}$.

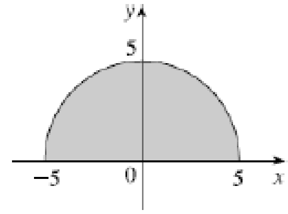


45. $\int_{-3}^3 \sqrt{9-x^2}\,dx$ can be interpreted as the area under the graph of $f(x) = \sqrt{9-x^2}$ between $x = -3$ and $x = 3$. This is equal to a semi-circle of radius 3; that is $\frac{1}{2}\pi \cdot 3^2 = \frac{9}{2}\pi$

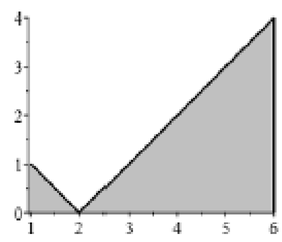


47. $\int_{-5}^{5}\left(x-\sqrt{25-x^2}\right)dx=\int_{-5}^{5}x\,dx-\int_{-5}^{5}\sqrt{25-x^2}\,dx.$ By symmetry, the value of the first integral is 0 since the shaded area above the x-axis equals the shaded areas below the x-axis. The second integral can be interpreted as one half the area of a circle with radius 5; that is, $\frac{1}{2}\pi(5)^2=\frac{25}{2}\pi$. Thus, the value of the original integral is $0-\frac{25}{2}\pi=-\frac{25}{2}\pi$.

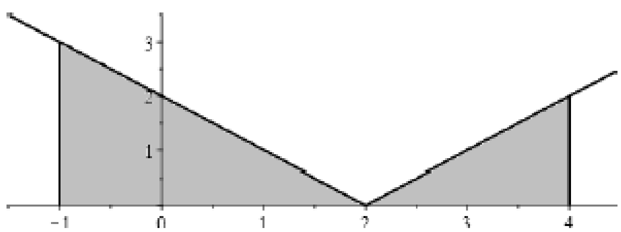




49. $\int_{1}^{6}|x-2|\,dx$ can be interpreted as the sum of the areas of the two shaded triangles; that is, $\frac{1}{2}(1)(1)+\frac{1}{2}(4)(4)=\frac{1}{2}+8=\frac{17}{2}$.



51. $\int_{-1}^{4}\sqrt{x^2-4x+4}\,dx$ can be interpreted as the sum of the areas of the two shaded triangles; that is, $\frac{1}{2}(3)(3)+\frac{1}{2}(2)(2)=\frac{9}{2}+2=\frac{13}{2}$.



53. $\int_{\pi}^{0}\sin^4\theta\,d\theta=-\int_{0}^{\pi}\sin^4\theta\,d\theta$

$$=-\int_{0}^{\pi}\sin^4 x\,dx$$

$$=-\tfrac{3}{8}\pi$$

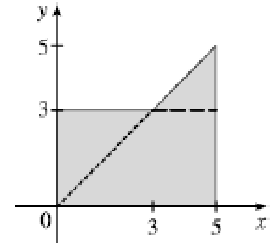
58. $\int_{-2}^{2}f(x)\,dx+\int_{2}^{5}f(x)\,dx-\int_{-2}^{-1}f(x)\,dx=\int_{-2}^{5}f(x)\,dx-\int_{-1}^{-2}f(x)\,dx$ [by Property 5 and reversing limits]

$$=\int_{-1}^{5}f(x)\,dx \qquad \text{[Property 5]}$$

59. $\int_{2}^{4}f(x)\,dx+\int_{4}^{8}f(x)\,dx=\int_{2}^{8}f(x)\,dx$, so $\int_{4}^{8}f(x)\,dx=\int_{2}^{8}f(x)\,dx-\int_{2}^{4}f(x)\,dx=7.3-5.9=1.4.$

60. $\int_{0}^{9}[2f(x)+3g(x)]\,dx=2\int_{0}^{9}f(x)\,dx+3\int_{0}^{9}g(x)\,dx=2(37)+3(16)=122$

61. If $f(x)=\begin{cases}3 & \text{for } x<3\\ x & \text{for } x\geq 3\end{cases}$, then $\int_0^5 f(x)\,dx$ can be interpreted as the area of the shaded region, which consists of a 5-by-3 rectangle surmounted by an isosceles right triangle whose legs have length 2. Thus, $\int_0^5 f(x)\,dx=5(3)+\frac{1}{2}(2)(2)=17$.



62. $\int_0^3 f(x)\,dx$ is clearly less than –1 and has the smallest value. The slope of the tangent line of f at $x=1$, $f'(1)$, has a value between –1 and 0, so it has the next smallest value. The largest value is $\int_3^8 f(x)dx$, followed by $\int_4^8 f(x)dx$, which has a value about 1 unit less than $\int_3^8 f(x)dx$. Still positive, but with a smaller value than $\int_4^8 f(x)dx$, is $\int_0^8 f(x)dx$. Ordering these quantities form smallest to largest gives us $\int_0^3 f(x)dx < f'(1) < \int_0^8 f(x)dx < \int_4^8 f(x)dx < \int_3^8 f(x)dx$ or B < E < A < D < C.

64. (a) $\int_0^e f(x)\,dx=\int_3^e f(x)\,dx-\int_0^d f(x)\,dx=10-7=3$

(b) $\int_b^d f(x)\,dx=\int_c^3 f(x)\,dx-\int_b^c f(x)\,dx-\int_0^d f(x)\,dx=3-5-7=-9$

(c) $\int_a^0 |f(x)|\,dx=\int_a^b f(x)\,dx-\int_b^c f(x)\,dx+\int_c^0 f(x)\,dx=15-(-5)+3=23$

(d) $\left|\int_a^0 f(x)\,dx\right|=|13|=13$

(e) $\int_{-e}^e f(|x|)\,dx=2\int_0^e f(x)\,dx=6$ because $f(|x|)$ is symmetric about the y-axis.

(f) $\int_{-c}^{-a} f(-x)\,dx=10$ because this is equivalent to reflecting the curve across the y-axis.

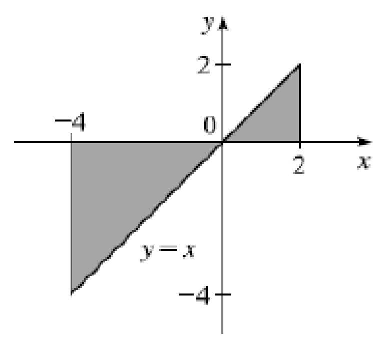
65. $I=\int_{-4}^2[f(x)+2x+5]dx\ =\int_{-4}^2 f(x)\,dx+2\int_{-4}^2 x\,dx+\int_{-4}^2 5\,dx=I_1+2I_2+I_3$

$I_1=3$ [area below x-axis] $+3-3=-3$

$I_2=-\frac{1}{2}(4)(4)$ [area of triangle, see figure] $+\frac{1}{2}(2)(2)=-8+2=-6$

$I_3=5[2-(-4)]=5(6)=30$

Thus, $I=-3+2(-6)+30=15$



67. If $\int_2^6 f(x)\,dx=7$ then $\int_2^6[f(x)+2]dx=\int_2^6 f(x)\,dx+\int_2^6 2\,dx=7+2(6-2)=7+2(4)=15$, which is option (**C**).

68. $\int_1^3 g(x)\,dx=\int_1^2 g(x)\,dx+\int_2^3 g(x)\,dx \Rightarrow \int_1^2 g(x)\,dx=\int_1^3 g(x)\,dx-\int_2^3 g(x)\,dx 1-(-3)=4$. This is option (**C**).

p. 456: 68-69, 72-73, 76

68. (a) Displacement = $\int_0^3 (3t-5)\,dt$ = -1.5

(b) Distance traveled = $\int_0^3 |3t-5|\,dt$

69. (a) Displacement = $\int_2^4 (t^2-2t-3)\,dt$

(b) Distance traveled = $\int_0^3 |t^2-2t-3|\,dt = 4$ m

72. $s(t) = \int v(t)\,dt = \int 2e^{t^2/10}dt;\ \ s(5) = s(2) + \int_2^5 2e^{t^2/10}\,dt = 5 + 26.619 = 31.619$

73. Distance traveled = $\int_1^4 |v(t)|\,dt = \int_1^4 |2t-3|\,dt$ = 6.5 which is option (**C**).

76. Let s be the position of the car. We know from Equation 2 that $s(100) - s(0) = \int_0^{100} v(t)\,dt$. We use the Midpoint Rule for $0 \le t \le 100$ with $n = 5$. Note that the length of each of the five time intervals is 20 seconds $= \frac{20}{3600}$ hour $= \frac{1}{180}$ hour. So the distance traveled is

$$\int_0^{100} v(t)\,dt \approx \tfrac{1}{180}\left[v(10)+v(30)+v(50)+v(70)+v(90)\right] = \tfrac{1}{180}(38+58+51+53+47) = \tfrac{247}{180} \approx 1.372 \text{ miles.}$$